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W_2-Flat Spacetime in FRW Cosmological Model

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Abstract: The object of the present paper is to investigate W_2-flat spacetime in the framework of FRW model. In this regard we have considered perfect fluid as a source of matter distribution of the universe. We have solved Einstein's field equations with variable cosmological term using special variational law $\Lambda = 3\beta H^2$, where β is a constant and H is the Hubble's parameter. Moreover, we have described a cosmological scenario of the universe with these solutions and shown that this model is in accordance with the recent day observations.

Keywords: W_2-flat, Einstein's field equation, Cosmic snap parameter, de-Sitter universe, FRW model, Λ CDM model.

2010 AMS Classification Number: 53C25, 53C35, 53C50, 53B30.

1. Introduction

For development of a physical theory, we need a model consistent with known facts of a system. Due to development in Cosmology and de-Sitter space-time, theoreticians are trying to solve Einstein's field equations to predict various phenomena of the universe. The most successful model, having tremendous prediction power for Cosmology, was obtained by Friedman in 1922 as well as by Robertson and Walker in 1930, which is termed as Friedman-Robertson-Walker (FRW) model. This is a non-static symmetric homogeneous model obeying the cosmological principle. This model is so successful that the standard hot big-bang theory is based on it.
In FRW model, the metric of the spacetime has the following form:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right], \tag{1.1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, $a(t)$ is the scale factor which describe how the distance between any two world lines changes with cosmic time t, r is the comoving radial coordinates. It is important to make some remarks on the scale factor $a(t)$ in FRW model given by (1.1). Its role is very important in the study of cosmic dynamics, in the sense, that its growth with time ensures expansion of the universe. It is obtained by solving Friedmann equation giving cosmological dynamics. Increasing $a(t)$ with time t shows that galaxies, at the $t =$ constant hypersurface, are moving away from each other. Obviously, it is possible when this hypersurface expands. This phenomenon is analogous to moving colored patches on the surface of an expanding balloon when air is pumped into it. The θ and ϕ parameters are the usual azimuthal and polar angles of spherical coordinates with $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. The coordinates (t, r, θ, ϕ) are called comoving coordinates. Also k is the curvature of the spacetime which take the values -1, 0, 1 representing open, flat and closed universe respectively. The spatial proper volume of this model is $V(t) = 2\Pi^2 a^3(t)$.

Moreover, it is to be noted that according to geometrical analysis, we can not say what should be the actual value of k. But we can think of only three possible values of k given above.

In 1970, Pokhariyal and Mishra[1] introduced some new curvature tensor fields, called W_2 and E tensor fields, in a Riemannian space and studied their properties. According to them a W_2 $-$curvature tensor on a Riemannian space is defined by

$$W_{2hijk} = R_{hijk} + \frac{1}{(n-1)}\left[g_{hj}R_{ik} - g_{ij}R_{hk}\right], \tag{1.2}$$

where R_{hijk} is the curvature tensor of type (0,4) and R_{ij} is the Ricci tensor of type (0,2). A space is called W_2-flat if W_2 curvature tensor vanishes at each point of the space. In this connection, it may be mentioned that Pokhariyal and Mishra[1,2] and Pokhariyal[3] introduced some new curvature tensors defined on the line of Weyl projective curvature tensor.

The W_2-curvature tensor was introduced on the line of Weyl projective curvature tensor and breaking W_2 into skew-symmetric parts, the tensor E has been defined. Rainich conditions for the existence of the non-null electrovariance can be obtained by W_2 and E, if we replace the matter tensor by the contracted part of these tensors. The tensor E enables to extend Pirani formulation of gravitational waves to Einstein space. It is shown that[1] except the vanishing of complexion vector and property of being identical in two spaces which are in geodesic correspondence, the W_2-curvature tensor possesses the properties almost similar to the Weyl projective curvature

tensor. Thus we can use W_2-curvature tensor in various physical and geometrical spheres in place of the Weyl projective curvature tensor.
The W_2-curvature tensor have also been studied by various authors in various ways such as De and Sarkar[4], Matsumoto, Ianus and Mihai[5], Pokhariyal[6,7], Taleshian and Hosseinzadeh[8], $\ddot{O}$zen[9], Yildiz and De[10] and many others.

We suppose that the spacetime is W_2-flat. Then from the definition of W_2-curvature tensor we infer that

$$R_{hijk} + \frac{1}{n-1}\left[g_{hj}R_{ik} - g_{ij}R_{hk}\right] = 0. \tag{1.3}$$

Now transvecting the above equation with g^{hk} we obtain

$$R_{ij} + \frac{1}{n-1}\left[R_{ij} - Rg_{ij}\right] = 0, \tag{1.4}$$

which infers that

$$R_{ij} = \frac{R}{n} g_{ij} \tag{1.5}$$

This means that in a W_2-flat spacetime the Ricci tensor is given by

$$R_{ij} = \frac{R}{4} g_{ij}. \tag{1.6}$$

Thus the spacetime is an Einstein spacetime.

Several authors[11-14] have studied FRW cosmological model in different ways. Motivated by the above studies in the present paper we characterize W_2-flat FRW cosmological model.
The present paper is organized as follows:
After introduction in Section 2 we study FRW metric and Einstein's field equations associated with special variational law and Friedmann equations. In the next section we consider the solutions of field equations and interpret the cosmological scenarios of the universe in different cases.
We estimate some cosmological parameters in Section 4. Finally, we drawn an overall conclusion regarding W_2-flat spacetime in the framework of FRW cosmological model with perfect fluid as a source of matter distribution of the universe.

2. Metric and Field Equations

With the metric (1.1), we compute the Christoffel symbols and the components of the Ricci tensor. Setting $\dot{a} = \frac{da}{dt}$, the Christoffel symbols are given by the following:

$$\Gamma^0_{11} = \frac{a\dot{a}}{1-kr^2}, \quad \Gamma^1_{11} = \frac{kr}{1-kr^2}, \quad \Gamma^0_{22} = a\dot{a}r^2, \quad \Gamma^0_{33} = a\dot{a}r^2\sin^2\theta,$$

$$\Gamma^1_{01} = \Gamma^2_{02} = \Gamma^3_{03} = \frac{\dot{a}}{a}, \; \Gamma^1_{22} = -r(1-kr^2), \; \Gamma^1_{33} = -r(1-kr^2)\sin^2\theta,$$

$$\Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r}, \quad \Gamma^2_{33} = -\sin\theta\cos\theta, \quad \Gamma^3_{23} = \cot\theta$$

The non-zero components of the Ricci tensor are

$$R_{00} = -3\frac{\ddot{a}}{a}, \quad R_{11} = \frac{a\ddot{a}+2\dot{a}^2+2k}{1-kr^2},$$

$$R_{22} = r^2(a\ddot{a} + 2\dot{a}^2 + 2k), \quad R_{33} = r^2(a\ddot{a} + 2\dot{a}^2 + 2k)\sin^2\theta$$

and the Ricci scalar is then

$$R = 6\left\{\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right\}.$$

The expression for Einstein's relativistic field equations ($c = 1$) is

(2.1) $$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi GT_{ij} + \Lambda g_{ij}.$$

We assume that the energy-momentum tensor of a perfect fluid is given by

(2.2) $$T_{ij} = (p + \rho)u_iu_j + pg_{ij}.$$

Also we take the equation of state as

(2.3) $$p = \omega\rho,$$

where $0 \leq \omega \leq 1$, p and ρ are respectively pressure and energy density of the cosmic fluid and u_i is unit flow vector such that $u_i = (1,0,0,0)$.

In view of equation (1.6), the Einstein's field equations (2.1) reduce to the form

(2.4) $$R_{ij} = 8\pi GT_{ij} - \Lambda g_{ij}.$$

For the metric (1.1) and the energy-momentum tensor (2.4), the Einstein's field equations (2.4) reduce to

(2.5) $$-3(\dot{H} + H^2) = 8\pi G\rho + \Lambda,$$

and

(2.6) $$\dot{H} + 3H^2 + \frac{2k}{a^2} = 8\pi Gp - \Lambda,$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Here and elsewhere an over-head dot denotes ordinary differentiation with respect to cosmic time t. For the flat($k = 0$) FRW metric (1.1), the equation (2.6) reduces to

(2.7) $$\dot{H} + 3H^2 = 8\pi Gp - \Lambda.$$

3. Solution of Field Equations

The system of equations (2.3), (2.5) and (2.7) give only three equations in four unknowns H, p, ρ and Λ. Hence one extra equation is needed to solve the system completely. There are significant theoretical evidence for the phenomenological Λ-decay scenarios which are considered by many authors in different ways such as Wang and Meng[15], Ray *et. al.*[16] Recently, Kumar and Srivastava[11] have studied FRW cosmological model for conharmonically flat spacetime. We now consider the following special law for the decay of Λ-cosmological term[17,18]

(3.1) $$\Lambda = 3\beta H^2,$$

where β is a constant. Here β represents the ratio between vacuum and critical densities.
From equations (2.3), (2.5), (2.7) and (3.1), we obtain

(3.2) $$\dot{H} + \frac{3(1+\beta)(1+\omega)}{1+3\omega} H^2 = 0.$$

Integrating (3.2) with respect to t yields

(3.3) $$H(t) = \frac{1+3\omega}{\{3(1+\beta)(1+\omega)t + c_1(1+3\omega)\}},$$

where c_1 is a constant of integration.
From equation (3.3), we can easily obtain the value of the scale factor $a(t)$ as follows:

(3.4) $$a(t) = c_2\{3(1+\beta)(1+\omega)t + c_1(1+3\omega)\}^{\frac{1+3\omega}{3(1+\beta)(1+\omega)}},$$

where c_2 is a constant of integration. Such constant of integration is related to the choice oforigin of cosmic time.

Now we analyze scenarios for different values of ω as follows:

A. Matter Dominated Era: For $\omega = 0$, from equation (3.4) we obtain the scale factor as

(3.5) $$a(t) = c_2\{3(1+\beta)t + c_1\}^{\frac{1}{3(1+\beta)}}.$$

In this case the spatial volume V, matter density ρ, isotropic pressure p and cosmological constant $\Lambda$are given by

(3.6) $$V = 2\pi^2 c_2^3\{3(1+\beta)t + c_1\}^{\frac{1}{(1+\beta)}},$$

(3.7) $$\rho = \frac{1}{8\pi G}\frac{6(1+\beta)}{\{3(1+\beta)t+c_1\}^2},$$

(3.8) $$p = 0,$$

(3.9) $$\Lambda = \frac{3\beta}{\{3(1+\beta)t+c_1\}^2}.$$

The expansion scalar Θ and density parameter Ω are given by

(3.10) $$\Theta = \frac{3}{3(1+\beta)t+c_1},$$

and

(3.11) $$\Omega = 2(1+\beta).$$

The deceleration parameter q is defined as $q = -1 - \frac{\dot{H}}{H^2}$. Thus we have the following

(3.12) $$q = 2 + 3\beta.$$

The deceleration parameter q is dimensionless and a negative value of q represents cosmic acceleration where as a positive q gives decelerating universe. Equations (3.11) and (3.12) together yield

(3.13) $$\Omega = \frac{2}{3}(q+1).$$

The vacuum energy density ρ_v and the critical energy density ρ_c are given by

(3.14) $$\rho_v = \frac{3\beta}{8\pi G\{3(1+\beta)t+c_1\}^2},$$

and

$$\rho_c = \frac{3}{8\pi G\{3(1+\beta)t+c_1\}^2}. \tag{3.15}$$

From equations (3.5) and (3.6) of the physical parameters of the model, we observe that the scale factor $a(t)$ and the spatial volume V are zero at $t = t_1$, where $t_1 = -\frac{c_1}{3(1+\beta)}$. It follows that the universe under the considered model has a finite-time big-bang singularity at $t = t_1$ which shifts to initial singularity by setting $c_1 = 0$. Here the Hubble parameter diverges at the time $t = t_1$. Moreover, the expansion scalar Θ, isotropic pressure p and matter density ρ all diverge at the time $t = t_1$. Also, in the limiting case for large t, all the physical parameters $H(t)$, Θ, p and ρ converge to zero where as the scale factor $a(t)$ and the volume V become infinite. Thus the considered model in this case represents a cosmological scenario in which the universe starts from a finite-time big-bang singular state and expands with cosmic time t. From equation (3.12) we observed that for $\beta > -\frac{2}{3}$ the value of the deceleration parameter q is positive which indicates the decelerating phase of expansion of the observed universe. This is mainly responsible for structure formation[19]. For $\beta < -\frac{2}{3}$ the current model indicates the accelerating phase of expansion. Recent observations favor accelerating models but they do not rule out decelerating models which are also consistent with present day observations.

B. Stiff Fluid Era: In this case, from equation (3.4), the scale factor $a(t)$ becomes

$$a(t) = c_2\{6(1+\beta)t + 4c_1\}^{\frac{2}{3(1+\beta)}}. \tag{3.16}$$

Here spatial volume V, matter density ρ, isotropic pressure p and cosmological constant Λ are as follows:

$$V = 2\pi^2 c_2^3\{6(1+\beta)t + 4c_1\}^{\frac{2}{(1+\beta)}}, \tag{3.17}$$

$$\rho = p = \frac{1}{8\pi G}\frac{6(1+\beta)}{\{3(1+\beta)t+2c_1\}^2}, \tag{3.18}$$

and

$$\Lambda = \frac{12\beta}{\{3(1+\beta)t+2c_1\}^2}. \tag{3.19}$$

The expansion scalar Θ and density parameter Ω are given by

(3.20) $$\Theta = \frac{6}{3(1+\beta)t+2c_1},$$

and

(3.21) $$\Omega = \frac{1}{2}(1+\beta).$$

The deceleration parameter q is defined as $q = -1 - \frac{\dot{H}}{H^2}$. Thus, in this case, we have the following

(3.22) $$q = \frac{1}{2} + \frac{3\beta}{2}.$$

Equations (3.21)and (3.22) together give

(3.23) $$\Omega = \frac{1}{3}(q+1).$$

From equations (3.16) and (3.17) of the physical parameters of the model, we observe that the scale factor $a(t)$ and the spatial volume V are zero at $t = t_2$, where $t_2 = -\frac{2c_1}{3(1+\beta)}$. It follows that the universe under the considered model has a finite-time big-bang singularity at $t = t_2$ which shifts to initial singularity by setting $c_1 = 0$. Here the Hubble parameter diverges at the time $t = t_2$. Moreover, the expansion scalar Θ, isotropic pressure p and matter density ρall diverge at the time $t = t_2$. Also, in the limiting case for large t, all the physical parameters $H(t)$, Θ, p and ρ converge to zero where as the scale factor $a(t)$ and the volume V become infinite. Thus the considered model in this case represents a cosmological scenario in which the universe starts from a finite-time big-bang singular state and expands with cosmic time t. From equation (3.22) we observed that for $\beta > -\frac{1}{3}$ the value of the deceleration parameter q is positive which indicates the decelerating phase of expansion of the observed universe. This is mainly responsible for structure formation[19]. For $\beta < -\frac{1}{3}$ the current model indicates the accelerating phase of expansion. Recent observations favor accelerating models but they do not rule out decelerating models which are also consistent with present day observations.

4. Some Cosmological Parameters

A. Cosmological Red-Shift: When an object goes away radiating signals (light or sound), frequency of signals decreases and the wavelength increases. In the visible region of a spectrum of light, red has the highest wavelength and blue has lowest wavelength. So, in case, the wavelength of pulses increases and we have red-shift. When wavelength decreases, we have blue-shift. Red-shift is a very useful parameter of observational cosmology. It gives spectral shift in wavelength, when a source of radiation moves away. Now we discuss red-shift in the cosmological context for conharmonically flat FRW model. Light travels from a galaxy to us. As the universe expands the wavelength of the light stretches in proportion to the amount of expansion of the universe. This means that if λ_e is the wavelength of the emitted light and λ_0 is the wavelength of light we observe, then

$$\frac{\lambda_0}{\lambda_e} = \frac{a(t_0)}{a(t_e)}, \tag{4.1}$$

where t_e is the time at which the galaxy emitted the light. Red-shift is quantified by the relative change in wavelength z, which is expressed as

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e}. \tag{4.2}$$

From (4.1) and (4.2), we obtain

$$1 + z = \frac{a(t_0)}{a(t_e)} = \frac{a_0}{a}. \tag{4.3}$$

B. Statefinder Parameters: Statefinder parameters $\{r, s\}$ play a significant role to describe the dynamics of the Universe in modern cosmological model. In this section we present the definition of the statefinder parameters and calculate them for projectively flat FRW model. It is to be noted that statefinder parameters solved the problem of discriminating between the various candidates of dark energy model. It is natural to look whether the above mentioned models agree with cosmological constant cold dark matter(ΛCDM) model which is considered as one of the most acceptable model in the present day observations. In an endeavor to discuss the cosmological scenarios of the universe Sahni[20] and Alam et al.[21] have introduced a pair of dimensionless geometrical parameters $\{r, s\}$ popularly known as Statefinder parameters. For a (ΛCDM) model, the Statefinder parameters $\{r, s\}$ have the value $\{1,0\}$. The Statefinder parameters may be defined as follows:

(4.4) $$r = \frac{\dddot{a}(t)}{a(t)H^3(t)},$$

and

(4.5) $$s = \frac{r-1}{3\left(q-\frac{1}{2}\right)},$$

where $a(t)$ is the expansion scale factor, $H(t)$ is the Hubble's parameter, q is the deceleration parameter and over head dot indicates differentiation with respect to the cosmic time t. The parameter r is also named as jerk parameter or as jolt or as super acceleration.

Hence for considered model, we have the jerk parameter as

(4.6) $$r = \frac{(2+3\beta+3\beta\omega)(5+6\beta+3\omega+6\beta\omega)}{(1+3\omega)^2}.$$

In the case of Radiation dominated solution we obtain

(4.7) $$r = (1+2\beta)(3+4\beta).$$

For $\beta = -1$, equation (4.7) yields $r = 1$. Consequently, from (4.5) we have $s = 0$. Thus, we obtain $\{r, s\} = \{1,0\}$ which shows that the model is in agreement with ΛCDM model.

C. Cosmic snap parameter: The cosmic snap parameter plays a key role in cosmology to obtain the models like ΛCDM. The present day value of cosmic snap parameter may be used to characterize the evolutionary status of the universe. This parameter is defined as[22,23]

(4.8) $$s^* = \frac{1}{a(t)H^4}\frac{d^4a(t)}{dt^4}.$$

The cosmic snap parameter is dimensionless and we can write

(4.9) $$a(t) = a_0\left\{1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2 + \frac{1}{6}r_0H_0^3(t-t_0)^3 + \frac{1}{24}s_0^*H_0^4(t-t_0)^4 + O[(t-t_0)^5]\right\},$$

where subscript 0 denotes the present day value of the concerned quantity. Equation (4.8) can also be represented as

(4.10) $$s^* = \frac{\dot{r}}{H} - r(2+3q),$$

where q is the deceleration parameter and r is the jerk parameter. The cosmic snap parameter (the fourth time derivative) is also sometimes called jounce.

Since in the considered model $r = 1$, then equation (4.10) yields $s^* = -(2 + 3q)$ and consequently $\frac{ds^*}{dq} = -3$, which implies a measure of the evolution of the universe deviating from ΛCDM model[24,25]. For the present model $\beta = -1$ implies $q = -1$. Consequently equation (4.10) gives $\dot{s^a} = 1$. Thus the considered model has too much similarity with recent day observations.

D. Om parameter: In 2003, Sahni et al[20] proposed a new cosmological parameter named Om which was introduced to differentiate ΛCDM from other dark energy models. The Om diagnostic method is actually a geometrical diagnostic which combines Hubble parameter and red-shift. Sahni and his collaborators demonstrated that irrespective to matter density content of universe acceleration probe can discriminate various dark energy models[26]. Om parameter is defined as follows:

$$Om(z) = \frac{\left[\frac{H(z)}{H_0}\right]^2 - 1}{(1+z)^3 - 1}.$$

Phantom like dark energy corresponds to the positive slope of $Om(z)$ whereas the negative slope means dark energy behaves like quintessence[27]. Also growth $Om(z)$ at late time favors the decaying dark energy models[20].

5. Conclusions

In the present paper we have investigated W_2-flat spacetimes in the framework of FRW cosmological model. We have considered here perfect fluid as a source of matter distribution of the universe. Einstein's field equations with variable cosmological constant are solved by using special variation law $\Lambda = 3\beta H^2$, where β is a constant. These solutions describe a cosmological scenario in which the universe in the present model has finite-time big-bang singularity at some finite times $t = -\frac{c_1}{3(1+\beta)}$ and $t = -\frac{2c_1}{3(1+\beta)}$ for matter dominated solution and stiff era respectively. Also we observed that the universe expands with cosmic time t. In this case we have found that Hubble parameter H, isotropic pressure p, energy density ρ, expansion scalar Θ and cosmological constant Λ all diverge at certain time with scale factor $a(t)$ andspatial volume V zero and the parameters $(p, \rho, H, \Theta, \Lambda)$ infinite whereas all these parameters $(p, \rho, H, \Theta, \Lambda)$ converge to zero as t tends to infinity with V and $a(t)$ infinite. These facts are in accordance with the present day observations. We also observe that the estimated values of

Statefinder parameters $\{r, s\}$ and cosmic snap parameter s^* are agree with Λ CDM model. Moreover, we characterize red-shift parameter and Om parameter in W_2-flat FRW spacetime model. Also different values of deceleration parameter indicates that the universe goes through deceleration and acceleration phase of expansion. These phenomena are consistent with the present day observations. Thus the present model has good consistency with recent cosmological studies and observations that make the model more objective.

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