ISSN 0974 - 9373

Vol. 24 No. 4 (2020) **Journal of International Academy of Physical Sciences** pp. 403- 418

Bianchi Type-V Model of the Universe with BVDP in *f (R, T)* Gravity

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(Received November 15, 2020)

Abstract: The present work deals with the dynamics of Bianchi type-V cosmological model in modified $f(R,T)$ theory of gravity. The solution of the field equations is obtained with the help of bilinear varying deceleration parameter (BVDP) $q = \frac{\alpha_1 - \alpha_2 t}{1+\alpha_2 t}$, where α_1 (dimensionless) and α_2 (dimension of inverse of time) are the positive constants. The behaviour of the BVDP shows early time cosmic deceleration and present cosmic acceleration which is in good harmony with the recent observational data. The time-variation of various cosmological parameters such as Hubble parameter, directional Hubble parameters, shear scalar and anisotropic parameter has been examined. For the stability of solution of the derived model, the behaviour of energy conditions has also been explored graphically. We have also observed the nature of statefinder pair (r,s).

Keywords: Bilinear Varying Deceleration Parameter, $f(R,T)$ Gravity Theory, Cosmological Model

2010 AMS Classification Number: 53B15, 53C15, 53C25.

1. Introduction

Since the beginning of human scientific endeavors, scientists have been eager to learn about the mysterious existence of the cosmos. Cosmologists have always been trying to investigate the origin, evolution and ultimate fate of the universe. Multiple cosmological observations made through

Supernovae type Ia[1-3], WMAP collaboration[4-5], Planck data[6-7] and other sources have confirmed accelerated expansion of the present universe. This fact of accelerated expansion of the universe begs us to question whether our fundamental theories are incomplete or if there is a need for new insights.

Some authors are of the opinion that various categories of dark energy such as Quintessence, k-essence, Phantom model, Chameleon fields, Chaplygin gas, Polytropic model and cosmological constant Λ could be responsible for the present scenario of the universe. Another group of scientists are making attempts to explain the present cosmic acceleration using modified theories of gravitation.

Einstein's General Theory of Relativity (GTR) is known to be, the fundamental theory to explain all the aspects of the universe. However, it has certain limitations such as- lack of compatibility with Mach's principle, singularity problem and its inability to explain the late time cosmic acceleration. Therefore, various modified theories have been proposed from time to time. Few of them are $f(R)$ gravity[8-9], $f(R,T)$ gravity[10] and scalar tensor theories of gravitation. All of the above mentioned theories have their own importance.

In the present communication, we have constructed cosmological model in $f(R,T)$ gravity formulated by Harko et al.[10]. In this gravity theory, the gravitational Lagrangian is described by an arbitrary function of Ricci scalar R and trace T of the energy-momentum tensor $T_{\mu\nu}$. This modification in the form of coupling between the matter and geometry gives the possibility to explain dark energy and late time accelerated expansion of the universe in a better way. Various authors including our peer research group have investigated cosmological models in $f(R,T)$ gravity[10]. Mishra et al.[11] have constructed dark energy cosmological model with variable deceleration parameter in $f(R,T)$ theory where the solution of the field equations is obtained with the help of suitable assumption of the scale factor $R(t)$. Tiwari et al.[12] have constructed cosmological model to study the time dependence of G and Λ in modified $f(R,T)$ theory of gravity. Reddy et al.[13] have examined the dynamics of five dimensional spherical and symmetric universe with cosmic strings in $f(R,T)$ gravity theory.

Hubble parameter and deceleration parameter (DP) are two basic parameters to discuss the cosmological evolution of the universe. Hubble parameter measures the expansion rate of the universe. However, DP infers about the nature of the expansion of the universe. The early time cosmic deceleration and present cosmic acceleration [1-7] infer that there must be variation in the sign of DP. Such fact encourages many authors to study cosmological models with variable DP[11,14-19]. Akarsu and Dereli[15] have investigated models of the universe with linear varying deceleration parameter (LVDP) i.e. $q = -kt + m - 1; k, m \geq 0$. Mishra et al.[20] have proposed a new form of DP as $q = \frac{\alpha(1-t)}{1+t}$, where $\alpha > 0$ is a constant and called it as bilinear varying deceleration parameter (BVDP). Thereafter, Singh et al.[21] have also studied cosmological model in Brans-Dicke theory of gravity with BVDP and some other similar forms of DP. They have examined the behaviour of bulk viscosity in the presence of the Chaplygin gas equation of state (EoS). Sahoo et al.[22] have studied Bianchi type-I cosmological model with magnetized strange quark matter distribution in $f(R, T)$ gravity where the exact solution of field equations is also obtained with the help of BVDP. Certainly, this reflect the importance of BVDP to study the various aspects of the cosmological models. Further, Mishra et al.[23] have also studied Bianchi-III cosmological model with BVDP in $f(R, T)$ gravity theory in which dynamics of the universe has been discussed in terms of BVDP $q = \frac{\alpha_1(1-t)}{1+\alpha_2 t}$, where α_1 and α_2 are positive constants. Mishra and Heena [24] have studied bulk viscous string models of the universe in alternate theory of gravity and field equtions have been solved with the help of BVDP. Under this motivation, in this communication, study of Bianchi type-V cosmological models with BVDP in $f(R, T)$ gravity theory has been done.

The outline of this communication is as follows: In section 2, formulation of $f(R, T)$ gravity theory has been presented. Section 3 deals with the basic equations governing the cosmological model. Solution of the field equations with the help of BVDP has been obtained in Section 4. The behaviour of the various physical and geometrical parameters has also been explored in section 4. Section 5 deals with the stability of the corresponding solutions. Finally, in last section i.e. section 6, results have been summarized. In the present study, we have used natural system of units with $c = G = 1$.

2. Review of $f(R,T)$ Gravity Theory

As discussed in introduction, $f(R,T)$ gravity theory formulated by Harko et al.[10] is obtained by modifying the geometric sector of the Einstein-Hilbert action. The gravitational field equations of this theory are formulated from the modified action in the following way

$$A=\int\left(\frac{f(R,T)}{16\pi}+\mathcal{L}_m\right)\sqrt{-g}\,d^4x\,, \tag{2.1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor $T_{\mu\nu}$, g is the determinant of metric tensor $g_{\mu\nu}$ and $\mathcal{L}_m$ is the Lagrangian density of matter source. As usual, the energy-momentum tensor of matter is defined by

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}} \tag{2.2}$$

and its trace is defined as $T=g^{\mu\nu}T_{\mu\nu}$. It is assumed that the Lagrangian density of matter $\mathcal{L}_m$ depends entirely on the metric tensor $g_{\mu\nu}$ and not on its derivatives. In such case, we obtain

$$T_{\mu\nu}=-2\frac{\partial\mathcal{L}_m}{\partial g^{\mu\nu}}+g_{\mu\nu}\mathcal{L}_m\,. \tag{2.3}$$

Following are the field equations of $f(R,T)$ gravity theory which are obtained by varying the action A given in Eq.(2.1) with respect to the metric tensor components $g^{\mu\nu}$

$$\begin{aligned}(g_{\mu\nu}W-\nabla_\mu\nabla_\nu)f_R(R,T)-\frac{1}{2}f(R,T)g_{\mu\nu}+f_R(R,T)R_{\mu\nu}\\ =8\pi T_{\mu\nu}-f_T(R,T)\Theta_{\mu\nu}-f_T(R,T)T_{\mu\nu}\,,\end{aligned} \tag{2.4}$$

where $f_R(R,T)=\dfrac{\partial f(R,T)}{\partial R}$, $f_T(R,T)=\dfrac{\partial f(R,T)}{\partial T}$, $W=\nabla^\mu\nabla_\mu=g^{\mu\nu}\nabla_\nu\nabla_\mu$, ∇_μ represents the covariant derivative and

$$\Theta_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2T_{\mu\nu} - 2g^{\eta\gamma}\frac{\partial^2\mathcal{L}_m}{\partial g^{\mu\nu}\partial g^{\eta\gamma}} . \tag{2.5}$$

Contraction of Eq.(2.4) gives an important relation between R and T as

$$\begin{aligned} &3Wf_R(R,T) - 2f(R,T) + f_R(R,T)R \\ &= 8\pi T - f_T(R,T)T - f_T(R,T)\Theta , \end{aligned} \tag{2.6}$$

where $\Theta = \Theta^{\mu}_{\mu}$.

It is assumed that matter behaves like a perfect fluid and hence energy-momentum tensor of matter is given by

$$T_{\mu\nu} = -pg_{\mu\nu} + (\rho + p)u_\mu u_\nu . \tag{2.7}$$

Here ρ and p are the energy density and pressure of the cosmic fluid respectively. $u^\mu = (0,0,0,1)$ is the four velocity vector in co-moving coordinate system satisfying the relations $u^\mu \nabla_\nu u_\mu = 0$ and $u_\mu u^\mu = 1$. Since there is no unique definition of the matter Lagrangian density, it can be taken as $\mathcal{L}_m = -p$. Eq.(2.5) now becomes

$$\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu} . \tag{2.8}$$

Various forms of $f(R,T)$ gravity have been given by Harko et al.[10], viz.,

$$f(R,T) = R + 2f(T) ,$$

$$f(R,T) = f_1(R) + f_2(T)$$

and

$$f(R,T) = f_1(R) + f_2(R)f_3(T) .$$

In this paper, we have considered

$$f(R,T) = f_1(R) + f_2(T) .$$

For this choice of $f(R,T)$, the gravitational field equations are given by

(2.9)
$$(g_{\mu\nu}W-\nabla_\mu\nabla_\nu)f_1^{'}(R)-\frac{1}{2}f_1(R)g_{\mu\nu}+f_1^{'}(R)R_{\mu\nu}$$
$$=8\pi T_{\mu\nu}+\left(f_2^{'}(T)p+\frac{1}{2}f_2(T)\right)g_{\mu\nu}+f_2^{'}(T)T_{\mu\nu}\,.$$

We have considered a particular forms of the functions $f_1(R)=\beta_1 R$ and $f_2(T)=\beta_2 T$, where β_1 and β_2 are arbitrary parameters. In this study, it has also been assumed that $\beta_1=\beta_2=\beta$ so that Eq.(2.9) becomes

(2.10)
$$(g_{\mu\nu}W-\nabla_\mu\nabla_\nu)\beta-\frac{1}{2}\beta(R+T)g_{\mu\nu}+\beta R_{\mu\nu}=8\pi T_{\mu\nu}+\beta T_{\mu\nu}+\beta p g_{\mu\nu}\,.$$

Setting $(g_{\mu\nu}W-\nabla_\mu\nabla_\nu)\beta=0$, Eq.(2.10) becomes

(2.11)
$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=\left(\frac{8\pi}{\beta}+1\right)T_{\mu\nu}+\left(\frac{1}{2}T+p\right)g_{\mu\nu}\,.$$

3. Equations Governing the Cosmological Model

We wish to consider a homogeneous and anisotropic universe described by a Bianchi type-V space-time metric as

(3.1)
$$ds^2=dt^2-r_1^2dx^2-e^{-2x}(r_2^2dy^2+r_3^2dz^2)\,,$$

where r_1, r_2 and r_3 are the scale factors in three different directions x, y and z respectively.

The field equations (2.11) for the above metric (3.1) and energy-momentum tensor (2.7) reduce to the following differential equations

(3.2)
$$H_1H_2+H_2H_3+H_3H_1-\frac{3}{r_1^2}=\left(\frac{8\pi}{\beta}+\frac{3}{2}\right)\rho-\frac{p}{2}\,,$$

(3.3)
$$(H_2+H_3)^2-H_2H_3+\dot{H}_2+\dot{H}_3-\frac{1}{r_1^2}=\frac{\rho}{2}-\left(\frac{8\pi}{\beta}+\frac{3}{2}\right)p\,,$$

(3.4) $$(H_1+H_3)^2 - H_1H_3 + \dot{H}_1 + \dot{H}_3 - \frac{1}{r_1^2} = \frac{\rho}{2} - \left(\frac{8\pi}{\beta}+\frac{3}{2}\right)p\,,$$

(3.5) $$(H_1+H_2)^2 - H_1H_2 + \dot{H}_1 + \dot{H}_2 - \frac{1}{r_1^2} = \frac{\rho}{2} - \left(\frac{8\pi}{\beta}+\frac{3}{2}\right)p\,,$$

(3.6) $$2H_1 - H_2 - H_3 = 0\,,$$

where $H_1 = \frac{\dot{r}_1}{r_1}$, $H_2 = \frac{\dot{r}_2}{r_2}$ and $H_3 = \frac{\dot{r}_3}{r_3}$ are the directional Hubble parameters in x, y and z direction respectively.

For our model, physical quantities such as spatial volume (V), Hubble parameter (H), the expansion scalar (θ), shear scalar (σ^2) and anisotropy parameter (A_m) are defined as

(3.7) $$V = r_1 r_2 r_3\,,$$

(3.8) $$H = \frac{1}{3}(H_1+H_2+H_3) = \frac{1}{3}\left(\frac{\dot{r}_1}{r_1}+\frac{\dot{r}_2}{r_2}+\frac{\dot{r}_3}{r_3}\right),$$

(3.9) $$\theta = 3H = \frac{\dot{r}_1}{r_1}+\frac{\dot{r}_2}{r_2}+\frac{\dot{r}_3}{r_3}\,,$$

(3.10) $$\sigma^2 = \frac{1}{2}\sum_{\mu=1}^{3} H_\mu^2 - \frac{3}{2}H^2\,,$$

(3.11) $$A_m = \frac{1}{3}\sum_{\mu=1}^{3}\left(\frac{H_\mu - H}{H}\right)^2.$$

In the next section, we have obtained the solution of the field equations (3.2)-(3.6) using a certain assumption.

4. Solution of the Gravitational Field Equations and Behaviour of Various Physical Parameters

DP plays an important role in the description of cosmic observations and in theoretical discussions. Being a first non-linear term of the Taylor series expansion of the scale factor $R(t)$, it helps to describe the kinematics of cosmic expansion in more detailed way. It is a dimensionless quantity and is defined as

$$q = \frac{-\ddot{R}R}{\dot{R}^2}. \tag{4.1}$$

Since we are constructing cosmological model with an anisotropic background, following form of the DP has been considered in the next proceeding of the paper

$$q = -\left(\frac{\dot{H} + H^2}{H^2}\right), \tag{4.2}$$

where $H = \frac{1}{3}\left(\frac{\dot{r}_1}{r_1} + \frac{\dot{r}_2}{r_2} + \frac{\dot{r}_3}{r_3}\right)$.

As discussed in introduction, we have found the solution of the gravitational field equations with the help of BVDP proposed by Mishra et al.[20] In this study, we have considered the following bilinear form of DP

$$q(t) = \frac{\alpha_1 - \alpha_2 t}{1 + \alpha_2 t}, \tag{4.3}$$

where α_1 (dimensionless) and α_2 (dimension of inverse of time) are positive constants. From Eq. (4.2)

$$H = \frac{1}{t + \int q(t)dt + k_1}. \tag{4.4}$$

Using above form of $q(t)$ (4.3), we obtain the following value of H

$$H = \frac{1}{\alpha_3 log(1 + \alpha_2 t) + k_2}, \tag{4.5}$$

where $\alpha_3 = \left(\dfrac{\alpha_1}{\alpha_2} + \dfrac{1}{\alpha_2}\right)$ and $k_2 = \left(k_1 - \dfrac{1}{\alpha_2}\right)$ is a constant.

It is well known that expansion rate of the universe is very high during the cosmic inflation, therefore, k_2 must be zero. The above expression of the Hubble parameter can be rewritten as

(4.6) $$H = \frac{1}{\alpha_3 log(1+\alpha_2 t)}$$

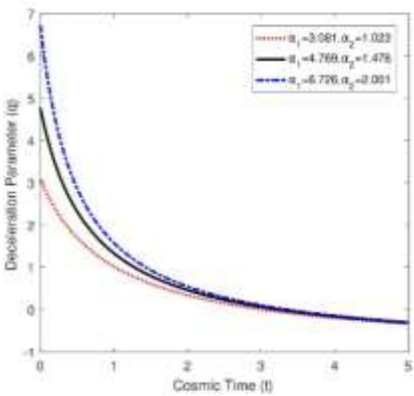

Figure 1. Plot of deceleration parameter (q) versus cosmic time (t).

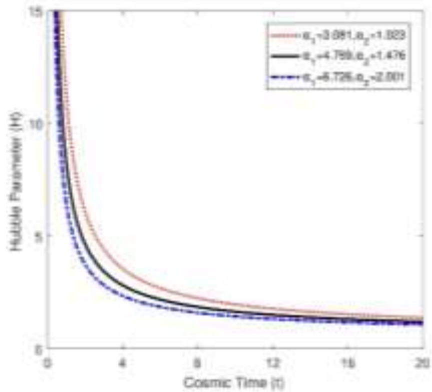

Figure 2. Plot of Hubble parameter (H) versus cosmic time (t).

This expression of H can be further simplified as

(4.7) $$H = \frac{1}{\alpha_2\alpha_3 t}\left[1 - \alpha_2 t\left(\frac{1}{2} - \frac{\alpha_2 t}{3} + \frac{\alpha_2^2 t^2}{4} - \frac{\alpha_2^3 t^3}{5} + \ldots\ldots\ldots\ldots\ldots\ldots\right)\right]^{-1}.$$

Since $H = \dfrac{1}{3}\left(\dfrac{\dot{r}_1}{r_1} + \dfrac{\dot{r}_2}{r_2} + \dfrac{\dot{r}_3}{r_3}\right)$, on integrating above expression, spatial volume for the model is obtained as

(4.8) $$V = r_1 r_2 r_3 = r_0^3\, t^{\frac{3}{1+\alpha_1}}\, exp(3M(t))$$

where

$$M(t)=\frac{\alpha_2 t}{2(1+\alpha_1)}-\frac{\alpha_2^2 t^2}{24(1+\alpha_1)}+\frac{\alpha_2^3 t^3}{72(1+\alpha_1)}-\frac{19\alpha_2^4 t^4}{2880(1+\alpha_1)}+O(t^5)$$

and r_0 is a constant of integration. Without any loss of generality, we take r_0 to be unity. Integration of Eq. (3.6) gives

(4.9) $$r_1^2=r_2 r_3 .$$

Using the above relation (4.9) and field equations (3.3)-(3.5) scale factors r_1, r_2 and r_3 are obtained as

(4.10) $$r_1=t^{\frac{1}{1+\alpha_1}}\,exp(M(t)),$$

(4.11) $$r_2=k_3\,t^{\frac{1}{1+\alpha_1}}\,exp(M(t))\,exp\left(k_4\int\frac{dt}{t^{\frac{3}{1+\alpha_1}}\,exp(3M(t))}\right),$$

(4.12) $$r_3=\frac{1}{k_3}\,t^{\frac{1}{1+\alpha_1}}\,exp(M(t))\,exp\left(-k_4\int\frac{dt}{t^{\frac{3}{1+\alpha_1}}\,exp(3M(t))}\right),$$

where k_3 and k_4 are constants.
Using above expressions of the scale factors (4.10)-(4.12), the directional Hubble parameters (H_μ, $\mu=1,2,3$) are obtained as

(4.13) $$H_1=\frac{\dot r_1}{r_1}=\frac{1}{\alpha_3 log(1+\alpha_2 t)},$$

(4.14) $$H_2=\frac{\dot r_2}{r_2}=\frac{1}{\alpha_3 log(1+\alpha_2 t)}+\frac{k_4}{t^{\frac{3}{1+\alpha_1}}exp(3M(t))},$$

(4.15) $$H_3=\frac{\dot r_3}{r_3}=\frac{1}{\alpha_3 log(1+\alpha_2 t)}-\frac{k_4}{t^{\frac{3}{1+\alpha_1}}exp(3M(t))}.$$

The other physical quantities such as expansion scalar (θ), shear scalar (σ^2) and anisotropic parameter (A_m) are found to be

(4.16) $$\theta = \frac{3}{\alpha_3 log(1+\alpha_2 t)},$$

(4.17) $$\sigma^2 = \frac{k_4^2}{t^{\frac{6}{1+\alpha_1}} exp(6M(t))},$$

(4.18) $$A_m = \frac{2}{3}\frac{k_4^2[\alpha_3 log(1+\alpha_2 t)]^2}{t^{\frac{6}{1+\alpha_1}} exp(6M(t))}.$$

It can be observed from the above set of equations that H_1, H_2, H_3 and θ approach to zero as $t \to \infty$. However, from Eq. (4.8), it is clear that spatial volume is zero at $t = 0$ (which is big-bang scenario) and thereafter it is increasing exponentially with the passage of time. Also, it becomes infinite as $t \to \infty$. From Eq. (4.18) it is observed that $A_m \to 0$ as $t \to \infty$. This indicates that our model of the universe shows transition from early anisotropic stage to present isotropic stage. Such behaviour of the model is in good agreement with the recent observational data.
Using eqs. (3.2)-(3.3) and (4.13)- (4.15), expressions for the energy density ρ and pressure p are obtained as

(4.19) $$\rho = \frac{1}{\left(\gamma^2 - \frac{1}{4}\right)}\left[\left(3\gamma - \frac{3}{2}\right)H^2 - \left(\gamma + \frac{1}{2}\right)\frac{k_4^2}{V^2} + \left(\frac{1}{2} - 3\gamma\right)\frac{1}{r_1^2} - \dot{H}\right],$$

where $\gamma = \left(\frac{8\pi}{\beta} + \frac{3}{2}\right)$ and $\dot{H} = -\left[\frac{\alpha_2}{(1+\alpha_2 t) log(1+\alpha_2 t)}\right]H$.

(4.20) $$p = \frac{1}{\left(\gamma^2 - \frac{1}{4}\right)}\left[\left(\frac{3}{2} - 3\gamma\right)H^2 - \left(\frac{1}{2} + \gamma\right)\frac{k_4^2}{V^2} + \left(\gamma - \frac{3}{2}\right)\frac{1}{r_1^2} - 2\gamma\dot{H}\right].$$

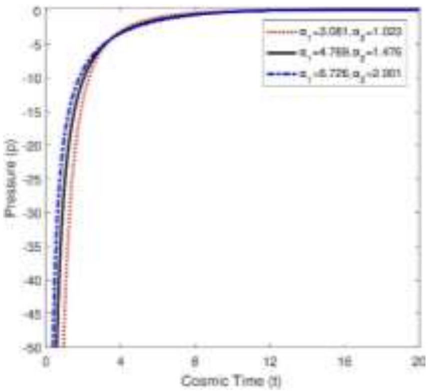

Figure3. Plot of energy density (ρ) versus cosmic time (t) $\beta = -15$ and $k_4 = 0.001$.

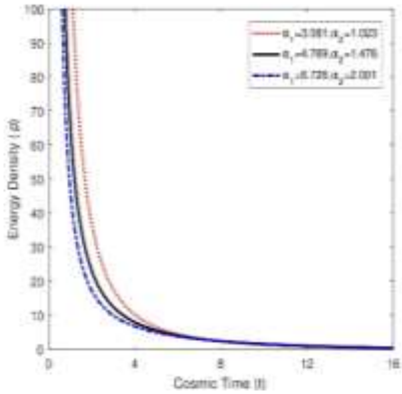

Figure 4. Plot of pressure (p) versus for cosmic time (t) for $\beta = -15$ and $k_4 = 0.001$.

5. Statefinder Diagnostic and Energy Conditions

5.1 Statefinder pair: Besides Hubble parameter and deceleration parameter, it is useful to consider the other parameters which involve higher order time derivatives of scale factor $R(t)$. They help us to get more detailed description of the cosmic expansion. Sahni et al.[25] have introduced a new diagnostic pair (r, s) called as Statefinder. This pair is constructed from the the scale factor $R(t)$ and its derivatives upto third order. Both the parameters r and s are dimensionless and allow us to differentiate between various forms of dark energy models. This statefinder diagnostic pair is defined as

$$r = 1 + \frac{1}{H^2}\left(3\dot{H} + \frac{\ddot{H}}{H}\right) \text{ and } s = \frac{1}{3}\left(\frac{r-1}{q-\frac{1}{2}}\right). \tag{5.1}$$

For the presented model, both the parameters are obtained as

$$r = 1 - \frac{3(1+\alpha_1)}{1+\alpha_2 t} + \frac{(1+\alpha_1)^2 log(1+\alpha_2 t)}{(1+\alpha_2 t)^2}\left[1 + \frac{2}{log(1+\alpha_2 t)}\right], \tag{5.2}$$

$$s = \frac{2(1+\alpha_1)^2 log(1+\alpha_2 t)}{3(1+\alpha_2 t)(2\alpha_1 - 3\alpha_2 t - 1)}\left[1 + \frac{2}{log(1+\alpha_2 t)}\right] - \frac{2(1+\alpha_1)}{2\alpha_1 - 3\alpha_2 t - 1}. \tag{5.3}$$

From Eq. (5.2) and Eq. (5.3), it is observed that the pair $(r,s)\rightarrow(1,0)$ as $t\rightarrow\infty$ i.e. our obtained model of the universe approaches to Λ CDM model at late times.

5.2 Energy Conditions: We have also explored the behaviour of some of the important energy conditions such as weak energy conditions (WEC), dominant energy condition (DEC) and strong energy condition (SEC). These are given by

- WEC: $\rho\geq 0$, $\rho-p\geq 0$
- DEC: $\rho+p\geq 0$
- SEC: $\rho+3p\geq 0$

It is observed that both the WEC and DEC are well satisfied in our model. But SEC is not satisfied in our model.

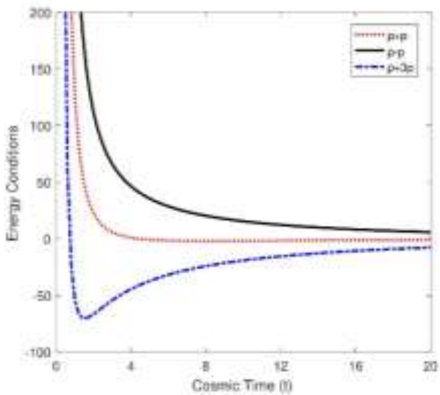

Figure 5. Plot of Energy Conditions versus cosmic time (t) for $\alpha_1=6.726$, $\alpha_2=2.001$, $\beta=-15$ and $k_4=0.001$

For the present universe, $t_0=13.8$ GYR and $q_0=-0.73$ [26]. Using these values, following relationship between α_1 and α_2 can be obtained from Eq. (4.3)

(5.4) $$\alpha_1-3.726\alpha_2=-0.73.$$

In all the graphical representations of the various physical parameters, the values of α_1 and α_2 are chosen in such a way that they satisfy the relation (5.4).

6. Concluding Remarks

In this communication, we have examined the dynamics of Bianchi type-V universe in $f(R,T)$ gravity theory. The gravitational field equations of the model have been solved with the help of BVDP proposed by Mishra et al.[20]. This choice of the DP yields spatial volume $V(t) = t^{\frac{3}{1+\alpha_1}} exp(3M(t))$, which shows that our model of the universe starts from initial big bang singularity (i.e. $V = 0$ at $t = 0$) and is expanding exponentially with cosmic time. Moreover, our model reaches isotropy at late times which is in good harmony with the present day data. The behaviour of various physical parameters have been analysed with the help of their pictorial representations. Following are the main features of our model

(i) From Eq. (4.3), it is observed that $q>0$ if $t < \frac{\alpha_1}{\alpha_2}$, $q = 0$ if $t = \frac{\alpha_1}{\alpha_2}$ and $q<0$ if $t > \frac{\alpha_1}{\alpha_2}$. This flipping nature of the sign of the DP indicates that our model of the universe undergoes phase transition from past decelerated phase to present accelerated phase which is in good agreement with the recent observational and theoretical data. Fig.1 also indicates same behaviour of the DP. Thus, our choice of considering BVDP is physically viable.

(ii) Fig.2 represents the variation of Hubble Parameter against time t. The graph also indicates that $H \to 0$ as $t \to \infty$. We have obtained the cosmological model which represents non-rotating, shearing and accelerating universe which starts with initial big bang singularity and gains isotropy at late times.

(iii) Fig.3 represents the variation of energy density ρ versus cosmic time t. It is noticed that energy density is a positive decreasing function of time as expected. Moreover, it converges to zero as $t \to \infty$.

(iv) Fig.4 represents the time-varying behaviour of pressure p. It is observed that in our model, pressure remains always in negative zone throughout the cosmic evolution which could be the indication of dark energy. It starts from very large negative value and is reaching to zero at late times.

(v) The behaviour of various energy conditions for $\alpha_1 = 6.726$ and $\alpha_2 = 2.001$ is represented in Fig.5. The graph indicates the same behaviour for other sets of values of α_1 and α_2. It is observed that weak and dominant energy conditions are well satisfied in our case. However, strong energy condition is violated at present epoch. Such violation of strong energy condition gives anti-gravitational effect which could be responsible for the current accelerated expansion of the universe[27].

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