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Lie-Derivative of a Linear Connexion and Various Kinds of Motions in a Kaehlerian Recurrent Space of First Order

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Abstract: Knebelman[1] studied and defined collineations and motions in generalized spaces. Levine[2] studied motions in linearly connected two dimensional spaces. Tokano[3] studied on the existence of Affine motion in a space with recurrent curvature tensor. Further, Negi and Rawat[4] studied Affine motion in an Almost Tachibana recurrent space. Rawat and Silswal[5] studied Theory of Lie-derivatives and motions in Tachibana space. In the present paper, we have studied Lie-derivative of a linear connexion and various kinds of motions (Affine motion, Projective motion and Conformal motion) in a Kaehlerian recurrent space of first order also several theorems have been established and proved therein.

Keywords: Collineation, Affine motion, Recurrent curvature tensor, Kaehlerian recurrent space
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1. Introduction

Let X_{2n} be a 2n-dimensional almost complex space and F_j^i its almost-complex structure, then by definition, we have

$$(1.1) \qquad F_j^s F_s^i = -\delta_j^i.$$

An almost- complex space with a positive definite Riemannian metric g_{ji} satisfying

$$(1.2) \qquad g_{rs} F_j^r F_i^s = g_{ji},$$

is called an almost- Hermitian space. From (1.2), it follows that $F_{ji} = g_{ri}F_j^r$ is skew- symmetric.
If an almost- Hermitian space satisfies

$$(1.3) \quad \nabla_j F_{ih} + \nabla_i F_{hj} + \nabla_h F_{ji} = 0,$$

where ∇_j denotes the operator of Covariant derivative with respect to the symmetric Riemannian Connection, then it is called an almost-Kaehlerian space and if it satisfies

$$(1.4) \quad \nabla_j F_{ih} + \nabla_i F_{hj} = 0,$$

then it is called a K- space. In an almost- Hermitian space, if

$$(1.5) \quad \nabla_j F_{ih} = 0 \text{ or } F_{ih,j} = 0,$$

then it is called a Kaehlerian space or, briefly a K_n - space.

A Kaehler space K_n satisfying the relation

$$R^h_{ijk.a} - \lambda_a R^h_{ijk} = 0 \text{ or}$$

$$(1.6) \quad \nabla_a R^h_{ijk} - \lambda_a R^h_{ijk} = 0,$$

for some non-zero vector λ_a, will be called a Kaehlerian recurrent space of first order. It is called Ricci-recurrent (or, semi-recurrent) space of first order, if it satisfies

$$R_{ij.a} - \lambda_a R_{ij} = 0 \text{ or}$$

$$(1.6a) \quad \nabla_a R_{ij} - \lambda_a R_{ij} = 0.$$

Multiplying the above equation by g^{ij}, we have

$$R_{.a} - \lambda_a R = 0 \text{ or}$$

(1.6b) $\nabla_a R - \lambda_a R = 0.$

Remark (1.1). From (1.6) and (1.6a), it follows that every Kaehlerian recurrent space of first order is Ricci-recurrent space of first order, but the converse is not necessarily true.

2. Lie-Derivative of a Linear Connexion

Let us consider a L_n (i.e. the space provided with a linear connexion $\Gamma^{\chi}_{\mu\lambda}(\xi)$ an infinitesimal point transformation

(2.1) $'\xi^{\chi} = \xi^{\chi} - v^{\chi}(\xi)dt$,

the deform of a contravariant vector u^{χ} is defined by

(2.2) $'u^{\chi'}(\xi) \overset{def}{=} u^{\chi}('\xi)$

and that of the linear connexion $\Gamma^{\chi}_{\mu\lambda}$ by

(2.3) $'\Gamma^{\chi'}_{\mu'\lambda'}(\xi) \overset{def}{=} \Gamma^{\chi}_{\mu\lambda}('\xi).$

If we now denote by, δ the covariant differential with respect to $\Gamma^{\chi}_{\mu\lambda}$ and by $'\delta$ the covariant differential with respect to $'\Gamma^{\chi}_{\mu\lambda}$, we have

$$\begin{aligned} '\delta' u^{\chi}(\xi) &= d'u^{\chi'}(\xi) + '\Gamma^{\chi'}_{\mu'\lambda}(\xi)u^{\lambda'}(\xi)d\xi^{\mu'} \\ &= du^{\chi}('\xi) + \Gamma^{\chi}_{\mu\lambda}('\xi)u^{\lambda}('\xi)d'\xi^{\mu} \\ &= \delta u^{\chi}('\xi). \end{aligned}$$

On the other hand, for the deform of δu^{χ}, we have

$$'\left(\delta' u^{\chi}(\xi)\right) = \delta u^{\chi}('\xi).$$

From these two equations, we have

(2.4) $\qquad {}'\delta' u^{\chi} = {}'\delta u^{\chi}{}'({}'\xi),$

holding with respect to every coordinate system and consequently

(2.5) $\qquad {}'\delta\left(u^{\chi} + \underset{\upsilon}{\pounds} u^{\chi}\right)dt = \delta u^{\chi} + \underset{\upsilon}{\pounds} u^{\chi} dt$

with respect to (x). Thus, we have

Theorem 2.1. *The covariant differential of the deform of a contravariant vector with respect to the deform linear connexion is equal to the deform of the covariant differential of the vector with respect to the original linear connexion.*

Since

$$'\delta\left(u^{\chi} + \underset{\upsilon}{\pounds} u^{\chi} dt\right) = d\left(u^{\chi} + \underset{\upsilon}{\pounds} u^{\chi} dt\right) + \left(\Gamma^{\chi}_{\mu\lambda} + \underset{\upsilon}{\pounds}\Gamma^{\chi}_{\mu\lambda} dt\right)\left(u^{\chi} + \underset{\upsilon}{\pounds} u^{\chi} dt\right) d\xi^{\mu}$$

$$= \delta u^{\chi} + \delta\underset{\upsilon}{\pounds} u^{\chi} dt + \left(\underset{\upsilon}{\pounds}\Gamma^{\chi}_{\mu\lambda}\right) u^{\lambda} d\xi^{\mu} dt.$$

We have from (2.5)

(2.6) $\qquad \underset{\upsilon}{\pounds}\delta u^{\chi} - \delta\underset{\upsilon}{\pounds} u^{\chi} = \left(\underset{\upsilon}{\pounds}\Gamma^{\chi}_{\mu\lambda}\right) u^{\lambda} d\xi^{\mu}.$

Taking account of $\underset{\upsilon}{\pounds}\xi^{\chi} = 0$, we have from (2.6)

(2.7) $\qquad \underset{\upsilon}{\pounds}\nabla_{\mu} u^{\chi} - \nabla_{\mu}\underset{\upsilon}{\pounds} u^{\chi} = \left(\underset{\upsilon}{\pounds}\Gamma^{\chi}_{\mu\lambda}\right) u^{\lambda}.$

Formula (2.7) can be generalized for a covariant vector ω_{λ} and for a general tensor $P^{\chi\lambda}_{\mu}$ as follows:

(2.8) $\qquad \underset{\upsilon}{\pounds}\nabla_{\mu}\omega_{\lambda} - \nabla_{\mu}\underset{\upsilon}{\pounds}\omega_{\lambda} = -\left(\underset{\upsilon}{\pounds}\Gamma^{\chi}_{\mu\lambda}\right)\omega_{\chi}.$

(2.9) $\qquad \underset{\upsilon}{\pounds}\nabla_{\mu} P^{\lambda\mu}_{\chi} - \nabla_{\mu}\underset{\upsilon}{\pounds} P^{\lambda\mu}_{\chi} = \left(\underset{\upsilon}{\pounds}\Gamma^{\chi}_{\nu\rho}\right) P^{\rho\lambda}_{\mu} + \left(\underset{\upsilon}{\pounds}\Gamma^{\lambda}_{\mu\rho}\right) P^{\chi\rho}_{\mu} - \left(\underset{\upsilon}{\pounds}\Gamma^{\rho}_{\nu\mu}\right) P^{\chi\lambda}_{\rho}.$

From these equations, we have

Theorem 2.2. *In order that (2.1) be an affine motion in an* L_n, *it is necessary and sufficient that the covariant differentiation and the Lie derivation with respect to (2.1) be commutative.*

Now since the deformed linear connexion is given by

$$(2.10)\qquad '\Gamma^{x}_{\mu\lambda} = \Gamma^{x}_{\mu\lambda} + \underset{\upsilon}{\pounds}\Gamma^{x}_{\mu\lambda}dt$$

it follows immediately that

$$(2.11)\qquad 'S^{x}_{\mu\lambda} = S^{x}_{\mu\lambda} + \underset{\upsilon}{\pounds}S^{x}_{\mu\lambda}dt$$

It is also evident that the deformed curvature tensor is given by

$$(2.12)\qquad 'R^{x}_{\nu\mu\lambda} = R^{x}_{\nu\mu\lambda} + \underset{\upsilon}{\pounds}R^{x}_{\nu\mu\lambda}dt.$$

In fact substituting (2.10) into

$$'R^{x}_{\nu\mu\lambda} = 2\partial_{\nu}\Gamma^{x}_{\mu\lambda} + 2'\Gamma^{x}_{\nu\rho}{}'\Gamma^{\rho}_{\mu\lambda}.$$

We find

$$(2.13)\qquad 'R^{x}_{\nu\mu\lambda} = R^{x}_{\nu\mu\lambda} + \left(\nabla_{\mu}\underset{\upsilon}{\pounds}\Gamma^{x}_{\mu\lambda} - \nabla_{\mu}\underset{\upsilon}{\pounds}\Gamma^{x}_{\nu\lambda} + 2S^{\rho}_{\nu\mu}\underset{\upsilon}{\pounds}\Gamma^{x}_{\mu\lambda}\right)dt.$$

On the other hand, by virtue of Ricci identity

$$2\nabla_{\nu}\nabla_{\mu}\upsilon^{x}_{\lambda} = R^{x}_{\nu\mu\rho}\upsilon^{\rho}_{\lambda} - R^{\rho}_{\nu\mu\lambda}\upsilon^{x}_{\rho} - 2S^{\rho}_{\nu\mu}\nabla_{\rho}\upsilon^{x}_{\lambda}$$

and of the second Bianchi identity

$$\nabla_{\nu}R^{x}_{\nu\mu\rho} = 2S^{\sigma}_{\nu\mu}R^{x}_{\mu\sigma\lambda}.$$

We find

$$\nabla_{\mu}\underset{\upsilon}{\pounds}\Gamma^{x}_{\mu\lambda} - \nabla_{\mu}\underset{\upsilon}{\pounds}\Gamma^{x}_{\nu\lambda} + 2S^{\rho}_{\nu\mu}\underset{\upsilon}{\pounds}\Gamma^{x}_{\mu\lambda} = \nabla^{\rho}\nabla_{\rho}R^{x}_{\nu\mu\lambda} - R^{\rho}_{\nu\mu\lambda}\upsilon^{x}_{\rho} + R^{x}_{\rho\mu\lambda}\upsilon^{\rho}_{\nu}$$
$$+ R^{x}_{\nu\rho\lambda}\upsilon^{\rho}_{\mu} + R^{x}_{\nu\mu\rho}\upsilon^{\rho}_{\lambda}$$

or

(2.14) $\nabla_\nu \underset{\upsilon}{\pounds}\Gamma^x_{\mu\lambda} - \nabla_\mu \underset{\upsilon}{\pounds}\Gamma^x_{\nu\lambda} - 2S^\rho_{\nu\mu}\underset{\upsilon}{\pounds}\Gamma^x_{\rho\lambda} = \underset{\upsilon}{\pounds}R^x_{\nu\mu\lambda}$.

3. Motions in a Kaehlerian Recurrent Space of First Order

Let us consider $n(=2m)$ dimensional Kaehlerian space K_n covered by a set of neighbourhoods with coordinates ξ^x and endowed with the fundamental quadratic differential form

(3.1) $ds^2 = g_{\lambda x}(\xi)d\xi^\lambda d\xi^x$,

where the indices x, λ, μ, ν ......, run over the range 1,2,3,.........., n. In the space K_n referred to ξ^x, we consider a point transformation

(3.2) $T: {'\xi^x} = f^x(\xi^\nu) : Det(\partial_\lambda \xi^x) \neq 0.$

which establishes a one-to-one correspondence between the points of a region R and those of some other region $*R$, where ∂_λ stands for the partial derivation $\frac{\partial}{\partial \xi^\lambda}$.

During this point transformation, a point ξ^x in R is carried to a point $^*\xi$ in $*R$ and a point $\xi^x + d\xi^x$ in R to a point $*\xi^x + d\xi^x$ in $*R$. If the distance d^*s between two displaced points $*\xi^x$ and $*\xi^x + d\xi^x$ is always equal to the distance between the two original points ξ^x and $\xi^x + d\xi^x$ the point transformation (3.2) is called a ***motion or an isometry*** in the space K_n .

(I). Affine Motion in K_n

Consider a Kaehlerian space K_n provided with a linear connexion $\Gamma^x_{\mu\lambda}(\xi)$. In a K_n the parallelism between a vector u^x at a point ξ^x and a vector $u^x + du^x$ at a point $\xi^x + d\xi^x$ is defined by

(3.3) $\delta u^x \overset{def}{=} du^x + \Gamma^x_{\mu\lambda}u^\lambda d\xi^\mu = 0.$

When we effect a point transformation (3.2), the differentials $d\xi^{x}$ at ξ^{x} are transformed into the differentials

$$(3.4) \qquad d^{*}\xi^{x} = \frac{\partial f^{x}}{\partial \xi^{\nu}} d\xi^{\nu}$$

at $^{*}\xi^{x}$. Now if we make the condition that the vector u^{x} at ξ^{x} is transformed from ξ^{x} to $^{*}\xi^{x}$ in the same way as the linear elements $d\xi^{x}$ at ξ^{x}, then the corresponding vector $*\xi^{x}$ is

$$(3.5) \qquad \overset{m}{u^{x}}\, {}^{*}\xi = \frac{\partial f^{x}}{\partial \xi^{\nu}} u^{\nu}(\xi).$$

Definition. When a point transformation (3.2) transforms any pair of parallel vector into a pair of parallel vectors, then (3.2) is called affine motion in K_n.

For an affine motion, we must have

$$(3.6) \qquad \delta^{m} u^{x}({}^{*}\xi) \overset{def}{=} d \overset{m}{u^{x}}({}^{*}\xi) + \Gamma^{x}_{\mu\lambda}({}^{*}\xi) \overset{m}{u^{\lambda}}({}^{*}\xi) d\xi^{\mu} = 0.$$

(ii). Projective Motion in K_n.

Let us consider a Kaehlerian space K_n with a symmetric linear connexion $\Gamma^{x}_{\mu\lambda}$. The geodesic of the space is given by

$$(3.7) \qquad \frac{d^2\xi^{x}}{dt^2} + \Gamma^{x}_{\mu\lambda} \frac{d\xi^{\mu}}{dt}\frac{d\xi^{\lambda}}{dt} = \alpha(t)\frac{d\xi^{x}}{dt}.$$

Definition. When a point transformation (3.2) transforms the system of geodesic into the same system, then (3.2) is called a projective motion in K_n.

The necessary and sufficient condition that (3.2) be a projective motion in a K_n is that the Lie-difference of $\Gamma^{x}_{\mu\lambda}$ with respect to (3.2) has the form

$$(3.8) \qquad {}^{*}\Gamma^{x}_{\mu\lambda} - \Gamma^{x}_{\mu\lambda} = A^{x}_{\mu} p_{\lambda} + A^{x}_{\lambda} p_{\mu},$$

where p_λ is a covariant vector.
When (3.2) is an infinitesimal transformation

$$(3.9) \qquad {}^*\xi^\chi = \xi^\chi + \upsilon^\chi(\xi)dt,$$

then the condition is

$$(3.10) \qquad \underset{\upsilon}{£}\Gamma^\chi_{\mu\lambda} = A^\chi_\mu p_\lambda + A^\chi_\lambda p_\mu.$$

(iii). Conformal Motion in K_n.

Definition. When a point transformation (3.2) does not change the angle between two directions at a point, then (3.2) is called a Conformal motion in the K_n.

The necessary and sufficient condition that (3.2) be a conformal motion in a K_n is that the Lie-difference of $g_{\lambda\chi}$ with respect to (3.2) be proportional to $g_{\lambda\chi}$.

$$(3.11) \qquad {}^*g_{\lambda\chi} - g_{\lambda\chi} = 2\phi g_{\lambda\chi},$$

where ϕ is a scalar.
When (3.2) is an infinitesimal transformation, then the condition is

$$(3.12) \qquad \underset{\upsilon}{£} g_{\lambda\chi} = 2\phi g_{\lambda\chi}.$$

Thus, we have

Theorem (3.1). *A necessary and sufficient condition that (3.9) be a conformal motion in a space K_n is that the Lie-derivative of $^*g_{\lambda\chi}$ be a multiple of $g_{\lambda\chi}$.*

Theorem (3.2). *A motion in a Kaehlerian space K_n is an affine motion.*

Proof. To prove this, we apply the formula (2.9) to the fundamental tensor $g_{\lambda\chi}$,

$$\underset{\upsilon}{\pounds}(\nabla_{\mu} g_{\lambda\chi}) - \nabla_{\mu}\left(\underset{\upsilon}{\pounds} g_{\lambda\chi}\right) = -\left(\underset{\upsilon}{\pounds}\left\{ {\rho \atop \mu\ \lambda} \right\}\right) g_{\rho\chi} - \left(\underset{\upsilon}{\pounds}\left\{ {\rho \atop \mu\ \chi} \right\}\right) g_{\lambda\rho},$$

from which

$$(3.13) \qquad \underset{\upsilon}{\pounds}\left\{ {\chi \atop \mu\ \lambda} \right\} = \frac{1}{2} g^{\chi\rho}\left[\nabla_{\mu} \underset{\upsilon}{\pounds} g_{\lambda\rho} + \nabla_{\lambda} \underset{\upsilon}{\pounds} g_{\mu\rho} - \nabla_{\rho} \underset{\upsilon}{\pounds} g_{\mu\lambda}\right].$$

This equation shows that $\underset{\upsilon}{\pounds} g_{\lambda\chi} = 0$ implies $\underset{\upsilon}{\pounds}\left\{ {\chi \atop \mu\ \lambda} \right\} = 0.$

Note: Under some global conditions $\underset{\upsilon}{\pounds}\left\{ {\chi \atop \mu\ \lambda} \right\} = 0.$ implies $\underset{\upsilon}{\pounds} g_{\lambda\chi} = 0.$

Theorem (3.3). *For a motion in a Kaehlerian space* K_n *the Lie-derivatives of the curvature tensor and its successive covariant derivatives vanish.*

Proof. Applying the formula (2.14) to the Christoffel symbol, we have

$$(3.14) \qquad \nabla_{\nu} \underset{\upsilon}{\pounds}\left\{ {\rho \atop \mu\ \lambda} \right\} - \nabla_{\mu} \underset{\upsilon}{\pounds}\left\{ {\rho \atop \nu\ \lambda} \right\} = \underset{\upsilon}{\pounds} K^{\rho}_{\nu\mu\lambda},$$

where $K^{\chi}_{\nu\mu\lambda}$ is the curvature tensor of K_n. Thus for a motion, we have

$$(3.15) \qquad \underset{\upsilon}{\pounds} K^{\chi}_{\nu\mu\lambda} = 0.$$

On the other hand since a motion is an affine motion, the covariant derivation and the Lie-derivation are commutative. Thus from (3.15) , we obtain

$$(3.16) \qquad \underset{\upsilon}{\pounds} \nabla_{\omega} K^{\chi}_{\nu\mu\lambda} = 0,\ \underset{\upsilon}{\pounds} \nabla_{\omega 1} \nabla_{\omega 2} K^{\chi}_{\nu\mu\lambda} = 0, \ldots\ldots\ldots$$

This proves the theorem.

4. Theorems on Projectively or Conformally Related Spaces

Theorem (4.1). *If two Kaehlerian spaces* K_n *and* $*K_n$ *are in geodesic correspondence and if* K_n *admits a group of motions,* $*K_n$ *also admits a group of motions.*

Proof. Considering two Kaehlerian spaces K_n and $*K_n$ which are in geodesic correspondence. Then denoting the Christoffel symbols of them by $\left\{ {\chi \atop \mu\ \lambda} \right\}$ and $*\left\{ {\chi \atop \mu\ \lambda} \right\}$ respectively, we have

$$(4.1)\qquad {}^{*}\left\{ {\chi \atop \mu\ \lambda} \right\} = \left\{ {\chi \atop \mu\ \lambda} \right\} + A_{\mu}^{\chi} p_{\lambda} + A_{\lambda}^{\chi} p_{\mu}.$$

But since K_n and $*K_n$ are both Kaehlerian, the vector ϕ_λ should be a gradient. Thus putting $p_\lambda = \frac{1}{2}\partial_\lambda \log\phi$, we get

$$(4.2)\qquad {}^{*}\left\{ {\chi \atop \mu\ \lambda} \right\} = \left\{ {\chi \atop \mu\ \lambda} \right\} + \frac{1}{2}A_{\mu}^{\chi}\partial_\lambda \log\phi + \frac{1}{2}A_{\lambda}^{\chi}\partial_\mu \log\phi.$$

We now assume that the space K_n admits a motion with symbol $\underset{v}{\pounds} f$. Then, we have

$$\underset{v}{\pounds} g_{\lambda\chi} = \nabla_\lambda v_\chi + \nabla_\chi v_\lambda = \partial_\lambda v_\chi + \partial_\chi v_\lambda - 2\left\{ {\rho \atop \lambda\ \chi} \right\} = 0.$$

Consequently on using (4.2), we have

$$\underset{v}{\pounds} g_{\lambda\chi} = \nabla_\lambda v_\chi + \nabla_\chi v_\lambda = \partial_\lambda v_\chi + \partial_\chi v_\lambda - \left[2\ {}^{*}\left\{ {\rho \atop \lambda\ x} \right\} - \frac{1}{2}A_{\chi}^{\rho}\partial_\lambda \log\phi - \frac{1}{2}A_{\lambda}^{\rho}\partial_\chi \log\phi \right] v_\rho$$

$$= \phi^{-1}\left[\partial_\lambda(\phi v_\chi) + \partial_\chi(\phi v_\lambda) - 2\ {}^{*}\left\{ {\rho \atop \lambda\ x} \right\} \phi v_\rho \right].$$

Thus denoting by $*g_{\lambda\chi}$ the fundamental tensor of $*K_n$ and $\underset{\upsilon}{{}^*\pounds} f = 0$ the symbol defined by $\phi\upsilon_\chi$ in $*K_n$, we have

$$\underset{\upsilon}{\pounds} g_{\lambda\chi} = \phi^{-1}\, {}^*\underset{\upsilon}{\pounds}\, {}^*g_{\lambda\chi}.$$

Theorem (4.2). *If a* K_n *admits a* G_r *of motions such that the rank of* υ_a^χ *in a neighborhood is equal to* $r < n$, *then there exist* $n-r$ K'^s *corresponding to* $n-r$ *independent solutions of* $\underset{\upsilon}{\pounds}\rho^2 = 0$, *which are conformal to the given* K_n *and admit the same group as a group of motions*

Proof. Let us consider a Kaehlerian space K_n which admits a r-parameter group G_r of motions such that the rank of υ_a^χ is in a certain neighbourhood is equal to $r < n$. Then, we have $\underset{\upsilon}{\pounds} g_{\lambda\chi} = 0$. In the same group G_r as a group of motions, it is necessary and sufficient that there exist a function ρ^2 such that $\underset{\upsilon}{\pounds}\left(\rho^2 g_{\lambda\chi}\right) = 0$ or $\underset{\upsilon}{\pounds}\rho^2 = 0$.

But on the other hand $\left(\underset{c}{\pounds}\underset{b}{\pounds}\right)\rho^2 = c^a_{cb}\underset{a}{\pounds}\rho^2$ and consequently $\underset{\upsilon}{\pounds}\rho^2 = 0$ admits $n-r$ independent solutions.

Note: The a-rank of the $\underset{\upsilon}{\pounds} g_{\lambda\chi}$ is the rank of the matrix $\underset{\upsilon}{\pounds} g_{\lambda\chi}$ where a denotes the rows and λ_χ denotes the columns.

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