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Conformal Screen on Lightlike Hypersurfaces of Almost Hyperbolic Hermitian Manifold

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Abstract: Object of present paper is to study the properties of conformal screen on lightlike hypersurfaces of almost hyperbolic Hermitian manifold with (*l, m*) –type connection.

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1. Introduction

A linear connection $\overline{\nabla}$ on a semi-Riemannian manifold $\overline{M}$ is called an (*l*,*m*) –type connection[1] if $\overline{\nabla}$ and its torsion tensor $\overline{T}$ satisfy

(1.1) $$(\overline{\nabla}_{\overline{X}}\overline{g})(\overline{Y},\overline{Z})=l\{\theta(\overline{Y})\overline{g}(\overline{X},\overline{Z})+\theta(\overline{Z})\overline{g}(\overline{X},\overline{Y})\}$$
$$-m\{\theta(\overline{Y})\overline{\mathrm{g}}(J\overline{X},\overline{Z})+\theta(\overline{Z})\overline{\mathrm{g}}(J\overline{X},\overline{Y})\}$$

and

(1.2) $$\overline{T}(\overline{\mathrm{X}},\overline{Y})=l\{\theta(\overline{Y})\overline{X}-\theta(\overline{X})Y)\},$$

where l and m are two smooth functions on $\overline{M}$, J is a tensor field of type (1,1) and θ is a 1-form associated with a smooth unit vector field ζ which is called the characteristic vector field of $\overline{M}$, given by $\theta(\overline{X})=\overline{g}(\overline{X},\zeta)$.

By direct calculation it can be seen that a linear connection $\bar{\nabla}$ on $\overline{M}$ is an (l, m) –type connection if and only if $\bar{\nabla}$ satisfies

$$\bar{\nabla}_{\overline{X}}\overline{Y} = \nabla_{\overline{X}}\overline{Y} + \theta(\overline{Y})\{l\overline{X} + mJ\overline{X}\}, \tag{1.3}$$

where ∇ is the Levi-Civita connection of a semi-Riemannian manifold $\overline{M}$ with respect to $\bar{g}$.

In case (l, m) = (1, 0): The above connection $\bar{\nabla}$ becomes a semi-symmetric non-metric connection. The notion of semisymmetric non-metric connection on a Riemannian manifold was introduced by Ageshe-Chafle[2,3] and later, studied by several authors[4,5]. In case (l, m) = (0, 1): The above connection $\bar{\nabla}$ reduces into a non-metric φ-symmetric connection such that $\varphi(\overline{X},\overline{Y}) = \bar{g}(J\overline{X},\overline{Y})$. The notion of the non-metric φ-symmetric connection was introduced by Jin[6,7,8].

In case (l,m) = (1,0) in equation (1.1) and (l,m) = (0,1) in equation (1.2): The above connection $\bar{\nabla}$ turns into a quarter-symmetric non-metric connection. The notion of quarter-symmetric non-metric connection was introduced by Golab[9] and then, studied by Sengupta-Biswas [10] and Ahmad-Haseeb[11]. In case (l,m) = (0, 0) in equation (1.1) and (l,m) = (0,1) in equation (1.2): The above connection $\bar{\nabla}$ reduces to a quarter-symmetric metric connection. The notion of quarter-symmetric metric connection was introduced Yano-Imai[12]. In case (l, m) = (0, 0) in equation (1.1) and (l, m) = (1, 0) in equation (1.2): The above connection $\bar{\nabla}$ will be a semi-symmetric metric connection. The notion of semi-symmetric metric connection was introduced by Hayden[13].

2. Preliminaries

Let us consider a differential manifold M_{2n} of class C^{∞} endowed with a tensor field of type (1, 1) F such that for an arbitrary vector field X.

$$\overline{\overline{X}} = X, \tag{2.1}$$

where $\overline{\overline{X}} \overset{\text{def}}{=} F(X)$, then F is called an almost hyperbolic Hermitian structure, and the differential manifold M^{2n} is called almost hyperbolic Hermitian manifold.

O n almost hyperbolic Hermitian manifold M^{2n}, if there exists a symmetric metric tensor g such that,

$$g(\overline{X}, \overline{Y}) + g(X, Y) = 0. \tag{2.2}$$

Then we say that g is compatible with almost complex structure and {F, g} is called an almost hyperbolic Hermitian structure. The manifold M^{2n} with an almost hyperbolic Hermitian structure is said to be an almost hyperbolic Hermitian manifold .

Let (M, g) be a lightlike hypersurface of $\overline{M}$. The normal bundle $TM^{\perp}$ of M is a subbundle of the tangent bundle TM of M, of rank 1, and coincides with the radical distribution Rad(TM) = TM $\cap TM^{\perp}$. Denote by F (M) the algebra of smooth functions on M and by T(E) the F (M) module of smooth sections of any vector bundle E over M.

A complementary vector bundle S(TM) of Rad(TM) in TM is non-degenerate distribution on M, which is called a screen distribution on M, such that

$$TM = Rad(TM) \oplus_{orth} S(TM),$$

where $\oplus_{orth}$ denotes the orthogonal direct sum. For any null section ξ of Rad(TM), there exists a unique null section N of a unique lightlike vector bundle tr(TM) in the orthogonal complement $S(TM)^{\perp}$ of S(TM) satisfying

$$\overline{g}(\xi, N) = 1, \quad \overline{g}(N; N) = \overline{g}(N; X) = 0; \quad \forall\, 8\, X \in T(S(TM)).$$

We call tr(TM) and N the transversal vector bundle and the null transversal vector field of M with respect to the screen distribution S(TM), respectively.

The tangent bundle $T\overline{M}$ of $\overline{M}$ is decomposed as follow:

$$T\overline{M} = TM \oplus tr(TM) = \{Rad(TM) \oplus tr(TM)\} \oplus_{orth} S(TM).$$

In the sequel, let X, Y, Z and W be the vector fields on M, unless otherwise specified. Let P be the projection morphism of TM on S(TM). Then the local Gauss and Weingartan formulas of M and S(TM) are given respectively by

$$\overline{\nabla}_X Y = \nabla_X Y + B(X.Y)N, \tag{2.3}$$

(2.4) $$\overline{\nabla}_X N = -A_N X + \tau(X)N,$$

(2.5) $$\nabla_X PY = \nabla^*_X PY + C(X.PY)\xi,$$

(2.6) $$\nabla_X \xi = -A^*_\xi X + \sigma(X)\xi.$$

where ∇ and ∇^* are the induced linear connections on TM and S(TM) respectively, B and C are the local second fundamental forms on TM and S(TM) respectively, A_N and A^*_ξ are the shape operators on TM and S(TM) respectively, and are 1-forms on TM.

For a lightlike hypersurface M of an almost Hermitian manifold ($\overline{M}$, $\overline{g}$), it is known[11] that J(Rad(TM)) and J(tr(TM)) are subbundles of S(TM), of rank 1 such that J(Rad(TM)) $\cap$ J(tr(TM)) = 0. Thus there exist two non-degenerate almost complex distributions D_o and D on M with respect to J, i.e., $J(D_o) = D_o$ and J(D) = D, such that

$$S(TM) = J(Rad(TM)) \oplus J(tr(TM)) \oplus_{orth} D_o,$$

$$D = \{Rad(TM) \oplus_{orth} J(Rad(TM))\} \oplus_{orth} D_o,$$

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(2.7) $$TM = D \oplus J(tr(TM)).$$

Consider two null vector fields U and V , and two 1-forms u and v such that

(2.8) $$U = -JN,\ V = J\xi,\ u(X) = g(X, V);\ v(X) = g(X, U).$$

Denote by S the projection morphism of TM on D. Any vector field X of M is expressed as X = SX + u(X)U. Applying J to this form, we have

(2.9) $$JX = FX + u(X)N,$$

where F is a tensor field of type (1, 1) globally defined on M by F = JoS. Applying J to (2.9) and using (1.2) , (1.3) and (2.8), we have

(2.10) $$F^2X = X - u(X)U.$$

As u(U) = 1 and FU = 0, the set (F, u, U) defines an indefinite almost contact structure on M and F is called the structure tensor field of M.

3. (*L*, *M*)-Type Connections

Let $(\overline{M}, \overline{g}, J)$ be an indefinite almost hyperbolic Hermitian manifold with a semi- symmetric metric connection $\overline{\nabla}$. Using (1.1), (2.1) and (2.7), we obtain

(3.1) $$(\nabla_X g)(Y, Z) = B(X,Y)\eta(Z) + B(X, Z)\eta(Y) - l\{\theta(Y)\mathrm{g}(X,Z) + \theta(\mathrm{Z})\mathrm{g}(X,Y)\} - m\{\theta(Y)\mathrm{g}(JX,Z) + \theta(\mathrm{Z})\mathrm{g}(JX,Y)\},$$

(3.2) $$T(X,Y) = l\{\theta(Y)X - \theta(X)\mathrm{Y}\} + m\{\theta(Y)\mathrm{FX} - \theta(\mathrm{X})\mathrm{FY}\},$$

(3.3) $$B(X,Y) - B(Y, X) = m\{\theta(Y)u(\mathrm{X}) - \theta(\mathrm{X})\mathrm{u}(\mathrm{Y})\},$$

where T is the torsion tensor with respect to ∇ and η is a 1-form such that

$$\eta(\mathrm{X}) = \overline{g}(\mathrm{X}, \mathrm{N}).$$

As B(X,Y) = $\overline{g}(\overline{\nabla}_X Y, \xi)$, so B is independent of the choice of S(TM) and satisfies

(3.4) $$\mathrm{B}(\mathrm{X}, \xi) = 0,\ \mathrm{B}(\xi, \mathrm{X}) = 0.$$

Local second fundamental forms are related to their shape operators by

(3.5) $$\mathrm{B}(\mathrm{X},\mathrm{Y}) = \mathrm{g}(A^*_\xi \mathrm{X}, \mathrm{Y}) + m u(X)\theta(Y), \quad \overline{g}(A^*_\xi \mathrm{X}, \mathrm{N}) = 0,$$

(3.6) $$\mathrm{C}(\mathrm{X}, \mathrm{PY}) = \mathrm{g}(\mathrm{A_N X}, \mathrm{PY}) + \{l\eta(X) + m u(X)\}\theta(\mathrm{P}Y), \overline{g}(\mathrm{A_N X}, \mathrm{N}) = 0.$$

S(TM) is non-degenerate,so using (3.4), (3.5), we have

(3.7) $$A^*_\xi \xi = 0.$$

Taking $\overline{\nabla}_X$ to(2.8) and (2.9) we have

(3.8) $$B(X,U)=u(A_N X)+m\theta(U)u(X)$$
$$=C(X,V)+m\{\theta(U)u(X)-\theta(V)\upsilon(X)\}-1\theta(V)\eta(X)$$

(3.9) $$\nabla_X U = F\left(A_N X\right)+\tau\left(X\right)U+\theta(U)\{lX+mFX\}$$

(3.10) $$\nabla_X V = F(A^*_\xi X)-\tau\left(X\right)V+\theta(V)\{lX+mFX\}$$

(3.11) $$(\nabla_X F)Y = u\left(Y\right)A_N X - B\left(X,Y\right)U$$
$$+l\{\theta(FY)X-\theta(Y)FX\}+m\{\theta(Y)X+\theta(FY)FX\}$$

4. Conformal Screen Distribution

On an almost hyperbolic Hermitian manifold $\overline{M}(c)$ of constant holomorphic sectional curvature c we have,

(4.1) $$\vec{R}(\overline{X},\overline{Y})\overline{Z}=\frac{c}{4}\{\overline{g}(\overline{Y},\overline{Z})\overline{X}-\overline{g}(\overline{X},\overline{Z})Y$$
$$+\overline{g}(J\overline{Y},\overline{Z})J\overline{X}-\overline{g}(J\overline{X},\overline{Z})J\overline{Y}+2\overline{g}(\overline{X},J\overline{Y})J\overline{Z}$$

where $\vec{R}$ is the curvature tensor of the (l, m)-type connection $\overline{\nabla}$ on $\overline{M}$.
Let $\overline{R}$ be the curvature tensor of the Levi-Civita connection $\overline{\nabla}$ on $\overline{M}$, then by (1.2) and (1.3) we have,

(4.2) $$\overline{R}(\overline{X},\overline{Y})\overline{Z}=\vec{R}(\overline{X},\overline{Y})\overline{Z}+(\nabla_{\overline{X}}\theta)(\overline{Z})\{l(\overline{Y}+mJ\overline{Y}\}$$
$$-(\nabla_{\overline{Y}}\theta)(\overline{Z})\{l(\overline{X}+mJ\overline{X}\}+\theta(\overline{Z})\{(\overline{X}l)\overline{Y}-(\overline{Y}l)\overline{X}+(\overline{X}m)J\overline{Y})-(\overline{Y}m)J\overline{X}).$$

Let R and R^* be the curvature tensor of induced connection ∇ and ∇^* on M and S(TM).By Gauss-Weingarten formula and (3.2), the Gauss equations for M and S(TM) are

(4.3) $$\overline{R}(X,Y)Z=R(X,Y)Z+B(X,Z)A_N Y-B(Y,Z)A_N X$$
$$+\{(\nabla_X B)(Y,Z)-(\nabla_Y B)(X,Z)+\tau(X)B(Y,Z)-\tau(Y)B(X,Z)$$

$$-l[\theta(X)B(Y,Z)-\theta(Y)B(X,Z)-m[\theta(X)B(\mathrm{F}Y,Z)-\theta(Y)B(\mathrm{F}X,Z)]\}\,\mathrm{N}\,,$$

(4.4) $$R(X,Y)\mathrm{P}Z=R^{*}(X,Y)\mathrm{P}Z+C(X,\mathrm{P}Z)A_{\xi}^{*}\mathrm{Y}-C(Y,\mathrm{P}Z)A_{\xi}^{*}X$$

$$+\{(\nabla_{X}C)(Y,Z)-(\nabla_{Y}C)(X,\mathrm{P}Z)-\tau(X)C(Y,\mathrm{P}Z)+\tau(Y)\mathrm{C}(X,\mathrm{P}Z)$$

$$-l[\theta(X)C(Y,\mathrm{P}Z)-\theta(Y)C(X,\mathrm{P}Z)-m[\theta(X)C(\mathrm{F}Y,\mathrm{P}Z)-\theta(Y)C(\mathrm{F}X,\mathrm{P}Z)]\}\xi\,.$$

Now comparing the tangential and transversal components (4.2) and using (4.1), and (4.3), we have

(4.5) $$R(X,Y)Z=B(Y,Z)A_{N}X-B(X,Z)A_{N}\mathrm{Y}$$

$$+(\overline{\nabla}_{X}\theta)(Z)\{1Y+mFY\}-(\overline{\nabla}_{Y}\theta)(Z)\{1X+mFX\}$$

$$+\theta(Z)\{(\mathrm{X}l)Y-(Yl)\mathrm{X}+(\mathrm{X}m)\mathrm{F}Y-(Ym)\mathrm{F}X\}$$

$$+\frac{c}{4}\{\mathrm{g}(Y,Z)\mathrm{X}-g(X,Z)\mathrm{Y}+\overline{\mathrm{g}}(\mathrm{JY},\mathrm{Z})\mathrm{F}$$

$$-\overline{\mathrm{g}}(\mathrm{JX},\mathrm{Z})\mathrm{FY}+2\overline{\mathrm{g}}(X,JY)\mathrm{F}Z),$$

(4.6) $$\nabla_{X}B(Y,Z)-\nabla_{Y}B(X,Z)+\{\tau(\mathrm{X})-1\theta(\mathrm{X})\}B(Y,Z)$$

$$-\{\tau(Y)-1\theta(Y)\}B(X,Z)-m\{\theta(\mathrm{X})\mathrm{B}(\mathrm{FY},\mathrm{Z})-\theta(Y)\mathrm{B}(\mathrm{F}X,\mathrm{Z})\}$$

$$-\mathrm{m}\{(\overline{\nabla}_{X}\theta)(Z)u(Y)-(\overline{\nabla}_{Y}\theta)(Z)u(X)-\theta(Z)\{(\mathrm{Xm})\mathrm{u}(Y)-(Ym)\mathrm{u}(\mathrm{X})\}$$

$$=\frac{c}{4}\{u(X)g(\mathrm{F}Y,Z)-u(Y)g(\mathrm{F}X,Z)+2\{u(\mathrm{Z})\overline{\mathrm{g}}(X,JY)\}.$$

Taking the scalar product with N to (4.2) with $\overline{Z}=PZ$ and using (4.1), (4.3), (4.4) and (3.6), we have

(4.7) $$(\nabla_{X}\mathrm{C})(Y,\mathrm{P}Z)-(\nabla_{Y}C)(X,\mathrm{P}Z)-\{\tau(\mathrm{X})+1\theta(\mathrm{X})\}C(Y,\mathrm{P}Z)$$

$$+\{\tau(Y)+1\theta(Y)\}C(X,\mathrm{P}Z)-m\{\theta(\mathrm{X})C(\mathrm{FY},\mathrm{PZ})-\theta(Y)C(\mathrm{F}X,\mathrm{PZ})\}$$

$$-\{(\overline{\nabla}_{X}\theta)(\mathrm{P}Z)\{1\eta(Y)+m\nu(Y)\}+(\overline{\nabla}_{Y}\theta)(\mathrm{P}Z)\{1\eta(\mathrm{X})+m\nu(X)\}$$

$$-\theta(\mathrm{P}Z)\{(\mathrm{X}l)\eta(Y)-(Yl)\eta(\mathrm{X})\}+(\mathrm{Xm})\nu(\mathrm{Y})-(Y\mathrm{m})\nu(X)\}$$

$$=\frac{c}{4}\{\eta(X)g(Y,\mathrm{P}Z)-\eta(Y)g(X,\mathrm{P}Z)+\nu(\mathrm{X})g(\mathrm{F}Y,\mathrm{P}Z)$$

$$-\nu(Y)g(\mathrm{F}X,\mathrm{P}Z)+2\nu(\mathrm{PZ})\overline{\mathrm{g}}(X,JY)\}$$

Definition 4.1: *A lightlike hypersurface M is said to be screen conformal*[14] *if there exist a non-vanishing smooth function* φ *on u such that*

(4.8) $C(X, PY) = \varphi B(X, PY).$

Theorem 4.1: *Let M be a lightlike hypersurface of an almost hyperbolic Hermitian manifold* $\overline{M}$ *(c) with an (l, m)-type connection such that* ζ *is tangent to M. Then the vector field* μ *defined as* $\mu = U - \varphi V$ *is an eigen vector of* A_ξ^* *corresponding to the eigenvector* $-m\theta(V)$. *If* $m\theta(V) = 0$, *then c = 0.*

Proof: Taking ξ in place of X to (4.8) and using (3.4), we have $C(\xi, PZ) = 0$. Hence $C(\xi, V) = 0$. Taking ξ in place of X to (3.8) and using $C(\xi, V) = 0$, we get

(4.9) $l\theta(V) = 0$.

Applying ∇_X to $C(Y, PZ) = \varphi B(Y, PZ)$, we get

$$(\nabla_X C)(Y, PZ) = (X\varphi)B(Y, PZ) + \varphi(\nabla_X B)(Y, PZ).$$

By this equation in (4.7) and using (4.6), we get

$$\{X\varphi - 2\varphi\tau(X)\}B(Y, PZ) - \{Y\varphi - 2\varphi\tau(Y)\}B(X, PZ)$$
$$-(\overline{\nabla}_X\theta)(PZ)\{l\eta(Y) + mg(Y, \mu)\} + (\overline{\nabla}_Y\theta)(PZ)\{l\eta(X) + mg(X, \mu)\}$$
$$-\theta(PZ)\{(Xl)\eta(Y) - (Yl)\eta(X) + (Xm)g(Y, \mu) - (Ym)g(X, \mu)$$
$$= \frac{c}{4}\{\eta(X)\}g(Y, PZ) - \eta(Y)\}g(X, PZ) + g(X, \mu)\}g(FY, PZ) - g(Y, \mu)\}$$
$$g(FX, PZ) + 2g(PZ, \mu)\overline{g}(X, JY)\}.$$

Taking $X = \xi$ and using (3.4) with (3.8), we get

(4.10) $$\{\xi\varphi - 2\varphi\tau(\xi)\}B(Y, PZ) + l(\overline{\nabla}_Y\theta)(PZ)$$
$$-(\overline{\nabla}_\xi\theta)(PZ)\{l\eta(Y) + mg(Y, \mu)\}$$
$$+\theta(PZ)\{(Yl) - \eta(Y)\xi l - g(Y, \mu)\xi m\}$$

$$= \frac{c}{4}\{g(Y, PZ) + \mathrm{g}(Y, \mu)\}u(\mathrm{PZ}) + 2\,\mathrm{g}(\mathrm{PZ}, \mu)\,\mathrm{u}(Y)\}.$$

Applying $\overline{\nabla}_X$ to $l\theta(V) = 0$ and using (3.5),(3.10) and (4.9),we have

(4.11) $$(Xl)\theta(V) + l(\overline{\nabla}_X \theta)(V) - \mathrm{lB}(\mathrm{X}, \mathrm{F}\zeta) = 0.$$

Since $\mu = U - \varphi V$ and $l\theta(V) = 0$, so by (3.8), we get

(4.12) $$B(X, \mu) = m\{\theta(U)\,\mathrm{u}(X) - \theta(\mathrm{V})\upsilon(X)\}.$$

Since $\theta(J\xi) = 0$ and $\theta(N) = 0$,so $\theta(F\zeta) = 0$ and also $\upsilon(F\zeta) = 0$ and $u(F\zeta) = 0$.

Taking μ in place of Y in (3.3) and using (4.12), we have

(4.13) $$B(\mu, X) = m\{\theta(X) - \theta(\mathrm{V})\,\mathrm{g}(X, \mu)\}.$$

Replacing X by V and $F\zeta$ one by one and using (4.11), we get

(4.14) $$B(\mu, V) = 0,\ B(\mu, F\zeta) = 0,\ (\mu l)\theta(V) + l(\overline{\nabla}_\mu \theta)(V) = 0.$$

Taking $Y = \mu$ and PZ= V to (4.10) and using (4.14), we get

(4.15) $$\varphi\{m(\overline{\nabla}_\xi \theta)(V) + \theta(V)\xi\,\mathrm{m}\} = \frac{3}{8}\mathrm{c}.$$

Taking X = μ in (3.5) and using (4.13), we get

(4.16) $$A^*_\xi \mu = -m\theta(V)\mu.$$

Therefore μ is an eigenvector of A^*_ξ corresponding to the eigenvector $-m\theta(V) = 0$. If $m\theta(V) = 0$,then applying $\overline{\nabla}_\xi$ to (4.16) and using (3.7) with (3.10), we get

$$(\xi m)\theta(\mathrm{V}) + \mathrm{m}(\overline{\nabla}_\xi \theta)(V) = 0.$$

By this and (4.15), we have c = 0.

Corollary 4.1: *Let M be a lightlike hypersurface of an almost hyperbolic Hermitian manifold* $\overline{M}$ *(c) with a semi-symmetric non metric connection.If M is screen conformal then* $A^{*}_{\xi}\mu = 0$ *and c = 0.*

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