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Anti-invariant Submanifold of K-manifold

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Abstract: In this paper we have studied hypersurface of K-manifold and obtained some of its basic properties. It has been proved that the manifold is cosymplectic if and only if second fundamental form is proportional to *A(X)A(Y)*. In the last section we have discussed some properties of Anti-invariant submanifold of a K-manifold.

1. Introduction

Let $\overline{M}$ be an m-dimensional complex K-manifold with GF-structure J and Hermitian metric G satisfying

(1.1a) $$J^2 = a^2 I,$$

(1.1b) $$G(JX,JY) = -a^2 G(X,Y),$$

(1.1c) $$\overline{\nabla}_X J = 0,$$

where $a \neq 0$ is constant real or imaginary, I is identity operator, $\overline{\nabla}$ is the Riemannian connexion in $\overline{M}$ and X, Y, Z are arbitrary vector fields on $\overline{M}$. $\overline{M}$ is called nearly K-manifold if (1.1a), (1.1b) and

(1.2) $$(\overline{\nabla}_X J)(Y) + (\overline{\nabla}_Y J)(X) = 0,$$

are satisfied[1] for arbitrary vector fields X and Y on $\overline{M}$.

2. Hypersurface

Let M be the hypersurface of $\overline{M}$ with the immersion map b and the corresponding Jacobian map B. If g is induced metric in M and N is unit normal vector to M, then

(2.1a) $$g(X,Y) = G(BX,BY)ob,$$

(2.1b) $$G(N,N) = 1,$$

(2.1c) $$G(N,BX) = 0,$$

for arbitrary vector fields X and Y in M.

Let ∇ be the Riemannian connexion on M. Then Gauss and Weingarten equation respectively are

(2.2a) $$\overline{\nabla}_{BX} BY = B\nabla_X Y + h(X,Y)N,$$

and

(2.2b) $$\overline{\nabla}_{BX} N = a^2 BH(X),$$

where h and H are the second fundamental forms on M such that

$$h(X,Y) = g(H(X),Y) = g(X,H(Y)) = h(Y,X),$$

i.e. h is symmetric.

It is known that hypersurface of a K-manifold admits a structure (F, T, A, g) satisfying[1]

(2.3a) $$F^2 X = a^2(X - A(X)T),$$

(2.3b) $$FT = 0,$$

(2.3c) $$A(T) = 1,$$

(2.3d) $$AF(X) = 0,$$

and

(2.4) $$g(FX,FY) = -a^2 g(X,Y) - A(X)A(Y),$$

where F is a tensor field of type (1,1), T is a vector field and A is an 1-form on M. We call such M with the structure (F, T, A, g) as ACM-manifold.

In ACM-manifold, we have

(2.5) $$g(FX,Y) = -g(X,FY)$$

and

(2.6) $$g(X,T) = A(X).$$

An ACM-manifold on which

(2.7) $$\nabla_X F = 0 \Leftrightarrow (\nabla_X F)(Y,Z) = 0,$$

is called Cosymplectic manifold.

An ACM-manifold on which

(2.8) $$(\nabla_X F)(Y,Z) = A(Y)(\nabla_X A)(FZ) - A(Z)(\nabla_X A)(FY),$$

is called a generalized cosymplectic manifold[2].

Let us put

(2.9) $$J(BX) = BFX + A(X)N$$

and

(2.10) $JN = a^2 BT$.

By Gauss and Weingarten formulae and by (2.9) and 2.10), we have[2]

$$\begin{aligned}\overline{\nabla}_{BX} JBY &= \overline{\nabla}_{BX}(BFY + A(Y)N)\\ &= B\nabla_X FY + h(X,FY)N + A(Y)\overline{\nabla}_{BX} N\\ &= B(\nabla_X F)Y + B(F\nabla_X Y) + h(X,FY)N + A(Y)a^2 BH(X),\end{aligned}$$

Also,

$$\begin{aligned}\overline{\nabla}_{BX} JBY &= (\overline{\nabla}_{BX} J)BY + J\overline{\nabla}_{BX} BY\\ &= J(B\nabla_X Y + h(X,Y)N)\\ &= BF\nabla_X Y + A(\nabla_X Y)N + h(X,Y)a^2 BT .\end{aligned}$$

Comparing the above two, and collecting the tangential and normal parts, we get

$$(\nabla_X F)Y + a^2 A(Y)H(X) = a^2 h(X,Y)T, \text{ i.e}$$

(2.11) $$(\nabla_X F)Y = a^2(h(X,Y)T - A(Y)H(X))$$

and

$$A(\nabla_X Y) = h(X,FY)$$

which implies

(2.12) $$(\nabla_X A)Y = -h(X,FY).$$

The equation (2.11) implies

(2.13) $$(\nabla_X F)(YZ) = a^2(h(X,Y)A(Z) - A(Y)h(X,Z)).$$

Now $(\nabla_X F)(YZ) = 0$ implies

(2.14) $$h(X,Y)A(Z) - A(Y)h(X,Z) = 0,$$

which implies

$$h(T,T)A(Z) - A(T)h(T,Z) = 0.$$

This, in view of (2.3c), gives

(2.15) $$h(T,T)A(Z) - h(T,Z) = 0.$$

Putting T for Z in (2.14), we get

$$h(X,Y)A(T) - A(Y)H(X,T) = 0, \text{ which implies}$$

(2.16) $$h(X,Y) - A(Y)H(X,T) = 0.$$

From (2.15) and (2.16), we get

(2.17) $h(X,Y) = A(X)A(Y)h(T,T)$.

On the other hand, if we suppose (2.17), then (2.12) implies

$$A(\nabla_X Y) = A(X)A(FY)h(T,T) = 0,$$

for $AF = 0$. This implies

$$(\nabla_X F)Y = 0.$$

Thus we have:

Theorem 2.1: *Hypersurface M of a K-manifold is cosymplectic if and only if the second fundamental form* $h(X,Y) = A(X)A(Y)$, *if* $h(T,T) = 1$.

Since $(\nabla_X F)(Y,Z) = a^2(h(X,Y)A(Z) - A(Y)h(X,Z))$ is equivalent to $(\nabla_X F)(Y) = a^2(h(X,Y)T - A(Y)HX)$, we find that if $HX = 0$, we have

(2.18) $\nabla_X F = 0$.

Hence M is Cosymplectic.

On the other hand, if $\nabla_X F = 0$, we have

$$h(X,Y)T = A(Y)HX,$$

but this is satisfied if and only if $HX = 0$.

Hence we have:

Theorem 2.2: *The generalized cosymplectic hypersurface of K-manifold is cosymplectic if and only if* $HX = 0$.

For a cosymplectic manifold the condition

$$(\nabla_T F)T = 0$$

implies

(2.19) $F(\nabla_T T) = 0$.

Operating with F on (2.19) and using (2.3a), we get

$$\nabla_T T - A(\nabla_T T)T = 0.$$

Using (2.6), we get

$$\nabla_T T - g(\nabla_T T, T)T = 0,$$

which implies

$$\nabla_T T = 0,$$

for $g(\nabla_T T, T) = 0$.

From (2.4) and (2.6), we have

(2.20) $$g(FX, FY) = -a^2 g(X,Y) - g(X,T)g(Y,T).$$

Differentiating (2.20) covariently with respect to T, we get

$$g(FX, (\nabla_T F)(Y) + F(\nabla_T Y)) + g((\nabla_T F)X + F(\nabla_T X), FY)$$

$$= -a^2(g(X, \nabla_T Y) + g(\nabla_T X, Y)) - g(\nabla_T X, T)g(Y,T) - g(X.T)g(\nabla_T Y, T),$$

which is equivalent to

$$g(FX, (\nabla_T F)Y) + g(FX, F(\nabla_T Y)) + g((\nabla_T F)X, FY) + g(F(\nabla_T X), FY)$$

$$= -a^2(g(X, \nabla_T Y) + g(\nabla_T X, Y)) - g(\nabla_T X, T)g(Y,T) - g(X,T)g(\nabla_T Y, T).$$

Since $\nabla_T X = 0$ for an arbitrary vector field, the above implies that[3]

$$g(FX, (\nabla_T F)Y) + g((\nabla_T F)X, FY) = 0.$$

Using the property of cosymplectic manifold, we get

$$g(FX, (\nabla_Y F)T) + g((\nabla_X F)T, FY) = 0$$

implies $g(FX, F\nabla_Y T) + g(F\nabla_X T, FY) = 0$.

Using (2.4), we get

$$-a^2 g(X, \nabla_Y T) - A(X)A(\nabla_Y T) - a^2 g(\nabla_X T, Y) - A(\nabla_X T)A(Y) = 0,$$

which implies

$$g(X, \nabla_Y T) + g(\nabla_X T, Y) = 0,$$

for $A(\nabla_X T) = g(\nabla_X T, T) = 0$. From the above equation, we get

$$(\nabla_Y A)X + (\nabla_X A)Y = 0.$$

Thus we conclude:

Theorem 2.3: *On a cosymplectic manifold*

$$(\nabla_X A)(Y) = -(\nabla_Y A)(X).$$

From (2.18)

$$(\nabla_X F)Y = a^2(h(X,Y)T - A(Y)HX).$$

Let $HX = X$, then we get

$$(\nabla_X F)(Y) = a^2(h(X,Y)T - A(Y)X).$$

Putting $Y = T$, we get

$$(\nabla_X F)(T) = a^2 h(X,T)T - a^2 A(T)X$$

$$= a^2 A(X)T - a^2 X,$$

for $A(T)=1$, $h(X,Y)=A(X)A(Y)h(T,T)$ and taking $h(T,T)=1$.

This implies $-F(\nabla_X T) = -a^2(X - A(X)T)$

$$= -F^2 X$$

Thus, we have[4]:

Theorem 2.4: *If a hypersurface M of K-manifold $\overline{M}$ is cosymplectic with the condition h(T, T) =1, and HX = X, then* $F(\nabla_X T) = F^2 X$.

Let M be a generalized cosymplectic manifold, then

$$(\nabla_X F)(Y,Z) = A(Y)(\nabla_X A)(FZ) - A(Z)(\nabla_X A)(FY).$$

From this, we have

$$(\nabla_X F)(Y,FZ) + (\nabla_Y F)(X,FZ)$$

$$= A(Y)(\nabla_X A)(F^2 Z) - A(FZ)(\nabla_X A)(FY) + A(X)(\nabla_Y A)(F^2 Z) - A(FZ)(\nabla_Y A)(FY),$$

$$= A(Y)(\nabla_X A)(F^2 Z) + A(X)(\nabla_Y A)(F^2 Z), \text{ (since } AF = 0)$$

using (2.12), we get

$$(\nabla_X F)(Y,FZ) + (\nabla_Y F)(X,FZ) = -A(Y)h(X,F^3 Z) - A(X)h(Y,F^3 Z)$$

using (2.3a), we find

$$(\nabla_X F)(Y,FZ) + (\nabla_Y F)(X,FZ) = -a^2(A(Y)h(X,FZ) + A(X)h(Y,FZ)).$$

If $A(X)h(y,FZ) + A(Y)h(X,FZ) = 0$ then $(\nabla_X F)(Y,FZ) + (\nabla_Y F)(X,FZ) = 0$.

This leads to:

Theorem 2.5: *On a generalized cosymplectic manifold*

$$(\nabla_X F)(Y,\overline{Z}) + (\nabla_Y F)(X,\overline{Z}) = 0$$

if and only if

$$A(X)h(Y,\overline{Z}) + A(Y)h(X,\overline{Z}) = 0,$$

where $\overline{X} = FX$.

Definition: An ACM-manifold for which

(2.21) $(\nabla_X F)(Y,Z) + (\nabla_X F)(\overline{Y},Z) = a^2 A(Z)\{A(\nabla_X Y) + A(\nabla_Y X)\} - a^2 A(Y)A(\nabla_Z X)$,

is called normal.

Suppose M is a hypersurface of K-manifold, then

$$(\nabla_X F)(Y,Z) = a^2 h(X,Y)A(Z) - a^2 A(Y)h(X,Z).$$

Therefore

$$(\nabla_X F)(Y,Z)+(\nabla_X F)(\bar{Y},Z)$$
$$= a^2 h(\bar{X},Y)A(Z)-a^2 A(Y)h(\bar{X},Z)+a^2 h(X,\bar{Y})A(Z)-a^2 A(\bar{Y})h(X,Z)$$
$$= a^2 A(\nabla_Y X)A(Z)-a^2 A(Y)A(\nabla_Z X)+a^2 A(\nabla_X Y)A(Z).$$
$$= a^2 A(Z)\{A(\nabla_Y X)+A(\nabla_X Y)\}-a^2 A(Y)A(\nabla_Z X),$$

for $A(\bar{Y})=AF(Y)=0$, $A(\nabla_X Y)=h(X,FY)$ and h is symmetric.

Hence we have:

Theorem 2.6: *The hyper surface of K-manifold is an ACM-normal manifold.*

Let M be a nearly K-manifold. Then from (2.12), we have

(2.22) $$(\nabla_X F)(Y,Z)+(\nabla_Y F)(X,Z)$$
$$= a^2\{h(X,Y)A(Z)-A(Y)h(X,Z)\}+a^2\{h(Y,X)A(Z)-A(X)h(Y,Z)\}$$
$$= 2a^2 h(X,Y)A(Z)-a^2\{A(Y)h(X,Z)+A(X)h(Y,Z)\}.$$

From (2.12), we find

(2.23) $$(\nabla_X A)(Y)+(\nabla_Y A)(X) = -h(X,\bar{Y})-h(Y,\bar{X}).$$

Taking $\bar{Y}$ in place of Y in (2.23), we get

$$(\nabla_X A)(\bar{Y})+(\nabla_Y A)(X) = -h(X,\bar{\bar{Y}})-h(\bar{Y},\bar{X})$$
$$= h(X,a^2(Y-A(Y)T))-h(\bar{Y},\bar{X})$$
$$= a^2 h(X,Y)-a^2 A(Y)h(X,T)-h(\bar{Y},\bar{X}),$$

this implies

$$h(X,Y)=a^2\{(\nabla_X A)(\bar{Y})+(\nabla_Y A)(X)+h(\bar{Y},\bar{X})\}+A(Y)h(X,T).$$

Putting it in (2.22), we get

$$(\nabla_X F)(Y,Z)+(\nabla_Y F)(X,Z)$$
$$= 2A(Z)\{(\nabla_X A)(\bar{Y})+(\nabla_{\bar{Y}} A)(X)+h(\bar{Y},\bar{X})\}$$
$$+\ 2a^2 A(Y)A(Z)h(X,T)-A(Y)\{(\nabla_X A)(\bar{Z})+(\nabla_Z A)(X)$$
$$+\ h(\bar{Z},\bar{X})\}-a^2 A(Z)A(Y)h(X,T)-A(X)\{(\nabla_Y A)(\bar{Z})$$
$$+\ (\nabla_Z A)(Y)+h(\bar{Z},\bar{Y})\}-a^2 A(X)A(Z)h(Y,T).$$

If $(\nabla_X A)Y + h(X,Y) = A(X)A(Y)$, we get

$$(\nabla_X F)(Y,Z) + (\nabla_Y F)(X,Z)$$

$$= -A(Y)((\nabla_X A)(\overline{Z}) - A(X)(\nabla_Y A)(\overline{Z}) + A(Z)\{(\nabla_X A)(\overline{Y}) + (\nabla_Y A)(\overline{X})\}$$

$$= \{A(Z)(\nabla_X A)(\overline{Y}) - A(Y)(\nabla_X A)(\overline{Z})\} + \{A(Z)\{(\nabla_Y A)(\overline{X}) - A(X)(\nabla_Y A)(\overline{Z})\},$$

for $A(\overline{X}) = 0$. Hence we have:

Theorem 2.7: *The hypersurface of a nearly K-manifold is generalized cosymplectic if and only if*

(2.24) $$(\nabla_X A)(Y) + h(X,Y) = A(X)A(Y).$$

Nijenhuis tensor corresponding to F is given by

(2.25) $$N_F(X,Y) = (\nabla_{FX} F)(Y) - (\nabla_{FY} F)(X) + F(\nabla_Y F)(X) - F(\nabla_X F)(Y).$$

From (2.11) and (2.12), we get

$$N_F(X,Y) = a^2\{h(FX,Y)T - A(Y)H(FX)\} - a^2\{h(X,FY)T - A(X)H(FY)\}$$

$$+ a^2\{-A(X)F(H(Y))\} - a^2\{-A(Y)F(H(X))\} \quad \text{(since } AF = 0\text{)}$$

$$= a^2\{A(\nabla_Y X)T - A(Y)H(FX)\} - a^2\{A(\nabla_X Y)T - A(X)H(FY)\}$$

$$- a^2 A(X)F(H(Y)) + a^2 A(Y)F(H(X)).$$

Since $AF = 0$, we have

$$F(N_F(X,Y)) = 0.$$

Hence we have:

Theorem 2.8: *On a hypersurface of K- manifold we have*

$$F(N_F(X,Y)) = 0.$$

Definition: If on an ACM-manifold, the following are hold

(2.26a) $$A(\nabla_X \overline{Y}) = A(\nabla_{\overline{X}} Y) = -A(\nabla_Y X),$$

(2.26b) $$A(\nabla_X Y) = -A(\nabla_{\overline{X}} \overline{Y}) = -A(\nabla_Y X),$$

(2.26c) $$\nabla_T F = 0,$$

and

(2.27) $$(\nabla_X F)(Y,Z) = -A(Y)A(\nabla_{\overline{X}} Z) - A(Z)A(\nabla_Y \overline{X}),$$

then the manifold is called generalized cosymplectic manifold of second class.

Let us consider a generalized cosymplectic manifold of second class which is also normal. This means, we have (2.21), (2.26a), (2.26b), (2.26c) and (2.27).

Using (2.26c) in (2.21), we get

$$(\nabla_{\overline{X}} F)(Y,Z) + (\nabla_X F)(\overline{Y},Z) = -a^2 A(Y) A(\nabla_Z X).$$

Using (2.27) in above, we get

$$-A(Y)A(\nabla_{\overline{X}} Z) - A(Z)A(\nabla_Y, \overline{\overline{X}}) - A(\overline{Y})A(\nabla_{\overline{X}} Z) - A(Z)A(\nabla_{\overline{Y}} \overline{X}) = -a^2 A(Y)A(\nabla_Z X).$$

In view of (2.26a), we have

$$-a^2 A(Y)A(\nabla_X(-Z + A(Z)T) - 2A(Z)a^2 A(\nabla_Y(-X + A(X)T)) = -a^2 A(Y)A(\nabla_Z X),$$

$$A(Y)A(\nabla_X Z) - A(Y)A(\nabla_X(A(Z)T)) + 2A(Z)A(\nabla_Y X) - 2A(Z)A(\nabla_Y(A(X)T))$$
$$= -A(Y)A(\nabla_Z X).$$

Utilizing (2.26b), we find

$$-A(Y)A(Z)A(\nabla_X T) + 2A(Z)A(\nabla_Y X) - 2A(Z)A(X)A(\nabla_Y T) = 0.$$

Using (2.12), we get

$$-A(Y)A(Z)h(X,FT) + 2A(Z)h(X,FY) - 2A(Z)A(X)h(X,YT) = 0,$$

which implies

$$A(Z)h(X,FY) = 0.$$

Thus, we have:

Theorem 2.9: *On a normal generalized cosymplectic manifold of second class*

(2.28) $$A(Z)h(X,FY) = 0.$$

3. Anti-invariant Submanifold of K-manifold

Let M be an anti-invariant submanifold immersed in a K-manifold $\overline{M}$, then $X = A(X)T$.

In view of this, we have

$$g(X,X) = g(A(X)T, A(X)T)$$
$$= (A(X))^2 g(T,T).$$

Since $g(T,T) = A(T) = 1$, we have

$$g(X,X) = (A(X))^2.$$

Putting T in place of Y, the equation (2.2a) may be written as

(3.1) $$\overline{\nabla}_{BX} BT = B\nabla_X T + h(X,T)N .$$

Multiplying (2.10) by a^2 and using $a^4 = 1$, we get

$$BT = a^2 JN .$$

Hence $$\overline{\nabla}_{BX} BT = a^2((\overline{\nabla}_{BX} J)N + J(\overline{\nabla}_{BX} N)) .$$

Using (2.2b), we get

$$\overline{\nabla}_{BX} BT = a^2((\overline{\nabla}_{BX} J)N + Ja^2 BH(X))$$

Since $\overline{M}$ is a K-manifold, $\overline{\nabla}_{BX} J = 0$. Therefore

$$\overline{\nabla}_{BX} BT = JBH(X) ,$$

which, in view of (2.9), gives

(3.2) $$\overline{\nabla}_{BX} BT = A(H(X))N .$$

From (3.1) and (3.2), we get $\nabla_X T = 0$ and $h(X,T) = A(H(X))$.

Thus we have:

Theorem 3.1: *The following hold in an anti-invariant submanifold of a K-manifold*

(3.3) $$\nabla_X T = 0 ,$$

(3.4) $$h(X,T) = A(H(X)) ,$$

and

(3.5) $$g(X,X) = (A(X))^2 .$$

Now

(3.6) $$\overline{\nabla}_{BX}(JBY) = (\overline{\nabla}_{BX} J)(BY) + J(\overline{\nabla}_{BX} BY) .$$

Interchanging X and Y, we get

(3.7) $$\overline{\nabla}_{BY}(JBX) = (\overline{\nabla}_{BY} J)(BX) + J(\overline{\nabla}_{BY} BX) .$$

Adding (3.6) and (3.7), we get

(3.8) $$\overline{\nabla}_{BX}(JBY) + \overline{\nabla}_{BY}(JBX) = (\overline{\nabla}_{BX} J)(BY) + (\overline{\nabla}_{BY} J)(BX) + J(\overline{\nabla}_{BX} BY) + J(\overline{\nabla}_{BY} BX) .$$

Let $\overline{M}$ be a nearly K-manifold, then

(3.9) $$(\overline{\nabla}_{BX} J)(BY) + (\overline{\nabla}_{BY} J)(BX) = 0 .$$

Using (2.2a), we get

$$\overline{\nabla}_{BX}(JBY) + \overline{\nabla}_{BY}(JBX)$$
$$= J(B\nabla_X Y)Jh(X,Y)N + J(B\nabla_Y X) + Jh(Y,X)N$$

$$= J(B\nabla_X Y) + J(B\nabla_Y X) + 2h(X,Y)JN .$$

From (2.9) and (2.10), we get

(3.10) $$\overline{\nabla}_{BX}(JBY) + \overline{\nabla}_{BY}(JBX)$$

$$= BF(\nabla_X Y) + A(\nabla_X Y)N + BF(\nabla_Y X) + A(\nabla_Y X)N + 2a^2 h(X,Y)BT$$

$$= A(\nabla_X Y)N + A(\nabla_Y X)N + 2a^2 h(X,Y)BT ,$$

for $F(\nabla_X Y) + F(\nabla_Y X) = 0$.

Also

(3.11) $$\overline{\nabla}_{BX}(JBY) + \overline{\nabla}_{BY}(JBX)$$

$$= (\nabla_X A)(Y)N + (\nabla_Y A)(X)N + a^2 A(Y)B(H(X)) + a^2 A(X)B(H(Y)),$$

Comparing (3.10) and (3.11), we get

(3.12) $$(\nabla_X A)Y + (\nabla_Y A)X = A(\nabla_X Y) + A(\nabla_Y X) ,$$

and

(3.13) $$A(Y)H(X) + A(X)H(Y) = 2h(X,Y)T .$$

Thus, we have:

Theorem 3.2: *Let M be an anti-invariant submanifold of a nearly K-manifold, then* (3.12) *and* (3.13) *hold for any vector field X, Y on* $\overline{M}$.

From (3.13), we have

(3.14) $$A(Y)h(X,Z) + A(X)h(Y,Z) = 2h(X,Y)A(Z) .$$

Putting T in place of Y and Z and using $A(T) = 1$, we get

(3.15) $$h(X,T) \quad A(X)h(T,T) .$$

Putting $Z = T$ in (3.14), we get

$$A(Y)h(X,T) + A(X)h(Y,T) = 2h(X,Y) .$$

Using (3.15), we get

$$A(Y)A(X)h(T,T) + A(X)A(Y)h(T,T) = 2h(X,Y) ,$$

which implies $h(X,Y) = A(X)A(Y)h(T,T)$.

If $h(T,T) = 1$, we have $h(X,Y) = A(X)A(Y)$.

Thus, we have:

Theorem 3.3: *In an anti-invariant submanifold of a K-manifold* $\overline{M}$, *the second fundamental form* $h(X,Y) = A(X)A(Y)$ *if* $h(T,T) = 1$.

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