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Host – Vector Model for Japanese Encephalitis

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Abstract: This paper is concerned with the infection of Japanese encephalitis by the host-vector model of transmission. In this model of transmission, the susceptible human acquires the infection only from infected mosquito and vice-versa. The analysis of the solution reveals an interesting fact that this disease was already present in the society. The disease can be controlled by the reduction of contact between man and mosquito, and decreasing the pool of infection i.e. mosquito population.

1. Introduction

Recognition of encephalitis in India based on serological survey, was first made in Tamil Nadu. Subsequent surveys carried out by National Institute of Virology, Pune indicated that about half of the population in South India had neutralizing antibodies to this virus. In the last decade there has been a major upsurage of encephalitis in Assam, Andhra Pradesh, Goa, Karnataka, Manipur, Maharashtra, Tamil Nadu, Uttar Pradesh, Pandicherry and West Bengal.

In Uttar Pradesh, the spread of disease is more in eastern part. Kushinagar, Deoria, Maharajganj, Gorakhpur are highly risky districts for the incidence of encephalitis. In these districts, the disease encephalitis is known as Navaki Bimary (New Disease). The case fatality rate of encephalitis is maximum comparison to other infectious disease in Gorakhpur Commissionerry.

Encephalitis is a mosquito-born disease caused by a group B arbovirus and transmitted by culicine mosquitoes. Female mosquitoes get infected after feeding on a viraemic host and after 9-12 days of incubation period they can transmit the virus to other hosts. Only Female mosquitoes are involved since only they suck blood; males take their liquid meals from fruit juices and elsewhere. The disease is transmitted to man by the bite of infected mosquitoes. Man to man transmission has not so far recorded. Among the animal host, pigs have been incriminated as the major vertebrate hosts for encephalitis virus. Infected Pigs circulate the virus so that mosquitoes get infected and can transmit the virus to man[1].

The general theory of host-vector diseases, already initiated by Ross (1911) in the "Theory of Happening", made a new step forward in 1927 when Kermack and Mckendrick generalized their single-population threshold to cover the case of a typical host-vector situation. Hethcote[2] examined certain general endemic model in which the human population was subjected to infection and recovery with immunity, while the mosquito vector injoyed no recovery. It was shown that below a certain threshold the null stable of no infection in either host or vector is the only equilibrium point which is asymptotically stable[2].

For developing the model for J-encephalitis, we suppose that there are two interesting populations - host population (man) and vector population (mosquito). Infection in man is due to the bite of the infected mosquito and vice-versa. Conversely Female mosquito can pick up the infection, when they bite infected human.

2. Formulation of Mathematical Model

For developing the model we suppose that all the infected individuals are equally infected and all susceptible individuals are equally susceptible.

Let us assume for human population

n = Total population size

y = Total number of infected individuals

x = Recovery rate

u = Birth rate

v = Death rate

A similar set of definitions n^1, y^1, x^1, u^1 and v^1 refers to the mosquito population. Since not all the individuals bitten by infected mosquito develop disease, so we suppose F and F^1 be the proportion of infected man and mosquito who are actually infectious. We also need a time variable given by t. Let b be the man bitting rate in mosquito. Then, it follows that in time Δt, y^1 mosquito will make $bF^1y^1\Delta t$ infectious bites, of which a fraction $\frac{(n-y)}{n}$ are on susceptible humans. The number of new infections in human in time Δt is then $\frac{bF^1y^1(n-y)\Delta t}{n}$. The number of recoveries and deaths are given by $xy\Delta t$ and $vy\Delta t$ respectively in time Δt. If all new births are assumed to be susceptible, the birth rate is does not appear explicitly at this stage. The basic differential equation describing the rate of change of the human infected population is given by

$$\frac{dy}{dt} = \frac{bF^1y^1(n-y)}{n} - (x+v)y. \tag{2.1}$$

A similar argument for the mosquito population in time Δt is $-\frac{bF\,y(n^1-y^1)\Delta t}{n^1}$, this yields the equation

(2.2) $$\frac{dy^1}{dt} = \frac{bFy(n^1 - y^1)}{n^1} - (x^1 + v^1)y^1 .$$

If a constant mosquito population is assumed, the birth and death rate must balance and v^1 can be replaced by u^1, with this approximation equation (2.2) become

(2.3) $$\frac{dy^1}{dt} = \frac{bFy(n^1 - y^1)}{n^1} - u^1y^1 .$$

Since there are two dependent variables y and y^1, it's convenient to visualize the solution as a curve or trajectory in the (y, y^1) plane called phase plane.

There are three types of trajectory:

(1) The solution will grow without bound as time increases.

(2) The solution can approach a fixed limit cycle around an equilibrium point.

(3) The solution can approach a fixed equilibrium point.

The first case is not a possible solution because the solution trajectory must remain within the rectangle $0 \le y \le n;\ 0 \le y^1 \le n^1$ in phase plane. There are also no periodic solution contained entirely within the relevant region given by $0 \le y \le n,\quad 0 \le y^1 \le n^1$. Thus for the encephalitis trajectories (1) and (2) are not possible.

3. Stability Analysis

We shall use stability analysis to locate the possible equilibrium points and then to decide which equilibrium points are stable.

The equilibrium points occur when

$$\frac{dy}{dt} = 0 \text{ and } \frac{dy^1}{dt} = 0 .$$

$\frac{dy}{dt} = 0$ implies $\frac{bF^1y^1(n-y)}{n} - (x+v)y = 0$, i.e.

(3.1) $$y^1 = \frac{n(x+v)y}{bF^1(n-y)},$$

and $\frac{dy^1}{dt} = 0$ implies $\frac{bFy(n^1 - y^1)}{n^1} - u^1y^1 = 0$, i.e.

(3.2) $$y^1 = \frac{bFyn^1}{bFy + n^1u^1} .$$

Form equations (3.1) and (3.2), we have

$$\frac{n(x+v)y}{bF^1(n-y)} = \frac{bFyn^1}{bFy+n^1u^1}.$$

Solving the above relation, we get

(3.3) $$y = \frac{nn^1b^2FF^1 - nn^1(x+v)u^1}{b^2FF^1n^1 + n(x+v)bF}.$$

Putting the value of y in (3.1), we get

(3.4) $$y^1 = \frac{nn^1b^2FF^1 - nn^1(x+v)u^1}{b^2FF^1n + n^1bF^1u^1}.$$

We find that there are two possible equilibrium points

$$\left| \begin{array}{l} y = \bar{y} = 0 \\ \\ y^1 = \overline{y^1} = 0 \end{array} \right| \quad \text{and} \quad \left| \begin{array}{l} y = \bar{\bar{y}} = \dfrac{nn^1b^2FF^1 - nn^1(x+v)u^1}{b^2FF^1n^1 + n(x+v)bF} \\ y^1 = \overline{\overline{y^1}} = \dfrac{nn^1b^2FF^1 - nn^1(x+v)u^1}{b^2FF^1n^1 + n^1bF^1u^1} \end{array} \right|.$$

Since all the solutions of above equations are always within a rectangle $0 \le y \le n$, $0 \le y^1 \le n^1$ and all parameters in the expression for y and y^1 are positive, it follows that

1. If $b^2FF^1 \le (x+v)u^1$, the only equilibrium points are at $\bar{y} = 0$ and $\overline{y^1} = 0$.
2. If $b^2FF^1 > (x+v)u^1$, then both equilibrium points are possible so long $y \le n$ and $y^1 \le n^1$.

4. Stability

We must decide whether the solution will move to $\left(\bar{y}, \overline{y^1}\right)$ or to $\left(\bar{\bar{y}}, \overline{\overline{y^1}}\right)$, when both are possible. To do so, we linearize the governing equations around the equilibrium point (0,0). Deleting the quadratic terms in the governing equations, we get

(4.1) $$\frac{dy}{dt} = -(x+v)y + bF^1y^1,$$

(4.2) $$\frac{dy^1}{dt} = bFy - u^1y^1.$$

Equations (4.1) and (4.2) may be written as

(4.3) $$\frac{dX}{dt} = MX,$$

where $X = \begin{pmatrix} y \\ y^1 \end{pmatrix}$ and $M = \begin{pmatrix} -(x+v) & bF^1 \\ bF & -u^1 \end{pmatrix}$.

Solving (4.3), we get

$$\log X = Mt + C .$$

Since $X = X_0$ at $t = 0$, therefore $C = \log X_0$, which yields

(4.4) $$X = X_0 e^{\lambda t} .$$

Hence, the solution is in exponential form.

From equations (4.3) and (4.4), we get

$$\frac{d}{dt}\left(X_0 e^{\lambda t}\right) = MX ,$$

$$X_0 e^{\lambda t} . \lambda = MX ,$$

(4.5) $$MX = \lambda X .$$

Non-trivial solution of equation (4.5) exists only for certain values, called eigen values of the matrix M. Equation (4.5) can be written as homogeneous equation

$$(M - \lambda I)X = 0 .$$

We know that non-trivial solution of homogeneous system exists if the determinant of the coefficients is equal to zero, i.e. $|M - \lambda I| = 0$. This is called characteristic equation and its roots determine the stability. Thus

$$\begin{vmatrix} -(x+v+\lambda) & bF^1 \\ bF & -u^1 - \lambda \end{vmatrix} = 0 , \text{ i.e.}$$

(4.6) $$\lambda^2 + \lambda(x+v+u^1) + (x+v)u^1 - b^2 FF^1 = 0 .$$

Equation (4.6) is quadratic in λ, hence the roots of the equation (4.6) are given by

(4.7) $$\lambda = \frac{1}{2}\left\{-\left(x+v+u^1\right) \pm \sqrt{\left(x+v+u^1\right)^2 + 4\left[b^2 FF^1 - (x+v)u^1\right]}\right\}$$

It is clear from equation (4.7) that both roots are always real and distinct.

Case 1: If $b^2 FF^1 \leq (x+v)u^1$ then

(4.8) $$\lambda_1 < 0, \ \lambda_2 < 0, \ \lambda_1 \neq \lambda_2 ,$$

i.e. the roots of the auxiliary equation are real, negative and distinct. Hence the solution is in the form

$$X = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \text{ or } X = \sum_{i=1}^{2} C_i e^{\lambda_i t} ,$$

where C_i is arbitrary constant i.e.

(4.9) $$\begin{vmatrix} y \\ y^1 \end{vmatrix} = \sum_{i=1}^{2} C_i e^{\lambda_i t},$$

where C_i is parameterized the initial small distribution form (0,0).

Case 2: If $b^2 FF^1 > (x+v)u^1$ then

(4.10) $$\lambda_1 > 0, \ \lambda_2 < 0,$$

i.e. the roots of the auxiliary equation are real and of unlike sign. In this case the solution of the equations (4.1) and (4.2) is also given by equation (4.9).

It is clear form the nature of the roots λ_1 and λ_2 that if $b^2 FF^1 \le (x+v)u^1$ then null solution is asymptotically stable, while if $b^2 FF^1 > (x+v)u^1$ then null position is unstable but the epidemic level is asymptotically stable.

5. Result and Discussion

The solution of equations (2.2) and (2.3) are given by equation (4.9) and the nature of the roots is discussed by equations (4.8) and (4.10). If we plot a graph of equation (4.9) then it is an asymptotic curve which shows that the disease was already present in the society from the very beginning, but we are unable to identify a particular disease at primary level. Thus encephalitis is not a Navaki Bimary (New Disease), it had been exist in old days.

In our mathematical analysis, null-solution (disease free equilibrium) exists when $b^2 FF^1 \le (x+v)u^1$. Thus, the control of disease is depend on reduction of biting rate b in mosquito. To minimize biting rate b, we must reduce contact between man and mosquito through proper setting of houses, use of mosquito proof screen and netting, the wearing of protective clothing and the application of insect repellants.

The reduction of mosquito population n^1 (pool of infection) is also major segment to control of disease. There are of course alternative approaches of taking measures directly against mosquito –

1. Residual insecticide likes DDT or BHC especially where the mosquito enters houses and feed on man.
2. Antilarval measures like draining swamps or spraying larvicides on pool of water which may be used as mosquito breeding sites.

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