

Strong Explosions in Two - Phase mixture of Radiating Gases

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Abstract: In present paper, strong – explosions about a line and a point in two-phase flows, when radiation effect is taken into account, are discussed. Similarity method discussed by Taylor and Sedov is used to find the solution which reduces the governing non- linear equations into ordinary differential equations. Using finite difference and Runge Kutta method, variation of pressure for cylindrical and spherical cases is obtained, and obtained results are compared through graphs.

Key words & Phases: strong – explosions, two- phase, radiation.

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Nomenclatures

u – velocity of medium

$\bar{\sigma}$ – gas concentration

$\bar{\sigma} P$ – particle concentration

ρ – gas density

ρ_p – particle density

ρ_M – mixture density

p_M – mixture pressure

p_R – radiation pressure = $R_p p_M$

R_p – radiation pressure number

p – total pressure = $p_M + p_R$

T – temperature

Φ – particle mass fraction

γ – ratio of specific heats

η – mass loading ratio

$\gamma_M = \gamma (1 + \zeta \eta) / (1 + \gamma \zeta \eta)$ - ratio of specific heats of mixture in equilibrium

ζ – relative specific heat (c / c_p)

c – specific heat of particle material

c_p – specific heat of gas at constant pressure

E – total energy = $E_R + E_p = [p_M / \{ \rho_M (\gamma_M - 1) \}] [1 + 3 R_p (\gamma_M - 1)]$

E_M – energy of mixture = $p_M / \{ \rho_M (\gamma_M - 1) \}$

E_R – radiation energy = $3 p_R / \rho_M$

K - $[1 + 3 R_p (\gamma_M - 1)] / (\gamma_M - 1)$

R – gas constant

R_M – effective gas constant = $(1 - \Phi) R \Gamma - (1+K) / K$

1. Introduction

Blast - wave theory was originally developed by Taylor¹ & Sedov² to study the effect of atom -bombs during the second world – war period. The method they employed is known as similarity method which is in good agreement with experimental results. Blast-waves are essentially, un-steady flow fields generated by explosions. When a small amount of energy is suddenly released in a relatively small - region, a disturbance headed by a strong shock - wave known as “blast-wave” is produced into surrounding medium. Blast - wave theory is attracting many investigators working in various fields such as theory of explosion, exploding wire phenomena, under water explosion, astrophysics (explosion in stars), atmosphere of earth, volcano, tsunami, medical sciences and sonoluminescence. A considerable number of publications, like those of Neumann³, Latter⁴, Taylor⁵, Sedov⁶, Sakurai^{7,8}, Krobeenikov⁹ etc., on blast - wave propagation, have appeared in literature. Analytical solutions of blast - wave propagation in homogeneous and non- homogeneous medium have been obtained by Rogers^{10,11}, Laumbach and Probstien¹², Sachdev¹³, Vishwakarma et.al¹⁴. Theory of blast - waves and related flows are of considerable physical interest in the theory of sonic booms, high altitude nuclear detonation, supernova explosions and sudden expansion corona into interplanetary space.

Effects of radiation are of great significance in astrophysical problems and nuclear explosions. On the flow field of a gas, radiation effect can be expressed in terms of radiation - pressure, radiation energy density and radiation flux. In extremely high speed of flight of a spacecraft re-entering

planetary atmosphere, radiation becomes an important mode of heat transfer and plays an important role not only in stellar- atmospheres but also in stellar - interiors. If medium is extremely rarified but extended, the energy and pressure of the radiation become comparable with those of matter and thus influences the thermodynamic properties of the medium.

By two-phase flows we mean a special flow problem in which we consider the mechanism of two phases of matter simultaneously. In general two-phase flows may be divided in two groups. The first group consists of flow problems of mixture of two-phases of four state—solids (pseudo – fluid), liquid, gas and plasma, where two-phases may be mixed homogeneously or in-homogeneously. In second group of flow problems, interaction between the two-phases of matter through their interface is important. In present article we consider first group of two-phase flows neglecting particle interaction, which has its importance in internal ballistic, lunar ash flow, exploding wire phenomena, under water explosion, astrophysics (explosion in stars), atmosphere of earth etc.

There are many engineering problems in which dilute phase of gas particle is a good approximation of actual conditions. In such cases due to the existence of solid particles in the gas, properties of mixture differ significantly from those of gas alone. Such types of studies have numerous application in underground explosions^{15,16,17}. In present analysis, we consider the mixture of two fluids – one is radiating-gas (including radiation energy and pressure but excluding radiation flux) and the other is pseudo – fluid of solid particles (which are spheres of identical mass, radius, and specific heat), the mixture developed by Rudinger¹⁸ for negligible particle pressure and particle volume–fraction (the case of moderate particle loading). Equation of state in this case is given as

$$(1.1) \quad P_M = R_M T \rho_M.$$

2. Basic equations

Basic equations governing flow field of spherical and cylindrical symmetry of two- phase flows when radiation - pressure and radiation - energy are included but radiation flux is neglected are given by Rudinger¹⁸ and Kruger et.al.¹⁹.

$$(2.1) \quad \rho_M (\partial u / \partial t + u \partial u / \partial r) + \partial p / \partial r = 0 ,$$

$$(2.2) \quad \partial \rho_M / \partial t + u \partial \rho_M / \partial r + \rho_M \partial u / \partial r + nu \rho_M / r = 0,$$

$$(2.3) \quad (\partial E / \partial t + u \partial E / \partial r) + p [\partial(1 / \rho_M) / \partial t + u \partial(1 / \rho_M) / \partial r] = 0 ,$$

r being distance from point of explosion . Equation (2.3) can be written as

$$(2.4) \quad \partial p / \partial t + u \partial p / \partial r + p \Gamma (\partial u / \partial r + nu / r) = 0,$$

n being equal to 1 and 2 according as wave is cylindrical or spherical. In case $R_p=0$, above equations correspond to particle mixture case and if $\gamma_M = \gamma$, $R_p \neq 0$, we have radiating gas.

If motion is assumed to be confined within the shock – front

$$r = S (t),$$

the velocity of shock – wave moving outwards is given by

$$V = dS / dt.$$

If density in undisturbed state is given by,

$$(2.5) \quad \rho_{M1}(r) = \beta r^{-\alpha},$$

α, β being positive constants, mass within the shock front is given by

$$(2.6) \quad m = \int_0^S 4 \pi r^2 \rho_{M1} dr = 4 \pi \beta S^{(3-\alpha)},$$

which is positive only when $0 < \alpha < 3$.

3. Similarity Transformations

Applying similarity transformations Rogers¹⁰

$$u = r t^{-1} U(\eta) ,$$

$$(3.1) \quad \rho_M = r^b \Omega (\eta),$$

$$P = r^{b+2} t^{-2} P(\eta) ,$$

where $\eta = r^{-\lambda} t$.

Equations of flows can be reduced into ordinary - differential equations and parameters b and λ can be fixed according to the physical requirements of the problems. If shock – surface be determined by

$$\eta = \eta_1, \text{ where } \eta_1 \text{ is constant, then}$$

$$\eta_1 = S^{-\lambda} t \text{ and } V = dS/dt = (S/\lambda t), S \text{ being shock-radius.}$$

Total energy within the shock-front

$$E = \int_0^R 4\pi r^2 \{ (1/2)\rho_M u^2 + pK \} dr$$

reduces to

$$(3.2) \quad E = (4\pi / \lambda) \int_{\eta_1}^{\infty} \left[(1/2)\Omega(\eta)U^2(\eta) + P(\eta) K \right] \eta^{(-5-b-\lambda)/\lambda} t^{(b+5-2\lambda)/\lambda} d\eta.$$

At this stage, we assume that the explosion is instantaneous, so that the total energy depends on only η . Hence from equation (3.2), we have

$$(3.3) \quad b + 5 = 2\lambda.$$

Thus boundary conditions become

$$(3.4) \quad U(\eta_1) = \{2/\lambda(\Gamma+1)\},$$

$$(3.5) \quad \Omega(\eta_1) = \rho_{M0}(S)S^{-\alpha}(\Gamma+1)/(\Gamma-1),$$

$$(3.6) \quad P(\eta_1) = 2\rho_0(S)S^{-\alpha}/\{\lambda^2(\Gamma+1)\}.$$

Since η_1 is constant, left as well as right hand side of equations (3.4) to (3.6) are constant which is ensured if

$$(3.7) \quad \rho_{M1}(S)S^{-b} = \beta_1 \text{ (constant),}$$

Comparing equations (2.5) and (3.7), we are in a position to fix b and λ , such that

$$(3.8) \quad b = -\alpha, \quad \beta_1 = \beta,$$

$$(3.9) \quad 5 - \alpha = 2\lambda.$$

4. Another Form of Similarity Solution

Now introducing new independent variable

$$(4.1) \quad x = (\eta_1 / \eta)^{\{2 / (5 - \alpha)\}},$$

Velocity of the shock front becomes

$$(4.2) \quad V = \{2 / (5 - \alpha)\} S / t,$$

$$(4.3) \quad V^2 = S^{(\alpha - 3)} \{2 / \eta_1 (5 - \alpha)\}^2.$$

Thus, we have

$$(4.4) \quad u = V f(x),$$

$$(4.5) \quad \rho_M = \rho_{M1}(S) h(x),$$

$$(4.6) \quad p = \rho_{M1}(S) V^2 g(x) / \Gamma.$$

Introducing above transformations in equations (2.2), (2.2) and (2.4), we have

$$(4.7) \quad \{x - f(x)\} f'(x) = [g'(x) / \{\Gamma h(x)\}] + f(x) \{(\alpha - 3) / 2\},$$

$$(4.8) \quad \{x - f(x)\} \{h'(x) / h(x)\} = f'(x) + \{f(x) / x\} n,$$

$$(4.9) \quad \{x - f(x)\} \{g'(x) / g(x)\} = \Gamma \{f'(x) + f(x) n / x\} + \alpha - 3,$$

with following boundary conditions

$$(4.10) \quad f(1) = 2 / (\Gamma + 1),$$

$$(4.11) \quad g(1) = 2 \Gamma / (\Gamma + 1),$$

$$(4.12) \quad h(1) = (\Gamma + 1) / (\Gamma - 1).$$

5. Results and Conclusion

Using central difference and Runge Kutta method variation of pressure for cylindrical and spherical waves is calculated and results are compared for different cases. Fig. 1 to 5 show variation of pressure for cylindrical waves. Fig. 1 shows ideal gas as well as radiation gas case for various radiation pressures no 0.5 & 1.00.

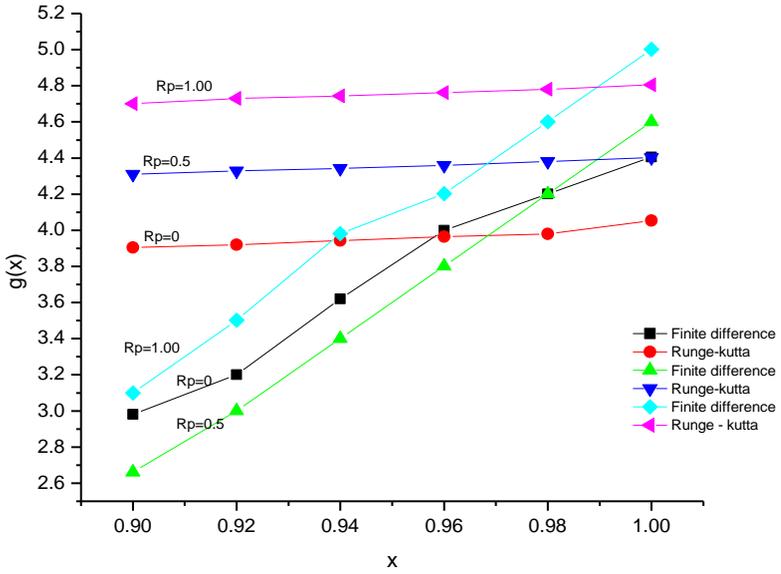


Figure 1: Variation of pressure for cylindrical wave

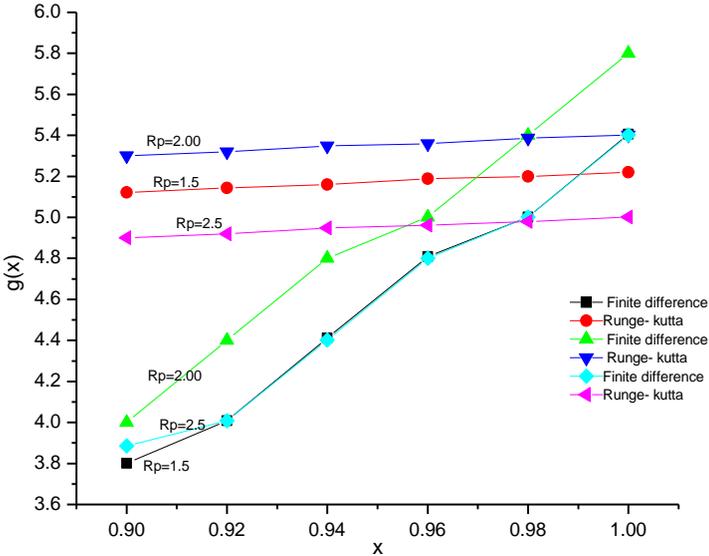


Figure 2: Variation of pressure for cylindrical wave

Fig.2 shows radiating gas case and is concluded that, pressure for $R_p = 1.5$ & 2.5 in case of finite difference method and for $R_p= 2.00$ by Runge Kutta method is same at $x = 1$.

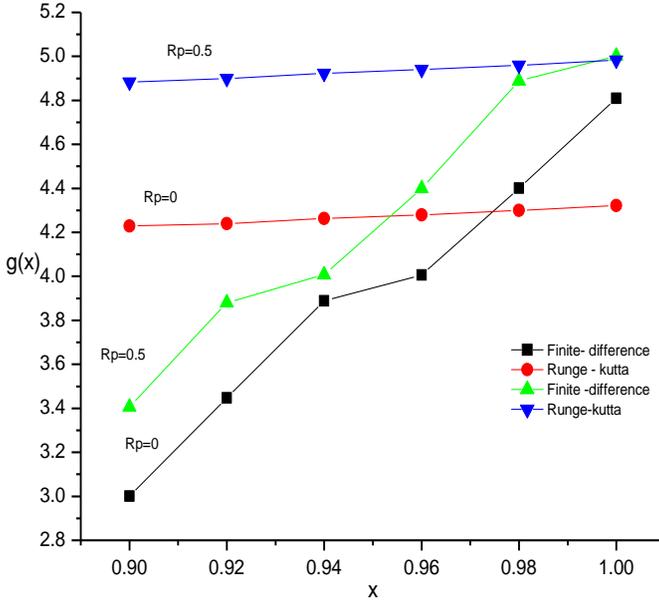


FIGURE 3: Variation of pressure for cylindrical waves where particle mass fraction=0.2

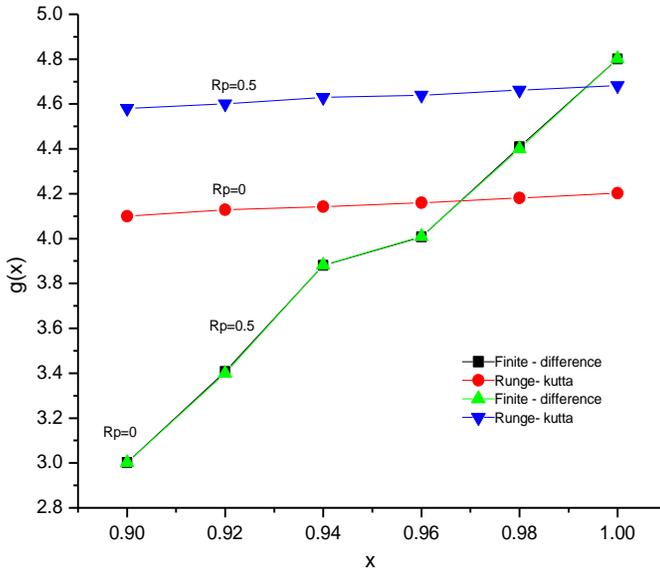


FIGURE 4: Variation of pressure for cylindrical wave where,particle mass fraction =0.4

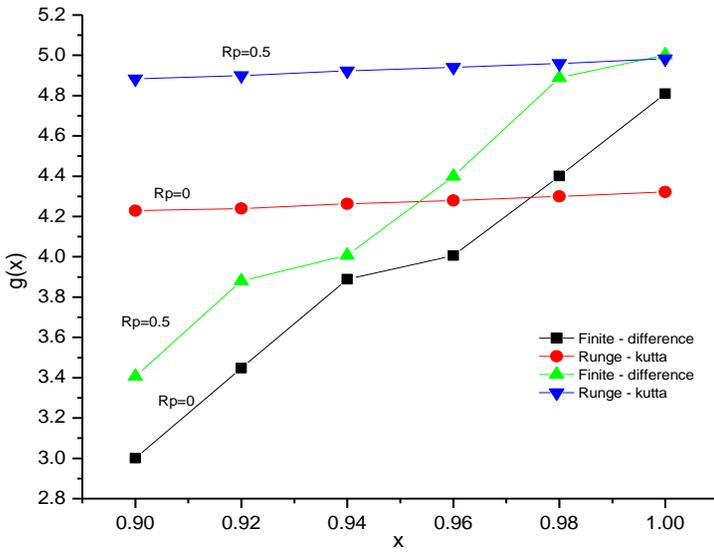


FIGURE 5: Variation of pressure for cylindrical wave where,particle mass fraction=0.6

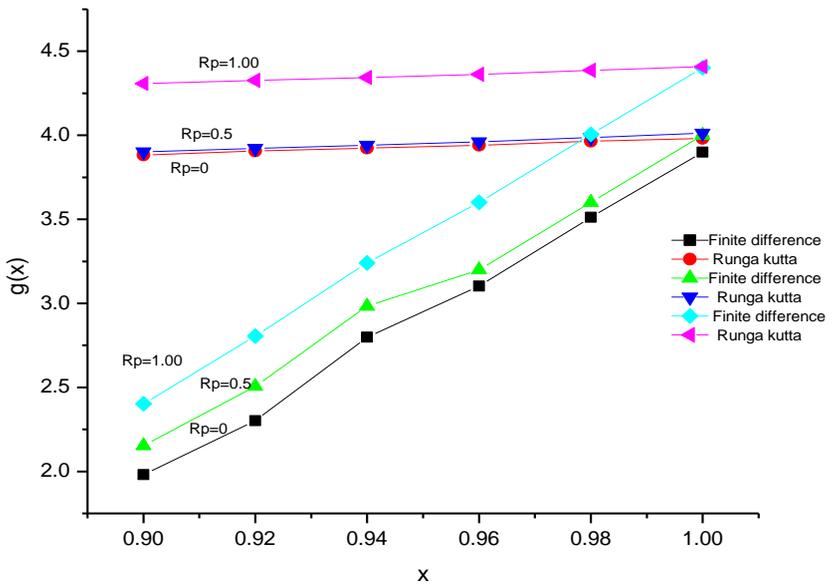


Figure6:Variation of pressure for spherical wave

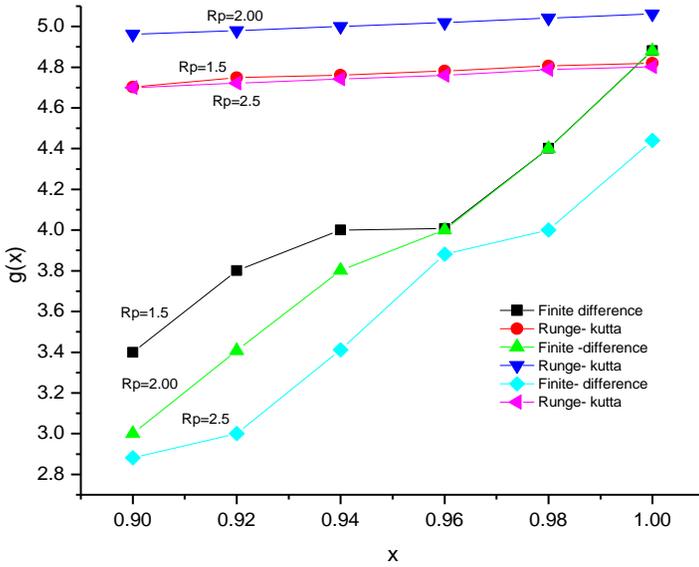


Figure 7: Variation of pressure for spherical wave

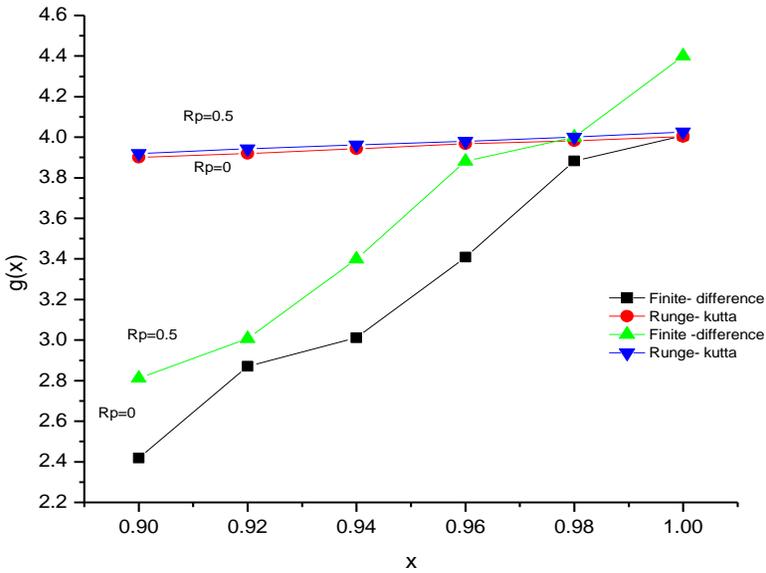


FIGURE 8: Variation of Pressure for Spherical Wave where,particle mass fraction=0.2

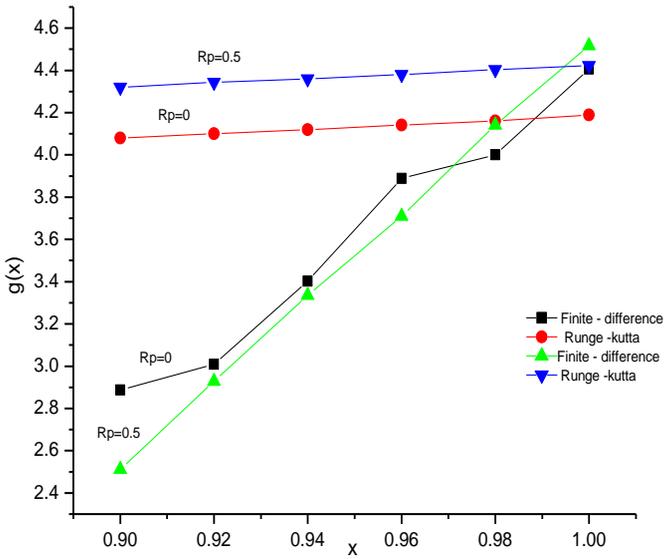


FIGURE 9: Variation of pressure for spherical wave where particle mass fraction=0.4

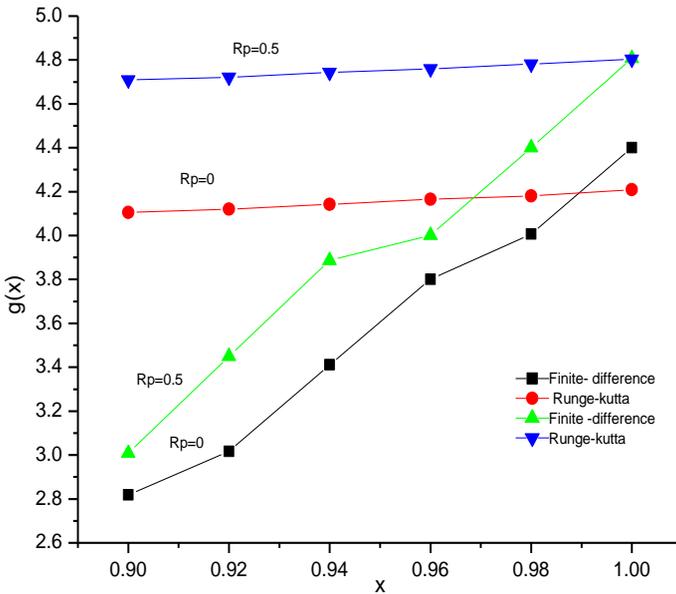


FIGURE 10: Variation of pressure for spherical wave where,particle mass fraction=0.6

Fig.3 to Fig. 5 show the variation of pressure for particle mass fraction 0.2, 0.4, 0.6 and $R_p=0$ & 0.5. Similarly Fig.6 to 10 show variation of pressure for spherical wave for ideal, radiation and radiating gas particle mixture.

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