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Fabrication of Optical Filters Using Photonic Layer Structures

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Abstract : This paper describes a new idea for the design of optical filters using Photonic Bandgap (PBG) materials in the ultraviolet region of the electromagnetic spectrum. The idea is based on the famous Kronig-Penny model in the band theory of solids. The periodic structures consisting of metal-dielectric and metal-semiconductor are considered. We must emphasize that the suggested filter may work at any region of the electromagnetic spectrum. and it may also act as a monochromatic source.

1. Introduction

Photonic band gap structures have recently developed a great deal of interest and emerged as a new multidisciplinary field of study[1-4]. Advances in photonic technology have generated the new trend in photonic devices, which have great advantages over conventional electronic devices. They can offer very high speed of operation, increased lifetime, smaller size, have tolerance to temperature fluctuation, and the capability to care high repetition rates. All these devices work on the principle of photonic band edge and they are extremely compact in structure. These structures are composed of thin dielectrics, semiconductor materials or metallic slabs surrounded by air or other materials of lower refractive index in order to confine the light wave. The waves incident on these materials will be reflected if their frequency lies within the gap. The existence of the spectral gap in such photonic crystals opens up variety of possible potential applications such as thresholdless semiconductor lasers[5], efficient optical filters[6-7], endlessly single-mode optical fibers etc.[8]. Conventional gratings have index modulations of few percent whereas PBG materials have large index contrasts in their indices to the extent of 4 : 1[9]. Due to this large index contrasts wide stop and pass bands are obtained.

Ojha *et al.*[10] suggested a method for the fabrication of optical filters in the near and far infrared region. This model was based on the weak guidance approximation such that $(n_1 - n_2)/n_1) \ll 1$and the working principle is analogous to that of the Kronig-Penny model in the band theory of solids[11]. Chen *et al.*[12] also suggested the design of an optical filter using photonic band gap air bridges and calculated the important results regarding filtering properties.

In the present communication, we suggest the fabrication of optical filters in the ultraviolet region using the periodic refractive index profile of the metal-dielectric and metal- semiconductor. Actually the filter designed in this way may work over a wide range of the wavelength by choosing the suitable values of the controlling parameters, which are refractive indices, and thickness of the layers. In our previous work we have also studied the operating characteristics of the optical filters using the alternate layers of dielectric-semiconductor materials and air-metals for the different regions of the electromagnetic spectrum and obtained the some interesting results[13, 14]. Introduction of metals to photonic band gap provide certain advantages, which include reduced size and weight, easy for fabrication and lower costs.

2. Theoretical Analysis

Kronig and Penny first discussed the essential behavior of electron waves through a periodic potential in solid-state theory. If we consider the motion of an electron wave in periodic potential, we may expect the existence of allowed and forbidden bands. The same idea may be applicable to the case of optical radiation if the electron waves are replaced by optical waves and the lattice periodicity structure is replaced by a periodic refractive index pattern. Instead of allowed and forbidden energy bands, we now obtain transmitting and forbidden bands of wavelengths or frequencies. By choosing a linearly periodic refractive index profile in the filter material one obtains a given set of wavelength ranges that are allowed or forbidden to pass through the filter material. We now choose a particular x-axis through the material and assume a periodic step function for the refractive index in the form[15, 9] given as

(1) $$n(x) = \begin{cases} n_1, & 0 \le x \le a; \\ n_2, & -b \le x \le 0 \end{cases}$$

where $n_1(x + td) = n_1$ and $n_2(x + td) = n_2$. Here t is the translational factor, which takes the values $t = 0, \pm 1, \pm 2, \pm 3, \ldots$ and $d = a + b$ is the period of the lattice with a and b being the width of the two regions having refractive indices (n_1) and (n_2) respectively. In case of metallic conductor n_2 is complex quantity and in polar form it can be written as

(2) $$n_2 = \alpha + i\beta = r(\cos\theta + i\sin\theta)$$

Thus, the absolute of value of n_2 is given as

(3a) $$|n_2| = r = \left(\alpha^2 + \beta^2\right)^{1/2}$$

and its phase is given by

(3b) $$\theta = \tan^{-1}\left(\frac{\beta}{a}\right)$$

The structure which have to study is shown in the Figure 1 in the form of rectangular symmetry.

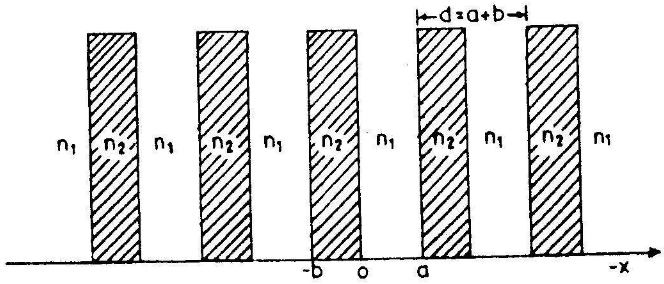

Fig 1 Shows the periodic variation of the refractive index profile in the form of rectangular structure.

The one-dimensional wave equation for lightwave propagating along the x-axis may be written as

(4) $$\frac{d^2\psi(x)}{dx^2} + \frac{n^2(x)\,\omega^2}{c^2}\,\psi(x) = 0$$

where $n(x)$ is given by Equation 1. Assuming that $n(x)$ is constant in the (n_1) and (n_2) regions, Equation 4 for wave equation may be written as

(5a) $$\frac{d^2\psi(x)}{dx^2} + \frac{n_1^2\,\omega^2}{c^2}\,\psi(x) = 0\,; \qquad 0 \le x \le a$$

(5b) $$\frac{d^2\psi(x)}{dx^2} + \frac{n_2^2\,\omega^2}{c^2}\,\psi(x) = 0\,; \qquad -b \le x \le 0$$

The solution of above equations must be Bloch functions of the form $\psi(x) = u_K(x)e^{iKx}$ where $u_k = u_K(x + d)$ and the quantity $K\,(= k + i\,k')$ is a complex quantity. Now applying boundary conditions as given below

(6a) $$u_1(x)\big|_{x=0} = u_2(x)\big|_{x=0}$$

(6b) $$u_1(x)\big|_{x=0} = u_2'(x)\big|_{x=0}$$

(6c) $$u_1(x)\big|_{x=a} = u_2(x)\big|_{x=-b}$$

and

(6d) $$u_1'(x)\big|_{x=a} = u_2'(x)\big|_{x=-b}$$

we get four by four homogeneous matrix equations having four unknown coefficients. In order to obtain a nontrivial solution for such a homogeneous system, the determinant of the coefficient matrix should be zero, which is given as

(7) $$\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} = 0$$

where

$$M_{11} = M_{12} = M_{13} = M_{14} = 1 ;$$

$$M_{21} = i(P_1 - K),\ M_{22} = -i(P_1 + K),\ M_{23} = i(P_2 - K),\ M_{24} = -i(P_2 + K);$$

$$M_{31} = e^{ia(P_1 - K)},\ M_{32} = e^{-ia(P_1 + K)},\ M_{33} = e^{-ib(P_2 - K)},\ M_{34} = e^{ib(P_2 + K)};$$

$$M_{41} = = i(P_1 - K)e^{ia(P_1 - K)}, \qquad M_{42} = -i(P_1 + K)e^{-ia(P_1 + K)};$$

$$M_{43} = i(P_2 - K)e^{-ib(P_2 - K)}, \qquad M_{44} = -i(P_2 + K)e^{ib(P_2 + K)};$$

where

$$P_1 = \left(\frac{n_1\omega}{c}\right) \text{ and } P_2 = \left(\frac{n_2\omega}{c}\right)$$

On solving equation 7 and equating real and imaginary parts of both sides of the equation we get two equations. In which only real part of equation represents the propagation of the light wave because in order to simplify the analysis, we consider the first approximation, accordingly we assume that the absorption in medium is very small i.e. $k' << k$. Under this approximation the final equation can be written as :

$$(8) \quad \cos\left(\frac{2\pi n_1 a}{\lambda}\right).\cos\left[\left(\frac{2\pi rb}{\lambda}\right).\cos\theta\right].\cosh\left[\left(\frac{2\pi rb}{\lambda}\right).\sin\theta\right]$$

$$-\sin\left(\frac{2\pi n_1 a}{\lambda}\right).\left[\left(\frac{n_1^2 + r^2}{2n_1 r}\right).\cos\theta.\sin\left[\left(\frac{2\pi rb}{\lambda}\right).\cos\theta\right].\cosh\left[\left(\frac{2\pi rb}{\lambda}\right).\sin\theta\right]\right]$$

$$+\sin\left(\frac{2\pi n_1 a}{\lambda}\right).\left[\left(\frac{n_1^2 - r^2}{2n_1 r}\right).\sin\theta.\cos\left[\left(\frac{2\pi rb}{\lambda}\right).\cos\theta\right].\sinh\left[\left(\frac{2\pi rb}{\lambda}\right).\sin\theta\right]\right] = \cos k\,(a + b)$$

Here we have used $\frac{\omega}{c} = \frac{2\pi}{\lambda}$, λ is the wavelength of the light wave propating through the medium.

The L.H.S. of equations (8) can be abbreviated and we can write as

$$(9) \qquad F_\lambda = \cos k\,(a + b)$$

Our aim is now to find the wavelength regions transmitted by the filter. To find this the characteristics (9) has been solved. For the illustration, we have considered the case of two metallic conductors (Copper and Zerconium) with three different kinds of materials (air, glass and zinc sulphate) for illustration. In first case we have alternate regions of copper-air, copper-glass and copper-zinc sulphate while in the second case we have zirconium-air, zirconium-glass and zirconium-zinc sulphate in alternative regions. We now consider these cases separately.

3. Results and Discussion

Fabrication of the Filter and Monochromator : We are now in a position to make some numerical estimates of the allowed and the forbidden bands. For the proposed filter, we have chosen the values of refractive indices in the first case as n_1 = 1.0 (air), 1.52 (glass), 2.2 (zinc sulphate) and n_2 = 1.03 + 0.98i (Cu) whereas in the second case n_2 has been chosen as 3.7 + 1.23i (Zr) and value of n_1 remain same as in the first case. The thickness of the alternate layers a and b are selected as 127.50 *nm* and 22.50 *nm*, which is taken according to Yablonovitch structure (a = 85% of d and b = 15% of d), respectively, in all the cases. Substituting all these values, Eqn. (9) is plotted against the wavelength and the curves so obtained are depicted in the Figures (2a), (2b), (2c) and (3a), (3b), (3c) respectively. Because of the existence of the cosine function on the right-hand side of the Eqn. (9), the upper and lower limiting values will be obviously + 1 and – 1 respectively. The portion of the curve lying between these limiting values will give the allowed ranges of wavelength and those outsides will show the forbidden ranges of transmission. Observation of Figs. (2a), (2b), (2c) and (3a), (3b), (3c) shows that the width of the allowed wavelength bands increases as we move towards the higher wavelength side which are shown in the Tables (1a), (1b), (1c) and (2a), (2b), (2c) respectively. Filters having alternate layers of Cu-air, Cu-glass and Cu-ZnS give five, eight and ten pass bands while the alternate layers of Zr-air, Zr-glass and Zr-ZnS structure give seven, nine and thirteen allowed respectively. Thus it is observed that the introduction of high index materials, in place of air, to the metals increase the number of bands with comparatively smaller bandwidth. One of the interesting features we find in the case of Cu-glass and *Zr*-glass filter is that Cu-glass filter passes maximum wavelengths i.e. has greater transmission where as the Zr-glass filter gives least transmission for the selected values of the parameters. At the same time Cu-glass filter acts as wide band frequently/wavelength selector in the near ultraviolet whereas the Zr-glass filter acts as narrow band frequency/wavelength selector. Finally, it is concluded that all the filters act as a wide band frequency or wavelength selector in the far ultraviolet region whereas very narrow band frequency or wavelength selectors in the near ultra violet region except Cu-air filter which is normally acts as wide band frequency or wavelength selector.

This suggested filter may be made to act as monochromator, which can pass the light of a single or nearly single wavelength in the desired optical region. This can be achieved by controlling the thickness of the two regions without changing the refractive indices. In our illustrative case we have selected n_1 = 1.0 (air), n_2 = 1.03 + 0.98i (Cu) with the values of lattice parameters a and b as given below :

(i) a = 150 *nm*, b = 68 *nm*

(ii) a = 240 *nm*, b = 86 *nm*.

Using these values in Eqation (9), we plot the curve that is depicted in the Figures (4a) and (4b). The allowed range of transmitted wavelength can be seen in these figures, which are approximately 799.13 – 800.20 Å (with the difference 1.07 Å) and 1009.90 – 1011.00 Å (with the difference 1.10 Å). This means that the filter allows nearly the single wavelength showing the property of a monochromator.

Table (1a) Ultraviolet region (500-3000 Å)

For ($n_1 = 1.0$, $n_2 = 1.03 + 0.98i$, $a = 127.50nm$ and $b = 22.50nm$)

No. Of Bands	Allowed Ranges (in A^0)	Bandwidth (in A^0)
1.	540-550	10
2.	659-685	26
3.	829-909	80
4.	1086-1345	259
5.	1566-2629	1063

Table (1b) Ultraviolet region (500-3000 Å)

For ($n_1 = 1.52$, $n_2 = 1.03 + 0.98i$, $a = 127.50nm$ and $b = 22.50nm$)

No. Of Bands	Allowed Ranges (in A^0)	Bandwidth (in A^0)
1.	510-515	5
2.	578-589	11
3.	663-685	22
4.	773-817	44
5.	924-1012	88
6.	1146-1334	188
7.	1521-1976	455
8.	2278-3000	722

Table (1c) Ultraviolet region (500-3000 Å)

For ($n_1 = 2.2$, $n_2 = 1.03 + 0.98i$, $a = 127.50nm$ and $b = 22.50nm$)

No. Of Bands	Allowed Ranges (in A^0)	Bandwidth (in A^0)
1.	531-536	5
2.	580-589	9
3.	639-653	14
4.	709-731	22
5.	796-830	34
6.	909-961	52
7.	1063-1146	83
8.	1281-1425	144
9.	1612-1889	277
10.	2166-2824	658

Table (2a) Ultraviolet region (500-3000 Å)

For ($n_1 = 1.0$, $n_2 = 3.7 + 1.23i$, $a = 127.50nm$ and $b = 22.50nm$)

No. Of Bands	Allowed Ranges (in A^0)	Bandwidth (in A^0)
1.	549-552	3
2.	641-647	6
3.	760-774	14
4.	891-916	25
5.	1191-1265	74
6.	1526-1712	186
7.	2442-3000	558

Table (2b) Ultraviolet region (500-3000 Å)

For ($n_1 = 1.52$, $n_2 = 3.7 + 1.23i$, $a = 127.50nm$ and $b = 22.50nm$)

No. Of Bands	Allowed Ranges (in A^0)	Bandwidth (in A^0)
1.	524-527	3
2.	574-578	4
3.	647-654	7
4.	735-745	10
5.	830-848	18
6.	982-1013	31
7.	1206-1279	73
8.	1484-1644	160
9.	1990-2333	343

Table (2c) Ultraviolet region (500-3000 Å)

For ($n_1 = 2.2$, $n_2 = 3.7 + 1.23i$, $a = 127.50nm$ and $b = 22.50nm$)

No. Of Bands	Allowed Ranges (in A^0)	Bandwidth (in A^0)
1.	501-503	2
2.	536-539	3
3.	577-580	3
4.	629-633	4
5.	689-697	8
6.	758-769	11
7.	842-858	16
8.	953-978	25
9.	1099-1142	43
10.	1284-1364	80
11.	1534-1679	145
12.	1931-2205	274
13.	2624-3000	376

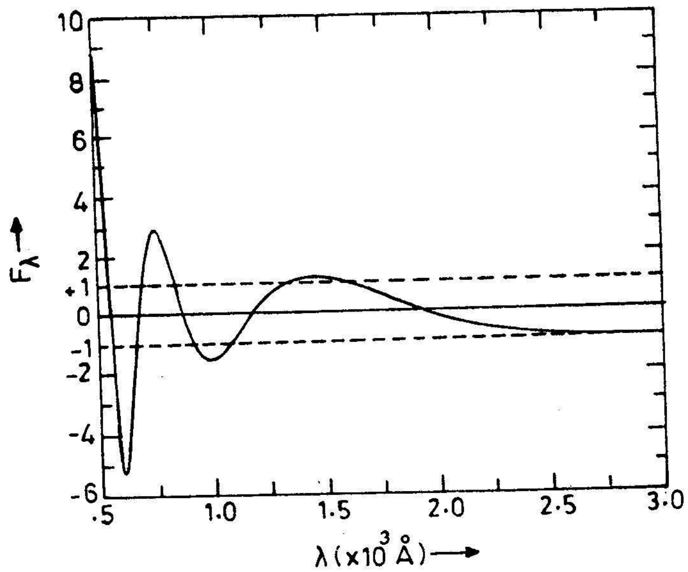

Figure 2a shows the variation of F_λ with the wavelength λ for $n_1 = 1.0$, $n_2 = 1.03 + 0.98i$ (Cu), $a = 127.50$ nm and $b = 22.50$ nm.

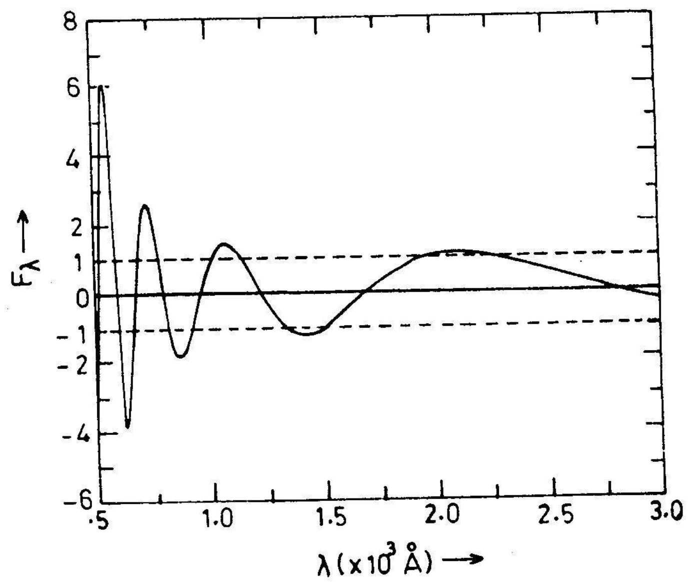

Figure 2b shows the variation of F_λ with the wavelength λ for $n_1 = 1.52$ (glass), $n_2 = 1.03 + 0.98i$ (Cu), $a = 127.50$ nm and $b = 22.50$ nm.

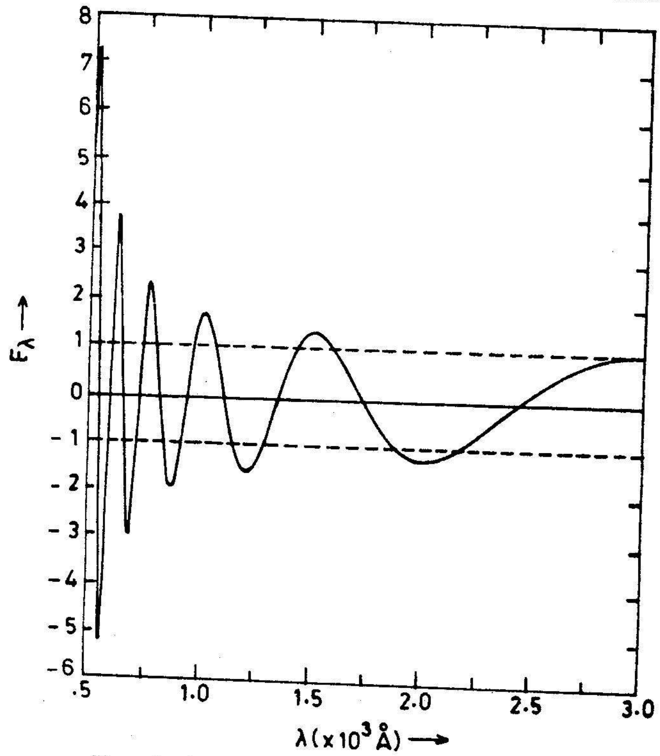

Figure 2c shows the variation of F_λ with the wavelength λ for $n_1 = 2.2$ (*ZnS*), $n_2 = 1.03 + 0.98i$ (*Cu*), $a = 127.50$ *nm* and $b = 22.50$ *nm*.

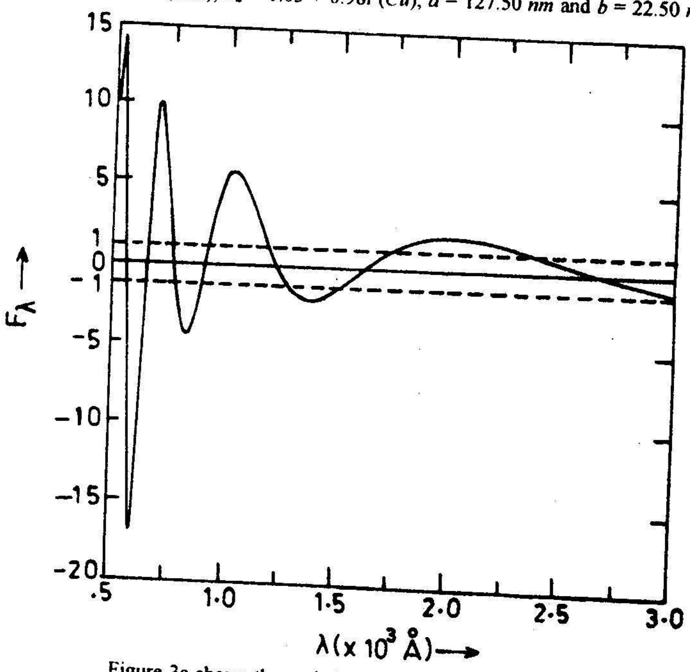

Figure 3a shows the variation of F_λ with the wavelength λ for $n_1 = 1.0$, $n_2 = 3.7 + 1.23i$ (*Zr*), $a = 127.50$ *nm* and b 22.50 *nm*.

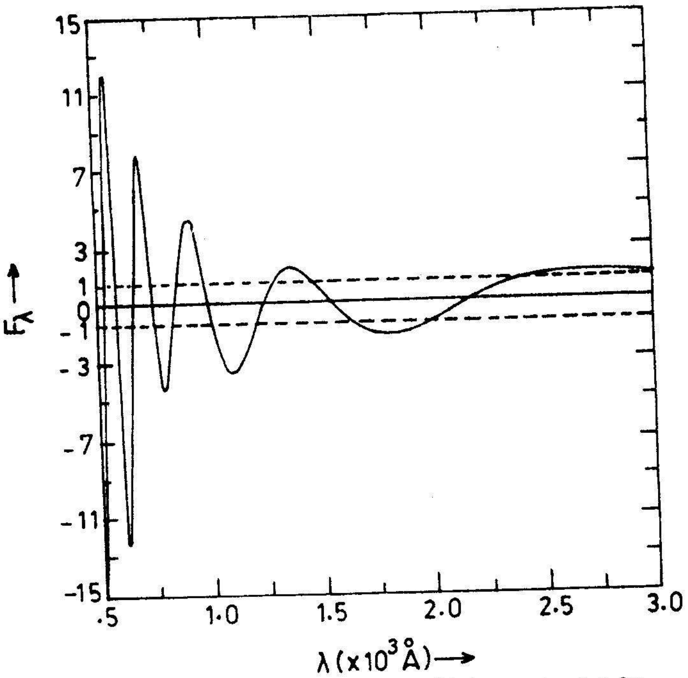

Figure 3b shows the variation of F_λ with the wavelength λ for n_1 = 1.52 (glass), n_2 = 3.7 + 1.23*i* (*Zr*), *a* = 127.50 *nm* and *b* = 22.50 *nm*.

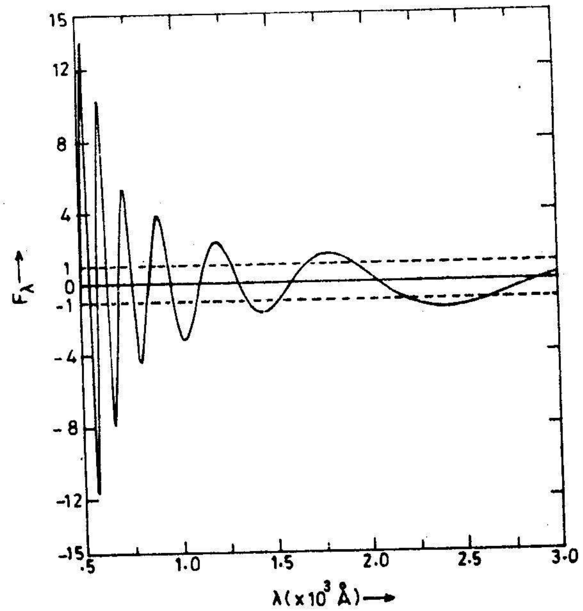

Figure 3c shows the variation of F_λ with the wavelength λ for n_1 = 2.2 (*ZnS*), n_2 = 3.7 + 1.23*i* (*Zr*), *a* = 127.50 *nm* and *b* = 22.50 *nm*.

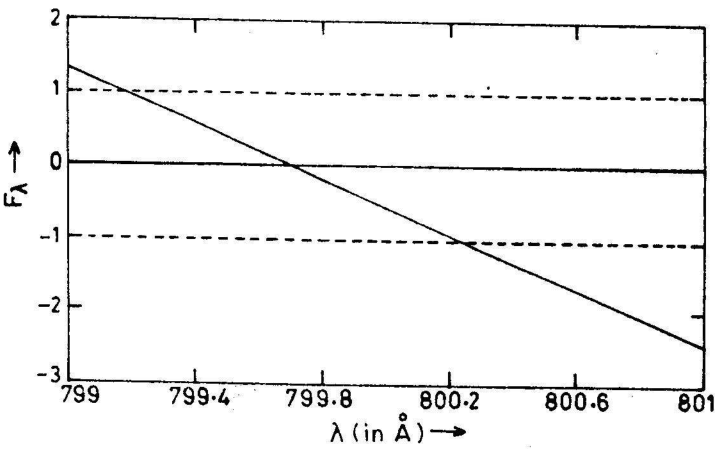

Figure 4a shows the monochromator for n_1 = 1.0 (air), n_2 = 1.03 + 0.98*i* (*Cu*), *a* = 150 *nm* and *b* = 68 *nm*.

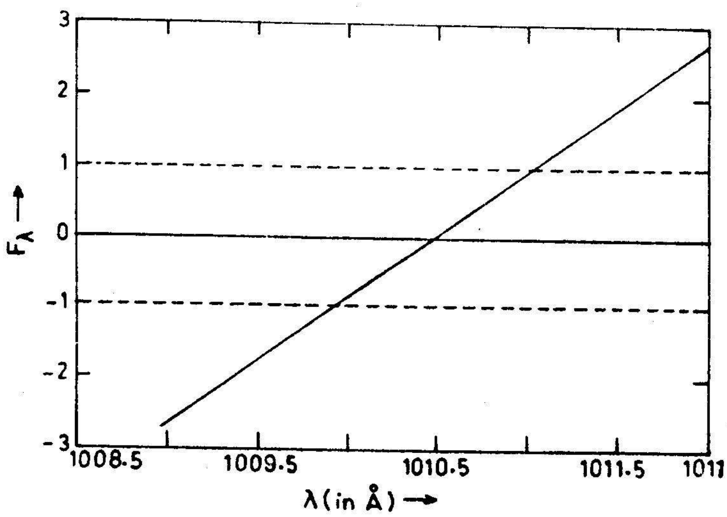

Figure 4b shows the monochromator for n_1 1.0 (air), n_2 = 1.03 + 0.98*i* (*Cu*), *a* = 240 *nm* and *b* = 86 *nm*.

It is mentioned here that the above calculations are made with particular values of parameters. The filter in other desired regions may also be designed by other choice of the various relevant parameters and has potential technical use.

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