

Adiabatic and Isothermal Cylindrical Shock in Uniform Medium

Yogember Singh

Department of Physics, Bareilly College, Bareilly

R.P. Yadav

Department of Physics, Govt., P.G. College, Bisalpur, Pilibhit

(Received October 22, 2004)

Abstract: Neglecting the effect of overtaking disturbances, study of propagation of cylindrical converging shock wave has been carried out by C.C.W. method, for two cases viz (1) when shock waves isothermally and (2) when they move adiabatically. Assuming the medium to be uniform, the expressions for flow variables are derived for both the cases. All the flow variables are numerically computed and discussed through figures. It is found that flow variables increase gradually for both isothermal and adiabatic processes, while they increase rapidly for large values of r , in case of adiabatic process.

1. Introduction

The study of propagation of converging shock waves is of immense importance in astrophysics and nuclear physics, specially for the production of high temperature and pressure. The analysis of converging shock waves has been given by Guderley¹, Butler², Stanyukovich³, Zeldovich and Raizer⁴, Lazarus⁵, Chisnell⁶, Mishkin and Fujimoto⁷ etc. Most of them exercised shockwaves under adiabatic conditions. Propagation of converging and diverging shock waves under isothermal conditions in case of spherical and cylindrical symmetry are examined by Levin and Zhuravskaya⁸. The case of adiabatic spherical converging shock waves in uniform and non uniform media has been discussed by Gangwar⁹. Since real processes neither follow adiabatic nor isothermal conditions, but lie between these two, therefore, the study of shock propagation in the two extreme cases are important to study simultaneously.

The aim of the present paper is to study the propagation of converging cylindrical shock wave in uniform medium, simultaneously for isothermal and adiabatic conditions. Neglecting the effect of overtaking disturbances on the motion of shockwaves, the analytical relations for flow variables have been obtained by C.C.W. method. All the flow variables are numerically computed and discussed through figures. The results obtained in both the cases are compared. It is found that flow variables increase with distances gradually for

isothermal and adiabatic conditions, while they increase drastically for large values of distance, in case of adiabatic processes.

2. Basic Equations

The equations governing the cylindrical symmetrical flow enclosed by the shock front are

$$(1) \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0$$

$$(2) \quad \left(\frac{\partial}{\partial t} + 4 \frac{\partial}{\partial r} \right) \rho + \rho \left(\frac{\partial u}{\partial r} + \frac{u}{r} \right) = 0$$

where, $u(r,t)$, $p(r,t)$ and $\rho(r,t)$ denote respectively the particle velocity, the pressure and density at a distance from the origin at time t .

3. Boundary Conditions

(A) Adiabatic Shock: Let P_0 and ρ_0 denote the unperturbed value of pressure and density in front of shock and p , ρ and u be the values of respective quantities at any point immediately after passage of shock, then the Rankine-Hugoniot conditions for adiabatic shock will permit us to express p , ρ and u in terms of unperturbed value of these quantities by means of the following equations.

$$(3) \quad p = \rho_0 a_0^2 \left[\frac{2M^2}{\gamma+1} - \frac{\gamma-1}{\gamma(\gamma+1)} \right]$$

$$(4) \quad \rho = \rho_0 \frac{(\gamma+1)M^2}{(\gamma-1)M^2 + 2}$$

$$(5) \quad u = \frac{2a_0}{\gamma+1} \left(M - \frac{1}{M} \right)$$

where γ is adiabatic index, $M = U/a_0$, is Mach number, U is the shock velocity, a and a_0 are the sound velocities in perturbed and unperturbed media respectively.

For Strong Shock i.e. $M \gg 1$ conditions (3), (4) and (5) reduce to

$$(6) \quad p = \frac{2\rho_0 U^2}{\gamma+1}$$

$$(7) \quad \rho = \frac{\gamma+1}{\gamma-1} \rho_0$$

$$(8) \quad u = \frac{2U}{\gamma+1}$$

(B) Isothermal Shock : In case of isothermal shock, the boundary conditions are

$$(9) \quad u = U - \frac{C_0^2}{\gamma U}$$

$$(10) \quad \rho = \rho_0 \gamma \frac{U^2}{c_0^2}$$

$$(11) \quad \rho = \rho_0 U^2$$

4. Theory

For cylindrical converging shocks, the characteristic form of the system of equations (1) & (2) i.e. the form in which each equation contains derivatives in only one direction in (r,t) plane, is

$$(12) \quad dp - \rho c du + \frac{\rho c^2 u}{(u-c)r} dr = 0$$

substituting equations (3) - (5) in equation (12), we get

$$(13) \quad dU + \frac{s^2(\gamma-1)}{(2-s)\{2-s(\gamma-1)\}} U \frac{dr}{r} = 0$$

$$\text{where } s^2 = \frac{2\gamma}{\gamma-1}$$

The equation (13) gives

$$(14) \quad U = Ar^{-s^2(\gamma-1)/(2-s)\{2-s(\gamma-1)\}}$$

where A is constant involved due to integration. This expression represents the velocity of the shock propagating adiabatically.

Similarly, on substituting shock conditions (9) to (11) in equation (12) and simplifying we get, the expression for velocity of shock propagating isothermally.

$$(15) \quad U = c_0 (\log r - A)$$

The expressions for shock strength for both the case can be written as

$$(16) \quad \frac{U}{c_0} = \frac{A}{c_0} r^{-s^2(\gamma-1)/(2-s)\{2-s(\gamma-1)\}}$$

$$(17) \quad \text{and } \frac{U}{c_0} = (\log r - A)$$

respectively, for adiabatic and isothermal shocks.

5. Results and Discussion

The expressions (14) & (16) and (15) & (17) for shock velocity and shock strength respectively for adiabatic and isothermal shock are used for computation. Initially taking $U/c_0 = 10$ at $r = 2.5$, variation of shock velocity with propagation distance has been shown in fig (1). From the graph, it is obvious that adiabatic shock velocity increases asymptotically with propagation distance r , where as isothermal shock strengthened logarithmically. Similiar variations in shock strength is also observed. [(c.f. fig. (2))]

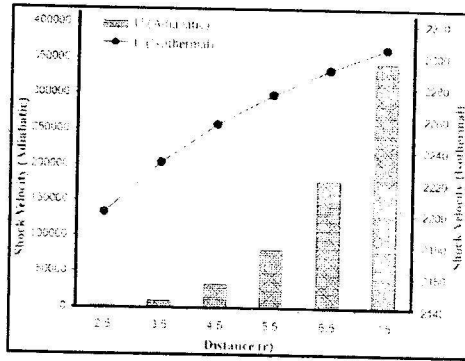


Fig. 1: Variation of Shock velocity with distance(r).

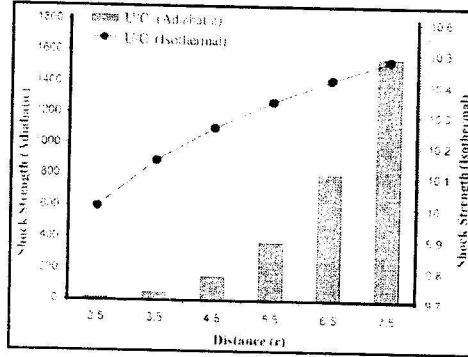


Fig. 2: Variation of Shock Strength with distance(r).

Finally, the expressions for pressure, particle velocity and the density can be written as

$$p = \frac{2\rho_0}{\gamma+1} A^2 r^{-2s^2(\gamma-1)/(2-s)\{2-s(\gamma-1)\}}$$

$$(18) \quad \rho = \rho_0 \left\{ \frac{\gamma+1}{\gamma-1} \right\}$$

$$u = 2Ar^{-s^2(\gamma-1)/(2-s)\{2-s(\gamma-1)\}}$$

and

$$p = \rho_0 [c_0(\log r - A)]^2$$

$$p = \rho_0 \gamma (\log r - A)$$

$$(19) \quad u = c_0(\log r - A) - \frac{c_0^2}{\gamma c_0(\log r - A)}$$

respectively for adiabatic and isothermal shocks.

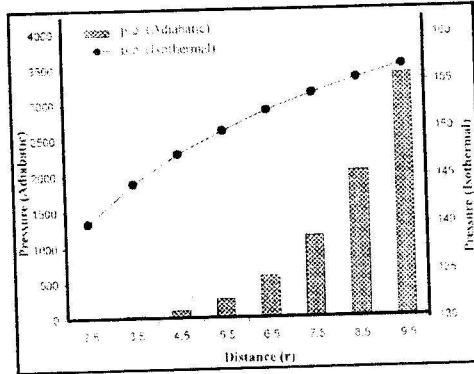


Fig. 3: Variation of Pressure with distance (r).

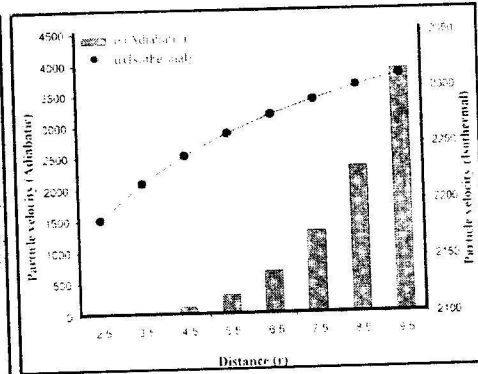


Fig. 4: Variation of Particle velocity with distance (r).

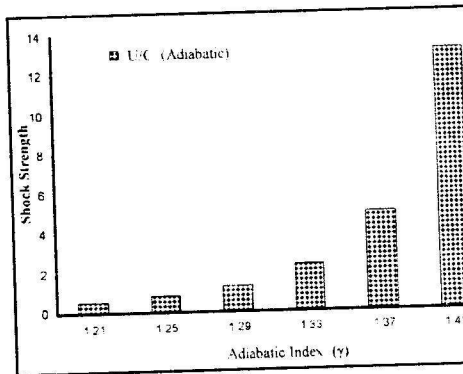


Fig. 5: Variation of Pressure with adiabatic index(γ).

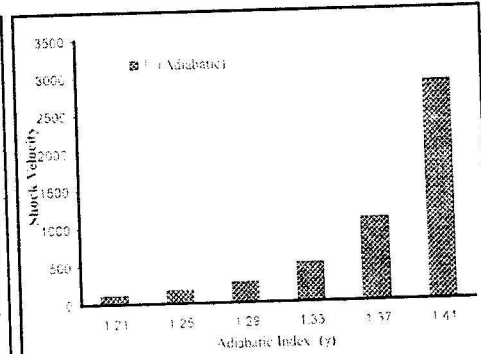


Fig. 6: Variation of Shock velocity with adiabatic index(γ)

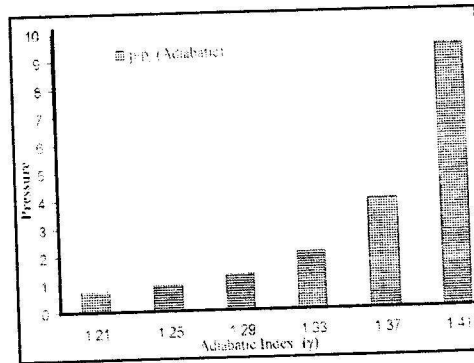


Fig. 7: Variation of Pressure with distance adiabatic index(γ).

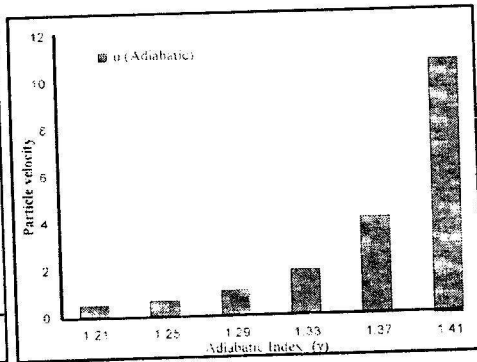


Fig. 8: Variation of Particle velocity with distance adiabatic index(γ)

The variation of these variables with propagation distance and adiabatic index γ are

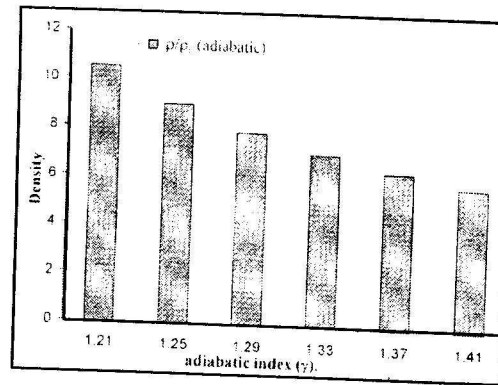


Fig. 9: Variation of density with adiabatic index (γ).

shown in figures (3) - (9). From these figures it may be concluded that flow variables increase gradually for both adiabatic and isothermal processes, while they increase drastically for large values of distance in the case of adiabatic process.

Aknowledgement

One of the authors (Y.S.) is thankful to C.S.I.R. for providing financial assistance in form of J.R.F. and principal, Bareilly College, Bareilly for giving research facilities in his institution.

References

1. G. Guderley: *Lufthftforsch.*, **19** (1942) 302.
2. D. Butler: Converging spherical and cylindrical shock, *Armanent Res. Estab. Rep.*, **54** (1954) 54.
3. K.P. Syanyukovich: *Unsteady motion of continuous media*, Academic Press, N.Y., 1960.
4. Ya.B. Zeldovich and Yu. P. Raizer: *Physics of shock waves and high temperature Hydrodynamic phenomenon*, Academic Press, 1967.
5. R.B. Lazarus: *Phys. Fluids*, **23** (1980) 844.
6. R.F. Chisnell: *Proc. IUPAP, IUTAM Symp. Nonlinear Acoustics, Novosibirsk*, (1987) 82.
7. E.A. Mishkin and Y. Fujimoto: *J. Fluid Mech.* (1978); *Phys.Fluids*, **21** (1978) 1933.
8. V.A. Levin and T.A. Zhuravskaya: *Shock waves*, **6(3)** (1996) 177.
9. P.K. Gangwar: Ph.D. Thesis, M.J.P. Rohilkhand Univ. Bareilly (2002).
10. W. Chester: *Phil. Mag.*, **45(7)** (1954) 1293.
11. R.F. Chishell: *Proc. Roy. Soc. London*, **A 232** (1955) 350.
12. G..B. Whitham: *J. Fluid Mech.*, **4** (1958) 337.