Vol. 9 (2005) **Journal of International Academy of Physical Sciences** pp.57 – 68

Quasi Einstein Kähler Manifold $(QEK)_n$

Dhruwa Narain and Akhilesh Ram Tripathi

Department of Mathematics and Statistics, D.D.U. Gorakhpur University, Gorakhpur

(Received August 5, 2004)

Abstract: The notion of Quasi Einstein manifold $(QE)_n$ was introduced in a recent paper[1] by M. C. Chaki and R. K. Maity. Further in paper[2] M. C. Chaki has introduced the notion of generalized Quasi Einstein manifold and manifold of generalized quasi-constant curvature with some properties. In present paper we have obtained relations of associated scalars and scalar curvature of Quasi-constant curvature in Kähler manifold and also obtained some result on specific curvature tensor.

1. Introduction

Let $\{M_{2n},g\}$ be 2n-dimensional Kähler manifold with respect to Levi-civita connection ∇, then we have

(1.1) $$\overline{\overline{X}} + X = 0, \qquad \overline{X} = FX$$

(1.2) $$g(\overline{X},\overline{Y}) = g(X,Y)$$

(1.3a) Let $'F(X,Y) \overset{def}{=} g(\overline{X},Y)$, then

(1.3b) $$'F(X,Y) + 'F(Y,X) = g(\overline{X},Y) + g(X,\overline{Y}) = 0$$

(1.4) $$(\nabla_X F)(Y) = 0$$

where X, Y are arbitrary vector field.

Let K be the curvature tensor on M_{2n},

(1.5) $$K(X,Y,Z) \overset{def}{=} \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$$

and we have

(1.6) $$'K(\overline{X},\overline{Y},\overline{Z},\overline{W}) = 'K(X,Y,Z,W) = 'K(X,Y,\overline{Z},\overline{W})$$

(1.7) $$'K(X,Y,\bar{Z},\bar{W})='K(\bar{X},\bar{Y},Z,W)$$

and Ricci tensor Ric

(1.8) $$Ric(\bar{X},\bar{Y})=Ric(X,Y)$$

(1.9) $$R\bar{X}=\overline{RX}$$

The Wyle's projective tensor denoted by W, defined as

(1.10) $$W(X,Y,Z)=K(X,Y,Z)+\frac{1}{(n-1)}\{Ric(X,Z)Y-Ric(Y,Z)X\}$$

and on Kähler manifold satisfy following properties

(1.11) $$'W(\bar{X},\bar{Y},\bar{Z},\bar{U})='W(X,Y,Z,U)$$

(1.12) $$'W(\bar{X},\bar{Y},Z,U)='W(X,Y,\bar{Z},\bar{U})$$

where

(1.13) $$'W(X,Y,Z,U)\overset{def}{=}g(W(X,Y,Z),U)$$

The Concircular curvature tensor denoted by C is defined as

(1.14) $$C(X,Y,Z)=K(X,Y,Z)+\frac{r}{(n-1)}\{g(Y,Z)X-g(X,Z)Y\}$$

and have

(1.15a) $$'C(\bar{X},\bar{Y},\bar{Z},\bar{U})='C(X,Y,Z,U)$$

(1.15b) $$^{*}C(\bar{Y},\bar{Z})={}^{*}C(Y,Z)$$

(1.15c) $$^{*}C(\bar{Y},Z)+{}^{*}C(Y,\bar{Z})=0$$

where

(1.16) $$^{*}C(Y,Z)=(trC)(Y,Z)$$

Conformal curvature tensor V is defined as,

(1.17)
$$V(X,Y,Z) \overset{def}{=} K(X,Y,Z) - \frac{1}{(n-2)}\{Ric(Y,Z)X - Ric(Z,X)Y - g(X,Z)RY + g(Y,Z)RX\} + \frac{r}{(n-1)(n-2)}\{g(Y,Z)X - g(X,Z)Y\}$$

and Conhormonic curvature tensor denoted by L is defined by,

(1.18)
$$L(X,Y,Z) \overset{def}{=} K(X,Y,Z) - \frac{1}{(n-2)}\{Ric(Y,Z)X - Ric(Z,X)Y - g(X,Z)RY + g(Y,Z)RX\}$$

A non-flat Riemannian manifold $\{M_{2n}, g\}$ is called quasi-Einstein if its Ricci tensor Ric of type (0,2) is not identically zero and satisfy the condition

(1.19)
$$Ric(X,Y) = ag(X,Y) + bA(X)A(Y)$$

where a, b are scalars of which $b \neq 0$ and A is nonzero 1-form such that,

(1.20) $g(X,U) = A(X)$; $\forall X$, and U is unit vector field.

In 1972 Chen and Yano[3] introduced the notion of a manifold of quasi-constant curvature according to them a Riemannian manifold $\{M^n, g\}$ $(n > 3)$ is said to be quasi-type (0,4) satisfies the condition

(1.21)
$$'K(X,Y,Z,W) = a[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)] + b[g(Y,Z)A(X)A(W) - g(X,Z)A(Y)A(W) + g(X,W)A(Y)A(Z) - g(Y,W)A(X)A(Z)]$$

where a, b are scalars of which $b \neq 0$ and A is nonzero 1-form such that,

(1.22)
$$g(X,U) = A(X); \qquad \forall X,$$

U is unit vector field. In such case a and b are called the associated scalars. A is called associated 1-form an U is called generator of manifold. Such an n-dimensional manifold was denoted by the symbol $(QC)_n$ in the paper[4].

2. Scalar curvature and associated scalars of $(QEK)_n$

Theorem (2.0): *On Quasi-Einstein Kähler manifold* $(QEK)_n$ *for* $(n \neq 2)$ *the* *associated scalar a and b are related by*

(2.1) $$a = \frac{(2n-3)b}{n(n-2)}$$

and the scalar curvature r will be

(2.2) $$r = \frac{(2n-5)b}{(n-2)}$$

Proof: From (1.21), we get

(2.3) $$K(X,Y,Z) = a[g(Y,Z)X - g(X,Z)Y] + b[g(Y,Z)A(X)U - g(X,Z)A(Y)U + A(Y)A(Z)X - A(X)A(Z)Y]$$

and from (2.3), we get

(2.4) $$K(\bar{X},\bar{Y},Z) = a[g(\bar{Y},Z)\bar{X} - g(\bar{X},Z)\bar{Y}] + b[g(\bar{Y},Z)A(\bar{X})U - g(\bar{X},Z)A(\bar{Y})U + A(\bar{Y})A(Z)\bar{X} - A(\bar{X})A(Z)\bar{Y}]$$

From (2.3) and (2.4), using (1.6) and (1.3a), we get

$$a[g(Y,Z)X - g(X,Z)Y] + b[g(Y,Z)A(X)U - g(X,Z)A(Y)U + A(Y)A(Z)X - A(X)A(Z)Y] = a['F(Y,Z)\bar{X} - 'F(X,Z)\bar{Y}] + b['F(Y,Z)A(\bar{X})U - 'F(X,Z)A(\bar{Y})U + A(\bar{Y})A(Z)\bar{X} - A(\bar{X})A(Z)\bar{Y}]$$

On contracting (2.5) over X and Y, we get

(2.6) n(n – 2)a + b(2n – 3) = 0 , which yields (2.1)

On contracting (1.19) over X and Y, we get

(2.7) r = na + b

Using (2.1) in (2.7), we get (2.2).

Theorem (2.1): *On Quasi-Einstein Kähler manifold the scalar curvature is given by*

(2.8) r = n[(n-1)a + 2b]

if quasi constant curvature 'K satisfy relation (1.6).

Proof: From (1.21) and (1.6), we get by using (1.2)

$$
\begin{aligned}
{}'K(X,Y,Z,W) = a[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)] \\
+ b[g(Y,Z)A(\bar{X})A(\bar{W}) - g(X,Z)A(\bar{Y})A(\bar{W}) \\
+ g(X,W)A(\bar{Y})A(\bar{Z}) - g(Y,W)A(\bar{X})A(\bar{Z})]
\end{aligned} \tag{2.9}
$$

From (2.9), we get

$$
\begin{aligned}
K(X,Y,Z) = a[g(Y,Z)X - g(X,Z)Y] - b[g(Y,Z)A(\bar{X})\bar{U} + g(X,Z)A(\bar{Y})\bar{U}] \\
+ b[A(\bar{Y})A(\bar{Z})X - A(\bar{X})A(\bar{Z})Y]
\end{aligned} \tag{2.10}
$$

On contracting (2.10) over X, and using (1.3) and (1.20)

$$
Ric(Y,Z) = [(n-1)a + b]g(Y,Z) + nbA(\bar{Y})A(\bar{Z}) \tag{2.11}
$$

Further contracting (2.11) over Y, we get (2.8).

3. Properties of Wyle's Projective curvature tensor

Theorem (3.1): *On quasi-Einstein Kähler manifold the Wyle's projective curvature tensor W satisfy following relations,*

$$
{}^{*}W(\bar{Y},\bar{Z}) = {}^{*}W(Y,Z) \tag{3.1}
$$

$$
{}^{*}W(\bar{Y},Z) + {}^{*}W(Y,\bar{Z}) = 0 \tag{3.2}
$$

iff

$$
A(\bar{Y})A(Z) + A(Y)A(\bar{Z}) = 0 \tag{3.3}
$$

Proof: From (1.10)

$$
W(X,Y,Z) = K(X,Y,Z) + \frac{1}{(n-1)}\{Ric(X,Z)Y - Ric(Y,Z)X\} \tag{3.4}
$$

and from (1.21), we get

$$
\begin{aligned}
K(X,Y,Z) = a[g(Y,Z)X - g(X,Z)Y] + b[g(Y,Z)A(X)U - g(X,Z)A(Y)U \\
+ A(Y)A(Z)X - A(X)A(Z)Y]
\end{aligned} \tag{3.5}
$$

using (3.5) and (1.19) in (3.4), we get

(3.6)
$$W(X,Y,Z)=\left\{\frac{a(n-2)}{(n-1)}\right\}[g(Y,Z)X-g(X,Z)Y]+\left\{\frac{b(n-2)}{(n-1)}\right\}[A(Y)A(Z)X-A(X)A(Z)Y]+b[g(Y,Z)A(X)U-g(X,Z)A(Y)U]$$

(3.7) Let ${}^{*}W(Y,Z)=(tr\,W)(Y,Z)$

Contracting (3.6) over X and using (3.7), we get

(3.8)
$${}^{*}W(Y,Z)=[a(n-2)+b]g(Y,Z)+b(n-2)A(Y)A(Z)$$

and

(3.9)
$${}^{*}W(\bar{Y},\bar{Z})=[a(n-2)+b]g(\bar{Y},\bar{Z})+b(n-2)A(\bar{Y})A(\bar{Z})$$

From (3.8) and (3.9), we get (3.3) and from (3.3) we get (3.1) and (3.2).

Corollary (3.2): On $(QEK)_n$ Wyle's scalar curvature tensor r^* is given by

(3.10)
$$r^* = b(4n - 5)$$

Proof: Contracting (3.8) over Y and using (2.1), we get (3.10).

Theorem (3.3): *On Quasi-Einstein Kähler manifold Wyle's projective curvature tensor W satisfy following,*

(3.11)
$$W(\bar{X},\bar{Y},\bar{Z})=W(\bar{X},Y,Z)+W(X,\bar{Y},Z)+W(X,Y,\bar{Z})$$

(3.12)
$$W(X,Y,Z)=W(X,\bar{Y},\bar{Z})+W(\bar{X},Y,\bar{Z})+W(\bar{X},\bar{Y},Z)$$

iff

(3.12) .
$$A(Y)A(Z)\bar{X}+A(\bar{Y})A(Z)X+A(Y)A(\bar{Z})X=A(\bar{Y})A(\bar{Z})\bar{X}$$

Proof: From (3.6), we get

(3.13)
$$W(\bar{X},\bar{Y},\bar{Z})=\left\{\frac{a(n-2)}{(n-1)}\right\}[g(Y,Z)\bar{X}-g(X,Z)\bar{Y}]+\left\{\frac{b(n-2)}{(n-1)}\right\}[A(\bar{Y})A(\bar{Z})\bar{X}-A(\bar{X})A(\bar{Z})\bar{Y}]+b[g(Y,Z)A(\bar{X})U-g(X,Z)A(\bar{Y})U]$$

(3.14)
$$W(\bar{X},Y,Z)=\left\{\frac{a(n-2)}{(n-1)}\right\}[g(Y,Z)\bar{X}-g(\bar{X},Z)Y] +\left\{\frac{b(n-2)}{(n-1)}\right\}[A(Y)A(Z)\bar{X}-A(\bar{X})A(Z)Y] +b[g(Y,Z)A(\bar{X})U-g(\bar{X},Z)A(Y)U]$$

(3.15)
$$W(X,\bar{Y},Z)=\left\{\frac{a(n-2)}{(n-1)}\right\}[g(\bar{Y},Z)X-g(X,Z)\bar{Y}] +\left\{\frac{b(n-2)}{(n-1)}\right\}[A(\bar{Y})A(Z)X-A(X)A(Z)\bar{Y}] +b[g(\bar{Y},Z)A(X)U-g(X,Z)A(\bar{Y})U]$$

and

(3.16)
$$W(X,Y,\bar{Z})=\left\{\frac{a(n-2)}{(n-1)}\right\}[g(Y,\bar{Z})X-g(X,\bar{Z})Y] +\left\{\frac{b(n-2)}{(n-1)}\right\}[A(Y)A(\bar{Z})X-A(X)A(\bar{Z})Y] +b[g(Y,\bar{Z})A(X)U-g(X,\bar{Z})A(Y)U]$$

Adding (3.14), (3.15) and (3.16) we get (3.12), Further if (3.12) holds then Wyle's projective curvature tensor W satisfy relation (3.11) and from (3.11) we get (3.12).

4. Properties of Concircular curvature tensor

Theorem (4.1): *On Quasi-Einstein Kähler manifold the Concircular curvature tensor C satisfy following condition if (3.12) holds*

(4.1)
$$C(\bar{X},\bar{Y},\bar{Z})=C(\bar{X},Y,Z)+C(X,\bar{Y},Z)+C(X,Y,\bar{Z})$$

(4.2)
$$C(X,Y,Z)=C(X,\bar{Y},\bar{Z})+C(\bar{X},Y,\bar{Z})+C(\bar{X},\bar{Y},Z)$$

Proof: From (1.21) we have

(4.3)
$$'K(X,Y,Z)=a[g(Y,Z)X-g(X,Z)Y]+b[g(Y,Z)A(X)U-g(X,Z)A(Y)U +A(Y)A(Z)X-A(X)A(Z)U]$$

Using (4.3), (2.7) in (1.4), we get

$$C(X,Y,Z)=a[g(Y,Z)X-g(X,Z)Y]+b[g(Y,Z)A(X)U-g(X,Z)A(Y)U+A(Y)A(Z)X-A(X)A(Z)Y]+\left(\frac{na+b}{n(n-1)}\right)[g(Y,Z)X-g(X,Z)Y] \tag{4.4}$$

After simplification from (4.4), we get

(4.5)

$$C(X,Y,Z)=\left(\frac{a(n-2)}{(n-1)}\right)[g(Y,Z)X-g(X,Z)Y]+b[g(Y,Z)A(X)U-g(X,Z)A(Y)U+A(Y)A(Z)X-A(X)A(Z)Y-g(Y,Z)X+g(X,Z)Y]$$

From (4.5), we get

(4.6)

$$C(\bar{X},\bar{Y},\bar{Z})=\left(\frac{a(n-2)}{(n-1)}\right)[g(Y,Z)\bar{X}-g(X,Z)\bar{Y}]+b[g(Y,Z)A(\bar{X})U-g(X,Z)A(\bar{Y})U+A(\bar{Y})A(\bar{Z})\bar{X}-A(\bar{X})A(\bar{Z})\bar{Y}-g(Y,Z)\bar{X}+g(X,Z)\bar{Y}]$$

(4.7)

$$C(\bar{X},Y,Z)=\left(\frac{a(n-2)}{(n-1)}\right)[g(Y,Z)\bar{X}-g(\bar{X},Z)Y]+b[g(Y,Z)A(\bar{X})U-g(\bar{X},Z)A(Y)U+A(Y)A(Z)\bar{X}-A(\bar{X})A(Z)Y-g(Y,Z)\bar{X}+g(\bar{X},Z)Y]$$

(4.8)

$$C(X,\bar{Y},Z)=\left(\frac{a(n-2)}{(n-1)}\right)[g(\bar{Y},Z)X-g(X,Z)\bar{Y}]+b[g(\bar{Y},Z)A(X)U-g(X,Z)A(\bar{Y})U+A(\bar{Y})A(Z)X-A(X)A(Z)\bar{Y}-g(\bar{Y},Z)X+g(X,Z)\bar{Y}]$$

(4.9)

$$C(X,Y,\bar{Z})=\left(\frac{a(n-2)}{(n-1)}\right)[g(Y,\bar{Z})X-g(X,\bar{Z})Y]+b[g(Y,\bar{Z})A(X)U-g(X,\bar{Z})A(Y)U+A(Y)A(\bar{Z})X-A(X)A(\bar{Z})Y-g(Y,\bar{Z})X+g(X,\bar{Z})Y]$$

Adding (4.7), (4.8), (4.9) and using (4.6) and (3.12), we get (4.1) and (4.1) yields (4.2).

Theorem (4.2): *On Quasi-Einstein Kähler manifold for $a \neq b$ the properties of Concircular curvature tensor C in Kähler manifold defined by (1.15b) and (1.15c) holds as well as in* $(QEK)_n$ *for $n > 2$ if (3.3) holds.*

Proof: Contracting (4.7) over X, we get

(4.10) $$^{*}C(Y,Z) = (n-2)[(a-b)g(Y,Z) + A(Y)A(Z)]$$

and

(4.11) $$^{*}C(\bar{Y},\bar{Z}) = (n-2)[(a-b)g(Y,Z) + A(\bar{Y})A(\bar{Z})]$$

From (4.10) and (4.11) we find that if (3.3) holds we get (1.15b) and further (1.15b) yield (1.15c) for $a \neq b$.

Corollary (4.3): On $(QEK)_n$ the scalar curvature of Concircular curvature tensor satisfy following relation

(4.12) $$r^{c} = na(n-2) - (n-1)^{2}b$$

Proof: From (4.5) using (1.15a) we get

(4.13)

$$C(X,Y,Z) = \left(\frac{a(n-2)}{(n-1)}\right)[g(Y,Z)X - g(X,Z)Y] + b[g(Y,Z)A(\bar{X})U$$
$$- g(X,Z)A(\bar{Y})U + A(\bar{Y})A(\bar{Z})X - A(\bar{X})A(\bar{Z})Y - g(Y,Z)X + g(X,Z)Y]$$

Contracting (4.13) over X and Y, we get (4.12).

5. Conformal curvature tensor

Theorem (5.1): If Quasi-Kähler manifold is conformally flat then scalar curvature tensor r satisfies following relation

(5.1) $$r = \left[\frac{na}{(n-1)} - \frac{(n-1)b}{(n-2)}\right]$$

Proof: Let

(5.2) $$V(X,Y,Z) = 0$$

Then from (1.17), we get

(5.2)
$$K(X,Y,Z)=\frac{1}{(n-2)}\{Ric(Y,Z)X-Ric(Z,X)Y-g(X,Z)RY+g(Y,Z)RX\}-\frac{r}{(n-1)(n-2)}\{g(Y,Z)X-g(X,Z)Y\}$$

Using (1.19) and (2.7) in (5.2), we get after simplification

(5.3)
$$K(X,Y,Z)=\frac{a}{(n-1)}[g(Y,Z)X-g(X,Z)Y]+\frac{b}{(n-1)(n-2)}[(n-1)\{g(Y,Z)A(X)U-g(X,Z)A(Y)U+A(Y)A(Z)X-A(X)A(Z)Y\}-\{g(Y,Z)X-g(X,Z)Y\}]$$

From (5.3), we get

(5.4)
$$K(\bar{X},\bar{Y},Z)=\frac{a}{(n-2)}[g(\bar{Y},Z)\bar{X}-g(\bar{X},Z)\bar{Y}]+\frac{b}{(n-1)(n-2)}[(n-1)\{g(\bar{Y},Z)A(\bar{X})U-g(\bar{X},Z)A(\bar{Y})U+A(\bar{Y})A(Z)\bar{X}-A(\bar{X})A(Z)\bar{Y}\}-\{g(\bar{Y},Z)\bar{X}-g(\bar{X},Z)\bar{Y}\}]$$

On Kähler manifold (5.3) and (5.4) yields.

(5.5)
$$\frac{a}{(n-2)}[g(Y,Z)X-g(X,Z)Y]+\frac{b}{(n-1)(n-2)}[(n-1)\{g(Y,Z)A(X)U-g(X,Z)A(Y)U+A(Y)A(Z)X-A(X)A(Z)Y\}-\{g(Y,Z)X-g(X,Z)Y\}]$$
$$=\frac{a}{(n-2)}[g(\bar{Y},Z)\bar{X}-g(\bar{X},Z)\bar{Y}]+\frac{b}{(n-1)(n-2)}[(n-1)\{g(\bar{Y},Z)A(\bar{X})U-g(\bar{X},Z)A(\bar{Y})U+A(\bar{Y})A(Z)\bar{X}-A(\bar{X})A(Z)\bar{Y}\}-\{g(\bar{Y},Z)\bar{X}-g(\bar{X},Z)\bar{Y}\}]$$

On contracting (5.5) over X, we get

(5.6)
$$ag(Y,Z)+bA(Y)A(Z)=\frac{a}{(n-1)}g(Y,Z)+\frac{b}{(n-2)}[A(\bar{Z})A(\bar{Y})+A(Y)A(Z)-g(Y,Z)]$$

Further contracting (5.6) over Y, we get (5.1).

Theorem (5.2): If on Quasi-Einstein Kähler manifold is conformally flat and satisfying the relation (3.12) then curvature tensor K satisfy the relation

(5.7) $$K(\bar{X},\bar{Y},Z) = K(\bar{X},Y,Z) + K(X,\bar{Y},Z) + K(X,Y,\bar{Z})$$

Proof: From (5.3), we get

(5.8) $$\begin{aligned} K(\bar{X},\bar{Y},Z) &= \frac{a}{(n-1)}[g(Y,Z)\bar{X} - g(X,Z)\bar{Y}] \\ &+ \frac{b}{(n-1)(n-2)}[(n-1)\{g(Y,Z)A(\bar{X})U - g(X,Z)A(\bar{Y})U \\ &+ A(\bar{Y})A(\bar{Z})\bar{X} - A(\bar{X})A(\bar{Z})\bar{Y}\} - \{g(Y,Z)\bar{X} - g(X,Z)\bar{Y}\}] \end{aligned}$$

(5.9) $$\begin{aligned} K(\bar{X},Y,Z) &= \frac{a}{(n-1)}[g(Y,Z)\bar{X} - g(\bar{X},Z)Y] \\ &+ \frac{b}{(n-1)(n-2)}[(n-1)\{g(Y,Z)A(\bar{X})U - g(\bar{X},Z)A(Y)U \\ &+ A(Y)A(Z)\bar{X} - A(\bar{X})A(Z)Y\} - \{g(Y,Z)\bar{X} - g(\bar{X},Z)Y\}] \end{aligned}$$

(5.10) $$\begin{aligned} K(X,\bar{Y},Z) &= \frac{a}{(n-1)}[g(\bar{Y},Z)X - g(X,Z)\bar{Y}] \\ &+ \frac{b}{(n-1)(n-2)}[(n-1)\{g(\bar{Y},Z)A(X)U - g(X,Z)A(\bar{Y})U \\ &+ A(\bar{Y})A(Z)X - A(X)A(Z)\bar{Y}\} - \{g(\bar{Y},Z)X - g(X,Z)\bar{Y}\}] \end{aligned}$$

and

(5.11) $$\begin{aligned} K(X,Y,\bar{Z}) &= \frac{a}{(n-1)}[g(Y,\bar{Z})X - g(X,\bar{Z})Y] \\ &+ \frac{b}{(n-1)(n-2)}[(n-1)\{g(Y,\bar{Z})A(X)U - g(X,\bar{Z})A(Y)U \\ &+ A(Y)A(\bar{Z})X - A(X)A(\bar{Z})Y\} - \{g(Y,\bar{Z})X - g(X,\bar{Z})Y\}] \end{aligned}$$

Adding (5.9), (5.10), (5.11) and using (5.8), (1.3b) and (3.12), we get (5.7).

References

1. M.C. Chak and R.K. Maity: On quasi-Einstein manifold, *Publ. Math. Debrecen*, **57**(2000) 297-306.

2. M. C. Chaki: On generlaised Quasi-Einstein manifold. *Publ. Math. Debrecen*, **58**(4) (2001) 683-691.

3. B. Chen and K. Yano: Hypersurfaces of conformally flat space, *Tensor N. S.*, **26**(1972) 315-321.

4. M. C. Chaki and M. L. Ghosh: On Quasi-Einstein manifold. Communicated.

5. K. Yano and M. Kon: Structures on manifolds, *World Scientific*, 1989.

6. R. S. Mishra: *Structures on a differentiable manifold and their application*. Chandrama Prakashan, Allahabad, 1984.