Gauge Unification of Fundamental Forces : the Story of Success*

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1. Introduction

Gauge theory in a form not much different from the modern gauge theory was created by Maxwell in 1866 to describe the gauge transformation of the electromagnetic field. To preserve the invariance of his electrodynamics under gauge transformation and to counteract the variation of "α" with x, y, z and t Maxwell wrote his transformation as

$$A_{\mu}' = A_{\mu} + (1/e)\,\partial\alpha/\partial x^{\mu}.$$

In 1954, Yang and Mills[1] in their fundamental paper, in an entirely identical style, introduced a B ** field in the case of the isotopic gauge transformation to counteract the dependence of S on x, y, z and t under an isotopic gauge transformation:

$$\psi \rightarrow \psi', \quad \psi' = S^{-1}\psi,$$

where S represents a space-time dependent isotopic spin*** rotation. This classical paper of Yang and Mills is one of the most important early attempts to unify weak and electromagnetic interactions.

Symmetry principles made their appearance in 20th century physics in 1905 with Einstein's identification of the invariance group of space and time. With this as a precedent, symmetries took on a character in physicist's minds as a priori principles of universal validity - expressions of simplicity of nature at its profoundest level. And as such, it was painfully difficult in the 1930's to discover that there are internal

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** Pauli was present in the audience when Yang was reading his paper at the Institute for Advanced Studie, in Princeton. On a query of Pauli about the nature of field B Yang replied not to have known it.

***The total isotopic spin τ was first introduced by Wigner[2].

symmetries such as isopin conservation having nothing to do with space and time symmetries which are far from self-evident governing the strong interactions only. The era of 1950's saw the discovery of another internal symmetry - the conservation of strangeness - which is not obeyed by weak interactions.

In 1956, it was discovered by Lee and Yang[3] that one of the supposedly sacred symmetries of space-time, i.e. parity was violated by weak interactions[4]. Instead of moving towards unification physicists were learning that different fundamental interactions are apparently governed by different symmetries. Matters became yet more baffling with the recognition of a symmetry group (the eightfold way) which is not even an exact symmetry of strong interactions[5].*

Above are "global" symmetries for which symmetry transformations do not depend on position in space and time. It had been recognized in the 1920's that quantum electrodynamics (QED) has another symmetry of a far more robust breed: a "local" symmetry under transformations in which the electron field suffers a phase change that can vary freely from point to point in space-time, in which the electromagnetic vector potential undergoes a corresponding gauge transformation. Currently, it is called a $U(1)$ gauge symmetry because a simple phase change can be thought of as multiplication by a unitary matrix 1×1.

2. Mesons

In the 1920's it was still believed that there were only two fundamental forces: gravitational and electromagnetic ones. In his efforts (spanning a period of 35 years) to unify them, Einstein might have hoped to formulate a universal theory of physics. Nevertheless, the study of the atomic nucleus in the 1930's revealed the need for two additional forces: the strong nuclear force to hold the nucleons together and the weak nuclear force to enable the nucleus to decay (β-radioactivity - then described by Fermi's non-renormalizable theory). Yukawa[6] raised a question in 1935 whether there might be a deep analogy between these nuclear forces and electromagnetism? He further argued that all forces result from the exchange of mesons. ** His π-mesons were originally intended to mediate both the strong and weak forces. They were strongly coupled to nucleons and weakly coupled to leptons. This was the first attempt to unify weak and strong forces and that was forty years premature. Yukawa's neutral mesons (to provide the charge independence to the neucleons) were also weakly coupled to pairs of leptons.

Not only is electromagnetism mediated by photons but it stems from the demand

* Also confirmed experimentally by the discovery of omega minus in February, 1964.

** Yukawa[6] predicted the existence of π-mesons which were discovered by Lattes et al.[7] in 1947. This discovery resulted in Yukawa's Nobel Prize in 1949.

of local gauge invariance. In 1954, this theory was generalized to non-abelian local symmetry groups by Yang and Mills[1], and also independently by Shaw[8]. The Yang-Mills theory, which matched Maxwell's gauge ideas with the internal symmetry $SU(2)$ (of which the proton-neutron system constituted a doublet), was independently discovered by Shaw. Yang and Mills had guessed the desirable renormalizability of their theory which was based on the masslessness of their spin-one intermediate mesons. The problem of mass was to be solved in the early 1960's with the understanding of the Higgs mechanism. Pauli was absolutely right in his letters written to Salam in 1957 blaming him of darkness about the problem of the masses of the Yang-Mills fields. It was not possible to obtain a mass without destroying the gauge symmetry.

All gauge mesons must be massless yet the photon is the only massless meson. The fundamental question was: how do the other gauge bosons get their masses? There was no acceptable answer to his question until the works of Salam[9] and Weinberg[10] appeared. Nevertheless, immediately after the experimental confirmation of Lee and Yang's parity violation theory[3] many ideas coming to fruition by the works of Glashow[11, 12], Weinberg[10], Salam[9], t'Hooft[13] and others had started to become crystal clear. Amongst these the first was the idea of theory of chiral symmetry that leads to a $(V-A)$ theory. In 1957, Salam's idea of chiral symmetry was limited to neutrinos, electrons and muons only[14]. In 1958, Feynman and Gell-Mann[15], Sakurai[16] and Sudarshan and Marshak[17] applied the idea of chiral (or γ_5) symmetry to baryons as well as leptons. Associated with the $(V-A)$ theory was the result:

"If weak interactions are mediated by intermediate mesons these mesons must carry spin-one".

Second was the idea of spontaneous breaking of chiral symmetry to generate electron and muon masses. This innovative idea was presented by Goldstone[18] and Nambu and Jona-Lasinio[19,20] independently in 1961.

Third was the application of Yang-Mills-Shaw's non-abelian gauge theory for describing spin-one intermediate charged mesons and generating masses to the intermediate bosons through spontaneous symmetry breaking in such a mathematical style so as to preserve the renormalizability of the theory. This was acheived only during 1963 and 1971 through the efforts of a number of talented workers *.

The question of the third component of $SU(2)$ triplet ** was raised in 1957-58. There were two alternatives: the electro-weak unification (where the electromagnetic current was assumed to be the third component of the ($SU(2)$ triplet); and the rival

* For a detailed bibliography cf.[2], page 3.

** The charged weak currents were the remaining two components.

suggestion of a neutral current unlinked with the electro-weak unification being the third component. Salam[9] called these two alternatives as Klein[22] and Kemmer[23] alternatives *. The Klein suggestion, made in the context of the Kaluza-Klein 5-dimensional pace-time, combined two hypothetical spin-one charged mesons with the photon in one multiplet, and as such deducing a theory (which has the semblance of Yang-Mills-Shaw's theory) from the compactification of the fifth dimension. On these non-abelian gauge aspects, the idea of uniting weak interactions with electromagnetism was developed by Glashow[24] and Salam and Ward[25] in 1959 while Schwinger[26] believed that the weak and electromagnetic interactions should be combined in a gauge theory. The rival Kemmer suggestion of a global $SU(2)$ invariant triplet of weak charged and neutral currents was independently suggested by Bludmann[27] in 1958 in a gauge context. And this is how the gauge unification theories stood in 1960. In 1961, Salam and Ward[28] concluded that "Our basic postulate is that it should be possible to generate strong, weak and electromagnetic interaction terms with all their correct symmetry properties (as well as with clues regarding their relative strengths) by making local gauge transformations on the kinetic energy terms in the free Lagrangian for all particles".

The gauge theory, uniting weak and electromagnetic interactions and incorporating the phenomenon of spontaneous symmetry breaking and the emergence of the $SU(2) \times U(1)$ group in a form vulnerable to experimental tests, was developed during the fruitful period between 1961-67.

3. Electron-Type Leptons

Things had to be so arranged that the charged current, but not the neutral (electromagnetic) current, would violate parity and strangeness. it is technically possible to construct such theory but it is both ugly and experimentally false. This theoretically consistent but experimentally false paper was written by Georgi and Glashow[29] in 1972. They soon discovered that the electroweak gauge group must be larger than their $SU(2)$ and neutral currents do exist.

Another electroweak synthesis without neutral currents was presented by Salam and Ward[25] in 1959; but they failed to know how to incorporate the experimental fact of parity violation. In 1961, Salam and Ward[28] presented a gauge theory of strong, weak and electromagnetic interactions based on the local symmetry group $SU(2) \times SU(2)$. This idea was an impressive portent of the $SU(3) \times SU(2) \times U(1)$ group which is now the standard model.

By 1967, it was not very difficult for Salam[9] and Weinberg[10] (who were

* It may be recalled that Salam worked for his doctoral dissertation at Cambridge under the supervision of Professor N. Kemmer.

working independently at Treiste and M. I. T.) to work out a mathematical model which could incorporate above mentioned ideas. There are two left-handed electron-type leptons: ν_{e_L} and e_L; and one right-handed electron-type lepton e_R. So, Weinberg started with the group $U(2) \times U(1)$: all unitary 2×2 matrices acting on the left-handed electron-type leptons together with all unitary 1×1 matrices acting on the right-handed electron-type leptons. Breaking up $U(2)$ into unimodular transformations and phase transformations, one could say that the group was $SU(2) \times U(1) \times U(1)$. But, then one of the $U(1)$'s could be identified with the ordinary lepton number and since lepton number appears to be conserved and there is no massless vector particle coupled to it, Weinberg decided to exclude the second $U(1)$ from the group. Thus, he was left with the four- parameter group $SU(2) \times U(1)$. Due to spontaneous breakdown of $SU(2) \times U(1)$ to $U(1)$ the electromagentic gauge invariance would impart massess to three of the four gauge bosons: the charged bosons W^+, W^- and a neutral boson Z^0. The fourth boson would automatically remain massless, and could be identified as the photon. Knowing the strength of the ordinary charged current weak interaction (like β–decay), the same being mediated by $W^\pm$, the masses of $W^\pm$ were predicted as about 40 GeV/sin θ, where θ is the γ- Z^0 mixing angle (also called as Weinberg-Salam angle)

To make the Salam-Weinberg-Glashow theory more coherent, it was necessary to invent some hypothesis about the mechanism for the breakdown of $SU(2) \times U(1)$. The only possible field in a renormalizable $SU(2) \times U(1)$ theory whose vacuum expectation values could give the electron a mass is a spin-zero $SU(2)$ doublet (ϕ^+, ϕ^0). For simplicity, Weinberg assumed that these were the only scalar fields in the theory. The mass of the Z^0 was then determined as about 80 GeV/sin 2θ, which fixed the strength of the neutral current weak interactions.

As in the case of QED once we decide the menu of the fields in the theory all details of the theory can be determined by symmetry principles and the constraints of renormalizability. It was vital to impose the constraint of renormalizability otherwise weak interactions would be received from $SU(2) \times U(1)$, i.e. invariant four fermion couplings, as well as from vector boson exchange. And as such, the theory would then loose most of its predictive power. Based on the above group theoretical logic Weinberg finalized his Nobel Prize winning paper[10] in September 1967.

4. Success of Salam-Weinberg-Glashow $SU(2) \times U(1)$ Group

Salam wrote his Nobel Prize winning paper[9] in November 1967. This excellent paper (written in the Fermi style) remained ignored for about four years and received recognition in 1971 only when a copy of the same was sent to Wigner. This paper incorporates some of the ideas of Higgs and Kibble, which Salam had previously presented in his lectures at Imperial Science College, London. In 1964, Salam and Ward[30] worked on the synthesis of electromagnetism and weak interactions. Their paper had all the ingredients of success in the synthesis of weak and electromagnetic interactions and yet it failed.

Salam's Nobel Prize winning paper[9] is based on group theoretical philosophy and the constraints of renormalization. Naturalness of the Salam-Weinberg-Glashow theory is proved by the fact that Salam, independently of Weinberg, developed much the same theory as was presented by Weinberg[10].

The next fundamental question was the renormalizability of the Salam-Weinberg model. The Feynman rules for Yang-Mills theories with unbroken gauge symmetries had been worked out by DeWitt[31] and Fadeev and Popov[32] independently, and it was proved by them that such theories are renormalizable. Salam and Weinberg were confident that their theories are renormalizable. Many persons including Weinberg tried to prove that Salam-Weinberg-Glashow theory is renormalizable, but finally t'Hoft[13] could establish it in 1971. It was finally completed by Lee and Zinn-Justin[33] and t'Hoft and Veltman[34] in 1972. In 1973, Gross and Wilczek[35] and Politzer[36] discovered a remarkable property of the Yang-Mills theories which they called "asymptotic freedom" where the effective coupling constant decreases to zero as the characteristic energy of a process goes to infinity. This might explain the experimental fact that the nucleon behaves in high energy deep inelastic electron scattering as if it consists of free quarks.

The theoretical efforts of Salam, Weinberg, Glashow and many others were illuminated by brilliant CERN experiment by Hasert et al.[37] in 1973. It discovered the neutral currents demanded by the Salam-Weinberg-Glashow theory of $SU(2) \times U(1)$ unification of weak and electromagnetic interactions. The later work (theoretical as well as experimental) on neutral currents at CERN, Fermilab, Brookhaven, Argonne and Serpukhov is now the subject matter of history. A highly sophisticated SLAC-Yale-CERN experiment by Taylor et al. proved in June 1978 the effective Z^0-photon interference in accordance with the predictions of the Salam-Weinberg theory. This was foreshadowed by Barkov et al.[38] in their experiments at Novosibirsk (Russia) in their exploration of parity-violation in the atomic potential of bismuth vapour.

The experimental success of the Salam-Weinberg-Glashow $SU(2) \times U(1)$ group is as dramatic and multi-dimensional as the successes of their theoretical papers. Nevertheless, Salam[9] in his Nobel lecture showed his preference for experiments (and also his starry-eyedness for Einstein) by refering to Einstein's Herbert Spencer lecture delivered in 1933 at Oxford.

"Pure logical thinking cannot yield us any knowledge of the emperical world; all knowledge starts from an experiment and ends in it".

5. From Electroweak to Electronuclear

Three main ideas leading to electronuclear or grand unification of the electroweak with the strong nuclear force are as follows.

First is the bold and adventurous paper of Pati and Salam[39] in which they grouped quarks and leptons in the same multiplet of unifying group *G*. They postulated that the group *G* must contain $SU(2) \times U(1) \times SU_c(3)$ and must be non-abelian if all the quantum numbers (flavor, color, lepton, quark and family numbers) are to be automatically quantized, and the resulting gauge theory is asymptotically free. They further elaborated their ideas in a mathematically elegant paper[40]. In this paper strong, weak and electromagnetic interactions of leptons and hadrons are generated by gauging a non-abelian renormalizable anomaly-fee subgroup of fundamental symmetry structure $SU(4)_L \times SU(4)_R \times SU(4')$. This structure unites three quartets of "colored" baryonic quarks and the quartets of known leptons into 16-folds of chiral fermionic multiplets with lepton number treated as the fourth "color" quantum number.

Second idea is an extension, proposed by Georgi and Glashow[41], which places not only left-handed quarks and leptons but also their antiparticles in the same multiplet of the unifying group. They presented a series of hypotheses and speculations leading (inescapably) to the conclusion that $SU(5)$ gauge group achieves the grand unification. Also, all elementary particle forces (strong, weak and electromagnetic) are different manifestations of the same fundamental interactions involving a single coupling strength. A gauge theory based on a "simple" or (with discrete symmetries) a "semi-simple" group *G* should contain one basic gauge constant. This constant would manifest itself physically above the "grand unification mass" *M*, exceeding all particle masses in the theory. These masses being generated, if possible, hierarchially through a suitable symmetry breaking mechanism.

The third important paper is by Georgi, Quinn and Weinberg[42] who showed how, using renormalization group ideas, one could relate the observed low-energy couplings:

$$\alpha(\mu) \text{ and } \alpha_s(\mu), \ (\mu \sim 100 GeV),$$

to the magnitude of the grand unifying mass M and the observed value of $\sin^2 \theta(\mu)$, where $\tan \theta(\mu)$ is the ratio of the $U(1)$ to the $SU(2)$ couplings.

If $\sin^2 \theta(\mu)$ is as large as 0.23 then the grand unifying mass M cannot be smaller than 1.3×10^{13} GeV. Planck's mass $m_p \approx 1.2 \times 10^{19}$ GeV is related to Newton's constant, where gravity must come in. This result follows from a formula by Marciano[43] and Salam[44]:

$$(11\alpha/3\pi) \ln(M/\mu) = \{\sin^2 \theta(M) - \sin^2 \theta(\mu)\} / \cos^2 \theta(M).$$

Salam proved that above result is a consequence of the assumption that $SU(2) \times U(1)$ symmetry survives intact from the low regime energies μ right up to grand unifying mass M.

6. Tests of Electronuclear Grand Unification

The most characteristic prediction for the validity and soundness of grand unification is proton decay. This was first discussed at the Second International Conference on Elementary Particles, Aix- en Provence (France), 1973 by Stech[45] quoting Pati and Salam's work[46] *. This implies that proton and, indeed, all nuclear matters must be inherently unstable. Sensitive searches are being conducted in the U.S.A., Russia, Switzerland, India and other places. Till April 1990 no success was achieved. If the proton lifetime is shorter than 10^{32} years, as theoretical calculations based on the standard model $SU(3) \times SU(2) \times U(1)$ indicate, it should be not long before the proton decay is observed.

7. The Discovery of $W^{\pm}$ and Z^0 Particles

In the early 1983, Rubbia and Van der Meer announced the discovery of W^+, W^- and Z^0 bosons. Their existence was predicted by the Salam-Weinberg-Glashow model which unified weak and electromagnetic interactions. This discovery is a great success of the $SU(2) \times U(1)$ gauge symmetry which has been extended to $SU(3) \times SU(2) \times U(1)$ symmetry that unifies strong, weak and electromagnetic interactions.

A highly sophisticated experimental complex was created by Rubbia, Van der Meer and a number of researchers from nore than ten countries at CERN, Geneva. The components of the huge experimental set-up were brought from more than six European countries and the experimental manifold was established at the cost of almost a billion U.S. Dollars. Rubbia, Van der Meer and other achieved this colossal

* Being post-deadline paper it could not be presented by Professor Salam at the Conference.

success after their real hard work covering a period of more than five years. Rubbia and Van der Meer shared the 1984 Physics Nobel Prize for this fundamental discovery.

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