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Two Unit System Requiring Separation of Units for Repair Under Circumstantial and Common Cause Failure

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Abstract : The present work is concerned with a two unit system under the effect of common cause and circumstantial failures in which manual operations are required for the separation of the failed unit form the system and installation of the unit after repairs. Supplementary variable technique has been used to derive expressions of interest and numerical computations have been performed to validate the results.

Introduction

One of the essential requirements for the estimation of reliability of a system is the observance of the conditions under which it operates. Most of the studies with redundant components are based on automatic replacement of failed components with the new one. Even in the present age of information technology, there are industries working with systems which adopt the manual replacement of the failed components. In such organizations, once a component fails, the system has to be stopped from functioning for the removal of the failed components. In a similar manner the system needs to be stopped when the component is to be reinstalled after completion of repairs. The time lost in replacement and installation has not been accounted for in most of the conducted studies. This over estimates the reliability of such systems.

Present paper is devoted to the study of a two-unit system under the effect of circumstantial and common cause failure that requires manual operations for the separation of failed unit from the system and installation of the unit after the repairs in an effort to obtain more realistic reliability estimates.

Model Description and Underlying Assumptions

The state transition diagram of the present system is given in Fig. 1. The state S_0 of the diagram consists of two units N_0 and N_s in perfectly operating condition. The system is assumed to remain in operative state as long as even a single unit remains operative. Once a unit fails, it can be repaired only after separating it from the main system. For this task, the system needs to be stopped for a constant time. After competition of repairs, once again system is made non-operational for sometime to reinstall the repaired unit. The above two situations are represented in the form of transition between the states S_1-S_2 and S_3-S_0 in the diagram.

The system besides its mechanical failure, may also suffer from the circumstantial and common cause failures. The failed states having failures on account of CF and CCF are represent by *Sc* and *Scc* respectively and these can be arrived from any of the up states S_F represents the state of mechanical failure. We assume a general repair time distribution from the states S_F and *Scc* for its transition to the perfectly normal state after the repairs. Ofcourse when the system is down under the effect of *CF*, it after recovery, transits back to the state from where it suffered *CF*.

Notations

N_0 : Operating unit

N_s : Standby unit

N_r : Failed unit under repair,

N_{ri}: Repaired unit to be installed

N_{fs} : Failed unit to be separated from the system

λ : Constant failure rate of the two identical units

λc : Constant failure rate due to circumstantial and common cause failure

p : Probability that the failure is *CF*

$q = (1 - p)$: Probability that the failure is *CCF*

w : Constant rate of separation

μ_1 : Constant repair rate of the unit

μ_2 : Constant repair time for *CF*

r : Constant rate for installation

$\alpha(x)/\beta(x)$: Repair rates from state *F* / *CC* to state 0.

$$S_k(s) = \int k(x) \exp\left[- sx - \int_0^x k(x)\, dx \right] dx,$$

$k = \alpha, \beta$: The integral $\int$ means definite integral from 0 to ∞, unless otherwise mentioned.

$$H_k(s) = \frac{1 - S_k(s)}{5}; \; k = \alpha, \beta$$

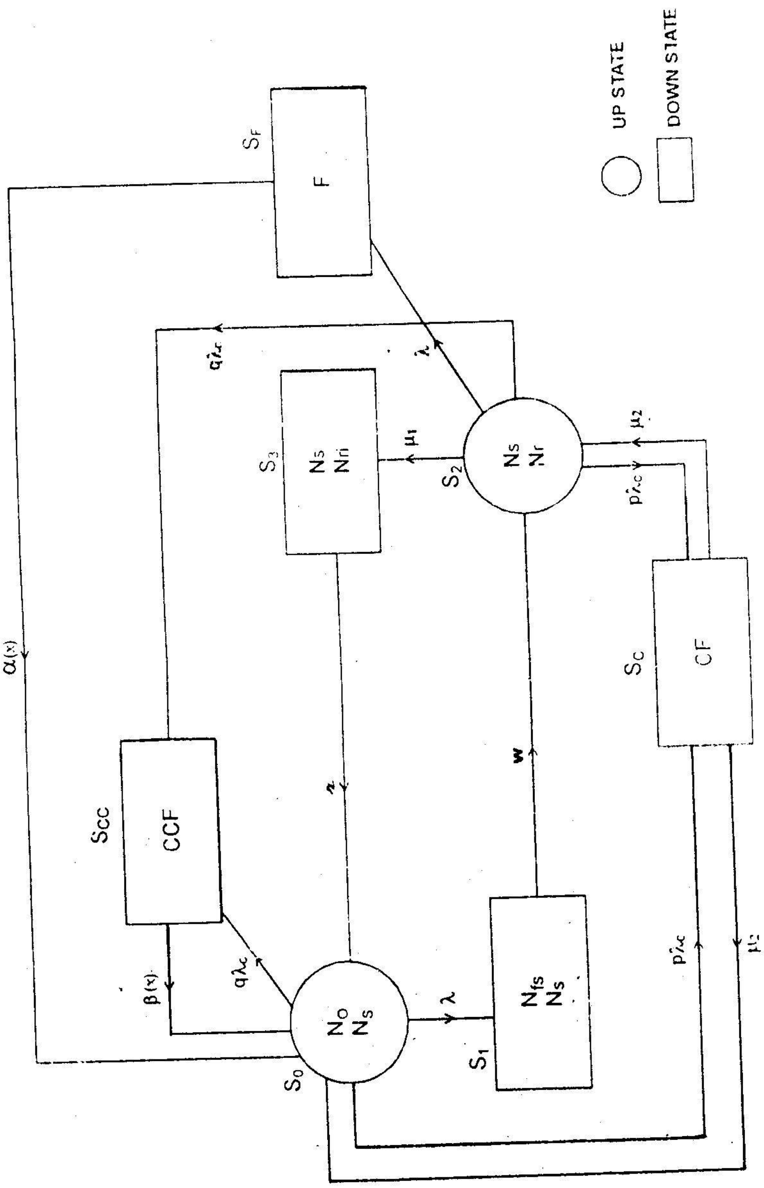

Fig. 1 : State Transition Diagram

$$I = \lambda + \lambda_c ; \qquad I_1 = \lambda + \lambda_c + \mu_1$$

$$A(s) = \frac{p\lambda_c \{(s + w)(s + I_1) + w\lambda\}}{\{(s + 2\mu_2)(s + I_1) - p\lambda_c \mu_2\}(s + w)}$$

$$B(s) = \frac{\mu_2 A(s)(s + w) + w\lambda}{(s + I_1)(s + w)}.$$

Mathematical Formulation of the Problem

Using above notations and the state space diagram of the model given in Fig. 1, one gets to the following system of differential equations for the process :

(1)
$$\frac{dP_0(t)}{dt} + \left(\lambda + p\lambda_c + q\lambda_c\right) P_0(t)$$

$$= rP_3(t) + \mu_2 P_c(t) + \int \alpha(x) P_F(x, t)\, dx + \int \beta(x) P_{cc}(x, t)\, dx$$

$$\frac{dP_1(t)}{dt} + w P_1(t) = \lambda P_0(t)$$

$$\frac{dP_2(t)}{dt} + \left(\lambda + p\lambda_c + q\lambda_c + \mu_1\right) P_2(t) = \mu_2 P_c(t) + w P_1(t)$$

$$\frac{dP_3(t)}{dt} + r P_3(t) = \mu_1 P_2(t)$$

$$\frac{dP_c(t)}{dt} + \left(\mu_2 + \mu_2\right) P_c(t) = p\lambda_c\left[P_0(t) + P_2(t)\right]$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x)\right] P_F(x, t) = 0 \qquad \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta(x)\right] P_{cc}(x, t) = 0.$$

With boundary condition

(2)
$$P_F(0, t) = \lambda P_2(t); \quad P_{cc}(0, t) = q\lambda_c\left[P_0(t) + P_2(t)\right].$$

Initial conditions

(3)
$$P_0(0) = 1; \; P_1(0) = P_2(0) = P_3(0) = P_c(0) = P_{cc}(0) = P_F(0) = 0.$$

Solution of the Differential Equations

To solve the system of differential equations alongwith the initial and boundary conditions given from (1 – 3), we take the Laplace transforms of these equations and the boundary conditions. This reduces the differential equations into the following system of algebraic equations :

(4) $$(s + 1) P_0(s) = 1 + rP_3(s) + \mu_2 P_c(s) + \int \alpha(x) P_F(x, s)\, dx + \int \beta(x) P_{cc}(x, s)\, dx$$

$$(s + w) P_1(s) = \lambda P_0(s)$$

$$(s + l_1) P_2(s) = \mu_2 P_c(s) + w P_1(s)$$

$$(s + r) P_3(s) = \mu_1 P_2(s)$$

$$(s + 2\mu_2) P_c(s) = p \lambda_c \left[P_0(s) + P_2(s) \right]$$

(5) $$\left[s + \frac{\partial}{\partial x} + \alpha(x) \right] P_F(x, s) = 0 \qquad \left[s + \frac{\partial}{\partial x} + \beta(x) \right] P_{cc}(x, s) = 0$$

(6) $$P_F(0, s) = \lambda P_2(s); \quad P_{cc}(0, s) = q \lambda_c \left[P_0(s) + P_2(s) \right].$$

Solving above equations for $P_j(s)$, $j = 0, 1, 2, 3, C, F, CC$ one gets the following Laplace transform of state probabilities :

(7) $$P_0(s) = \frac{1}{K(s)} \qquad P_1(s) = \frac{\lambda}{(s + w) K(s)} \qquad P_2(s) = \frac{B(s)}{K(s)}$$

$$P_3(s) = \frac{\mu_1 B(s)}{(s + r) K(s)} \qquad P_c(s) = \frac{A(s)}{K(s)}$$

$$P_F(s) = \frac{\lambda B(s) H_\alpha(s)}{K(s)} \qquad P_{cc}(s) = \frac{q \lambda_c [1 + B(s)] H_\beta(s)}{K(s)}$$

where

(8) $$K(s) = (s + 1) - r \mu_1 B(s)/(s + r) - \mu_2 A(s) - \lambda B(s) S\alpha(s) - q\lambda_c [1 + B(s)] S_\beta(s).$$

Laplace Transforms of Up and Down State Probabilities

The laplace transforms of the probability that at time t, the system remains in up state (operable state) is given by following expressions :

$$P_{up}(s) = P_0(s) + P_2(s)$$

$$= \frac{[1 + B(s)]}{K(s)}$$

$$P_{down}(s) = P_1(s) + P_3(s) + P_c(s) + P_F(s) + P_{cc}(s)$$

$$= \frac{1}{K(s)}\left[\frac{\lambda}{(s + w)} + \frac{\mu_1 B(s)}{(s + r)} + A(s) + \lambda B(s) H_\alpha(s) + q\lambda_c \{1 + B(s)\} H_\beta(s)\right]$$

It can easily be verified that

$$P_{up}(s) + P_{down}(s) = \frac{1}{s}$$

Availability Analysis

As a particular case, let us assume that the repairs follow exponential time distributions. We can then write

$$S_\alpha(s) = \frac{\alpha}{s + \alpha} \qquad S_\beta(s) = \frac{\beta}{s + \beta}$$

Setting $\alpha = \beta = \phi$, say, and using equation (7) and (8), the Laplace transform of state probabilities reduce to the following expressions :

(9) $$\bar{P}_0(s) = \frac{1}{\bar{K}(s)} \qquad \bar{P}_2(s) = \frac{B(s)}{\bar{K}(s)}$$

$$\bar{P}_1(s) = \frac{\lambda}{(s + w)\bar{K}(s)} \qquad \bar{P}_3(s) = \frac{\mu_1 B(s)}{(s + r)\bar{K}(s)}$$

$$\bar{P}_c(s) = \frac{A(s)}{\bar{K}(s)} \qquad \bar{P}_F(s) = \frac{\lambda B(s) H_\alpha(s)}{\bar{K}(s)}$$

(10) $$\bar{P}_{cc}(s) = \frac{q\lambda_c[1 + B(s)] H_\beta(s)}{\bar{K}(s)}$$

where

(11) $$\overline{K}(s) = (s + 1) - r\mu_1 B(s)/(s + r) - \mu_2 A(s) - \lambda B(s)\phi/(s + \phi)$$

$$- q\lambda_c[1 + B(s)]\phi/(s + \phi).$$

For obtaining $P_{up}(s)$ we use the following additional notations :

$$M_1 = 2\mu_2 + I_1;\ M = r + M_1;\ N = I_1 r + 2\mu_2\left(I_1 + r\right);\ T_1 = 2\mu_2 I_1;$$

$$T = T_1 r;\ Y = I_1 + w;\ T_7 = I_1 + r;\ T_8 = w\lambda;\ T_9 = T_8 r\mu_1$$

$$G = wI_1 + T_8;\ J_1 = Y + M;\ J_2 = G + MY + N;\ J_3 = MG + NY + T;$$

$$J_4 = NG + YT;\ T_2 = \mu_2 p\lambda_c;\ T_3 = q\lambda_c;\ T_6 = I_1 r;\ N_2 = T_6 + wT_7$$

$$N_1 = T_7 + w;\ N_3 = T_6 w;\ T_4 = \lambda + T_3;\ T_5 = T_1 - T_2.$$

From equation (9) and (11), one can obtain

(12) $$R(s) = \overline{P}_0(s) + \overline{P}(s)$$

$$= \frac{[1 + B(s)]}{\overline{K}(s)}$$

$$= \frac{s^6 + c_1 s^5 + c_2 s^4 + c_3 s^3 + c_4 s^2 + c_4 s^2 + c_5 s + c_6}{s^7 + d_1 s^6 + d_2 s^5 + d_3 s^4 + d_4 s^3 + d_5 s^2 + d_6 s + d_7}$$

where

(13) $$c_1 = J_1 + \phi;\ c_2 = J_2 + J_1\phi;\ c_3 = J_3 + J_2\phi$$

$$c_4 = J_4 + J_3\phi;\ c_5 = TG + J_4\phi;\ c_6 = TG\phi$$

$$d_1 = N_1 + M_1 I + \phi$$

$$d_2 = N_2 + M_1 N_1 + T_5 + I\left(N_1 + M_1\right) - T_2 + \phi\left(N_1 + M_1 + I - T_3\right)$$

$$d_3 = N_3 + M_1 N_2 + N_1 T_5 + I\left(N_2 + M_1 N_1 + T_5\right) - T_2\left(T_7 + Y\right)$$

$$+ \phi\left(N_2 + M_1 N_1 + T_5 + N_1 I + M_1 I - T_2 - T_3 M_1 - T_3 N_1\right)$$

$$d_4 = N_3 M_1 + N_2 T_5 + I\left(N_3 + M_1 N_2 + N_1 T_5\right) - T_2\left(T_6 + T_7 Y + G + r\mu_1\right)$$

$$- T_9 + \phi\left\{N_3 + M_1 N_2 + N_1 T_5 + I\left(N_2 + M_1 N_1 + T_5\right)\right.$$

$$\left. - T_2\left(T_7 + Y + T_4\right) - T_8 T_4 - T_3 N_2 - T_3 M_1 N_1 - T_3 T_5\right\}$$

$$d_5 = N_3 T_5 + I\left(N_3 M_1 + N_2 T_5\right) - T_2\left(Y T_6 + T_7 G + Y r \mu_1\right) - T_9 M_1$$

$$+ \phi\left\{N_3 M_1 + N_2 T_5 + I\left(N_3 + M_1 N_2 + N_1 T_5\right)\right.$$

$$- T_2\left(T_6 + T_7 Y + G + I + T_4 Y + T_4 r\right) - T_9 - T_8 T_4 M_1 (1 + r)$$

$$\left. - T_3 N_3 - T_3 M_1 N_2 - T_3 T_5 N_1\right\}$$

$$d_6 = N_3 T_5 I - T_2 G\left(\mu_1 r + T_6\right) - T_9 T_5 + \phi\left\{N_3 T_5 + I\left(N_3 M_1 + N_2 T_5\right)\right.$$

$$- T_2\left(Y T_6 + T_7 G + Y + T_4 G + T_4 Y r\right)$$

$$\left. - T_9 M_1 - T_8 T_4 T_5 - T_3 M_1 N_3 - T_3 T_5 N_2\right\}$$

$$d_7 = \phi\left\{T_5\left(N_3 I + T_9 + T_8 T_4 r\right) - T_2 G\left(T_6 + T_4 r + 1\right) - T_3 T_5 N_3\right\}$$

Now substituting $\phi = 1$ in equations (12) and (13), one gets the following expression for $P_{up}(s)$

(14) $$P_{up}(s) = \frac{s^6 + \bar{c}_1 s^5 + \bar{c}_2 s^4 + \bar{c}_3 s^3 + \bar{c}_4 s^2 + \bar{c}_5 s + \bar{c}_6}{s^7 + \bar{d}_1 s^6 + \bar{d}_2 s^5 + \bar{d}_3 s^4 + \bar{d}_4 s^3 + \bar{d}_5 s + \bar{d}_6 s + \bar{d}_7}$$

$$= \frac{F(s)}{G(s)}, \text{ say}$$

where the coefficients $\bar{c}_1$ to $\bar{c}_6$ and $\bar{d}_1$ to $\bar{d}_7$ are obtained by putting $\phi = 1$, in equation (13). To obtain the operational availability $P_{up}(t)$ of the system we shall have to take the inverse Laplace transform of $P_{up}(s)$.

Steady State Behaviour of the System

Using corollary to Abel's lemma, one gets

$$P_{up} = \lim_{s \to 0} s\, P_{up}(s)$$

$$= \lim_{r \to 0} s\left[\bar{P}_0(s) + \bar{P}_2(s)\right]$$

$$= \lim_{s \to 0} \frac{s\,[1 + B(s)]}{\bar{K}(s)}$$

$$= \frac{[1 + B(s)]}{\bar{K}(0)}$$

where

$$\bar{K}'(0) = \left[\frac{d}{ds}\{\bar{K}(s)\}\right]_{s=0}$$

$$= 1 - \mu_1 B'(0) + \frac{\mu_1 B(0)}{r} - \mu_2 A'(0) - B'(0)\left(\lambda + q\lambda_c\right)$$

$$+ \frac{\{(\lambda + q\lambda_c) B(0) + q\lambda_c\}}{\phi}$$

where $A'(0)$and $B'(0)$ are obtained by differentiating $A(s)$ and $B(s)$ respectively at $s = 0$ $P_{down} = 1 - P_{up}$.

Time and Variance of the Time to Failure

To obtain the expression for MTTF and variance to the time to system failure we put $\phi = 0$, in equations (12) and (13). Therefore

$$R(s) = \frac{s^6 + c_1^* s^5 + c_2^* s^4 + c_3^* s^3 + c_4^* s^2 + c_5^* s + c_6^*}{s^7 + d^* s^6 + d^* s^5 + d^* s^4 + d^* s^3 + d^* s^2 + d^* s + d^*}$$

where c_1^* to c_6^* and d_1^* to d_7^* are obtained by putting $\phi = 0$, in equation (13).

(15) $$MTTF = \lim_{s \to 0} R(s) = \frac{c_5^*}{d_6^*}$$

$$\sigma^2 = -2 \lim_{s \to 0} \frac{dR(s)}{ds} - (MTTF)^2$$

(16) $$= \frac{2c_5^* d_5^* - 2c_4^* d_6^* - c_5^{*2}}{d_6^{*2}}.$$

Numerical Computations

Letting $\lambda = 0.17$, $\lambda_c = 0.13$, $w = 0.1$, $\mu_1 = 0.21$, $\mu_2 = 0.2$, $r = 0.3$, $p = q = 0.5$. We get the following expression for $P_{up}(t)$

(17) $$P_{up}(t) = 0.8642758 - 1.238981 * 10^{-3} \exp(-0.2716\, t) - 0.4885545 \exp(-0.0457\, t)$$

$$+ \exp(-1.1713\, t)\,[0.3786046 \cos(0.5006\, t) + 0.1176984 \sin(0.5006\, t)$$

$$+ \exp(-0.2300\, t)\,[0.246913 \cos(0.4315\, t) + 0.27552403 \sin(0.4315\, t)]$$

Table 1 lists values for the availability of the system for different values of time parameter ranging from $t = 0$ to $t = 7$ with step size 0.2. Availability vs time graph has been given in Fig. 2. It can be observed that the availability deceases with increase in time. The decrease is sharp initially and with further increase in time it stabilizes.

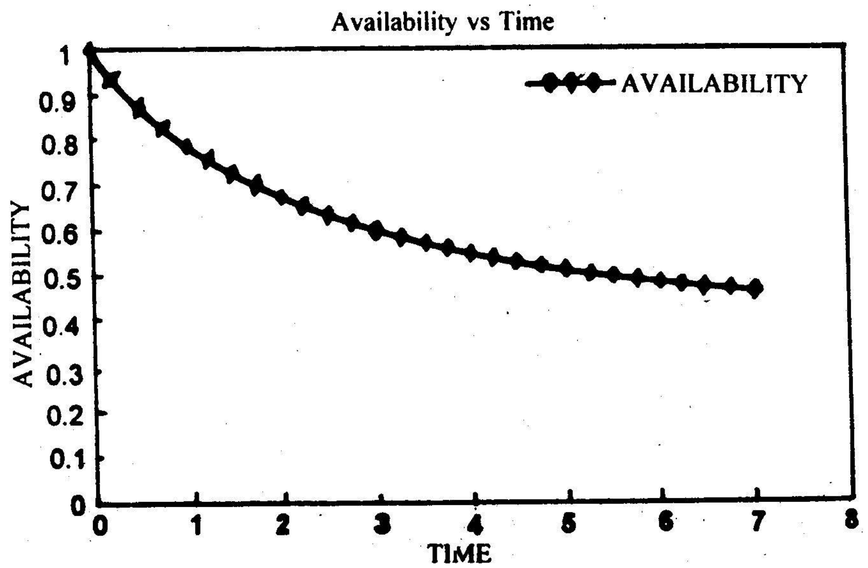

Table 1

Availability vs Time

Time	Availability	Time	availability
0	1	3.6	0.5718948
0.2	0.9439275	3.8	0.5605106
0.4	0.8953375	4.0	0.5496079
0.6	0.8535214	4.2	0.5392288
0.8	0.8176178	4.4	0.5294183
1.0	0.7867262	4.6	0.5202207
1.2	0.7599801	4.8	0.5116778
1.4	0.7365892	5.0	0.5038265
1.6	0.7158630	5.2	0.4966980
1.8	0.6972176	5.4	0.4903169
2.0	0.6801756	5.6	0.4847009
2.2	0.6643587	5.8	0.4798604
2.4	0.6494776	6.0	0.4757988
2.6	0.6353200	6.2	0.4725126
2.8	0.6217391	6.4	0.4699919
3.0	0.6086413	6.6	0.4682206
3.2	0.5959758	6.8	0.4671772
3.4	0.5837246	7.0	0.4668356

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