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GERT Analysis of Process Control-Plan : Two-Phase Inspection

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Abstract : This paper presents a two-phase inspection process control plan to maintain high quality level of (not necessarily) production processes which are not well-behaved and/or subject to some deterioration. Graphical Evaluation and Review Technique (GERT) has been applied to model and analyse the dynamic characteristics of the plan.

1. Introduction

A production process, and for that matter any other process, generally tends to loose some of its efficiency in service because of deterioration of machines and equipments as time elapses. As a consequence, the quality of products from the process decreases in time and therefore, some corrective action should be taken. Not having full information of actual state of process, the decision is to be based on sampling. Possible corrective actions are (minimal) repairs, adjustment, renewals, which all can be performed after detection of the state of deterioration or as preventive actions. In order to minimise the chances of making these types of errors, we need to use the control chart and other statistical tools.

Recently, Shanker[2] developed a corrective action plan for the production processes using the repeated sample results of one inspection cycle (i.e. from in-control state to the out-of-control or deterioration state) of the process. Later on,the concept of two-phase inspection scheme introduced by Shanker[9] has been utilised by Shankar and Srivastava[12] to provide further modification to the Shankar's[2] plan. Subsequently Shankar and Sahu[11] developed a process control procedure in which the decision of corrective action is deferred. This paper presents a modification over two-phase inspection plan using different sample sizes in two-phases of inspection. The basis of the development of plan is differentiation of cause of variation in quality viz. variation due to deterioation of machines or variation produced by any potential assignable factor causing abrupt changes in product quality.

The main objective of this paper is to model and analyse the dynamic characteristic of processes which are not well-behaved and/or subject to some deterioration. An attempt has been made to model and analyse the dynamics of the plan through GERT (Graphical Evalution and Review Technique). A brief account of GERT methodlogy and its application in quality control has been given by Shankar[9]. The formula for OC and other performance characteristics of the plan have been derived by applying Mason's[6] rule on the GERT network representation of the inspection system. Numerical examples have been included to illustrate the mathematical findings. Lastly, Poisson unity value has been tabulated to facilitate the construction and operation of the plan. It is found through numerical calculations that a consecutive sequence of two marginals is optimum in sample size efficiency to control a process.

The inherent mechanism of deterioration of the process is shown in Fig.1 There is a very thin control line between the good and the bad quality performance of the process. Any improvement from in-control state towards control line and/or out-of-control state is referred to as *Deterioration of the Process.* A deterioration in in-control is not of much importance because they belong to the category of chance variation about which little can be done other than to revise the process

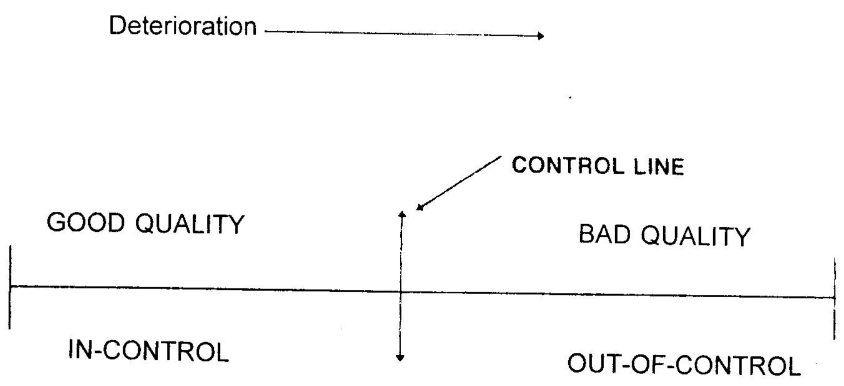

Fig. 1 : Deterioration Structure of the Process

2. Development of the Plan

Following the notations and concepts similar to those of Shankar2 the proposed process control plan characterised by n_1, n_2, i, c_1 and c_2 proceeds as follows :

Step 1 : Draw a sample n_1 successive units for normal inspection from the production line at more or less regular intervals of time and determine the numbere d of defectives found therein.

Step 2 : At any stage of Sampling/inspection, if

(a) $d \leq c_1$, Continue production. The process is in-control

(b) $d > c_2$, Stop process and hunt for potential assignable causes of variation. The process is out-of-control

(c) $c_1 < d \leq c_2$, Then switch to step 3 for tightened inspection.

Step 3 : Draw another sample of n_2 ($>n_1$) successive units from the production line and determine the number d of defectives found therein. At any stage of sampling under tightened inspection, if

(a) $d \le c_1$, Continue production. The process is in-control

(b) $d > c_2$. Stop process. The process is out-of-control

(c) $c_1 < d \le c_2$. Then defer the decision of corrective action until next ($i - 1$) sample under tightened inspection also alarm a state of corrective action i.e., in each of the ($i - 1$) succeeding samples $c_1 < d \le c_2$; otherwise repeat the Step 3.

Here, it may be noted that the inspection process is automatically switched to normal inspection (Step1) after each corrective action (maintenance) and/or improvement in the production process (due to out-of-control state).

3. GERT Analysis of the Plan

The possible states of the inspection system described above can be defined as follows :

S_0 : Initial state of the plan.

S_1 : State in which normal inspection is performed.

$S_2(k)$: State in which k^{th} [$k = 1, 2, 3, \ldots \ldots (i - 1)$] succeding sample also alarm a state of corrective action

S_{CA} : State in which corrective action is performed

SR : State in which process is interrupted for serious assignable causes.

The above states enable us to construct GERT network representation of the inspection system as shown in Fig. 2. In the present study reference has been made to following parameters:

(i) Sample size n (parameter θ)

(ii) Transition time t (parameter s)

The transition from initial in-control state to the deterioration state can be looked upon as a failure of the process. As gently known, the failure distribution in reliability theory is postulated as an exponential distribution. Therefore, holding time density function, $h(t)$, is taken as exponential with the parameter λ.

(3.1) $$h(t) = \lambda \exp(-\lambda t)$$

As a consequence, the Moment Generating Function (M.G.F.) of duration t assoociated with a branch is characterised by the M.G.F. of the form

$$M_t(s) = \int_0^\infty e^{st} e^{-\lambda t}\, dt = \frac{\lambda}{\lambda - s}$$

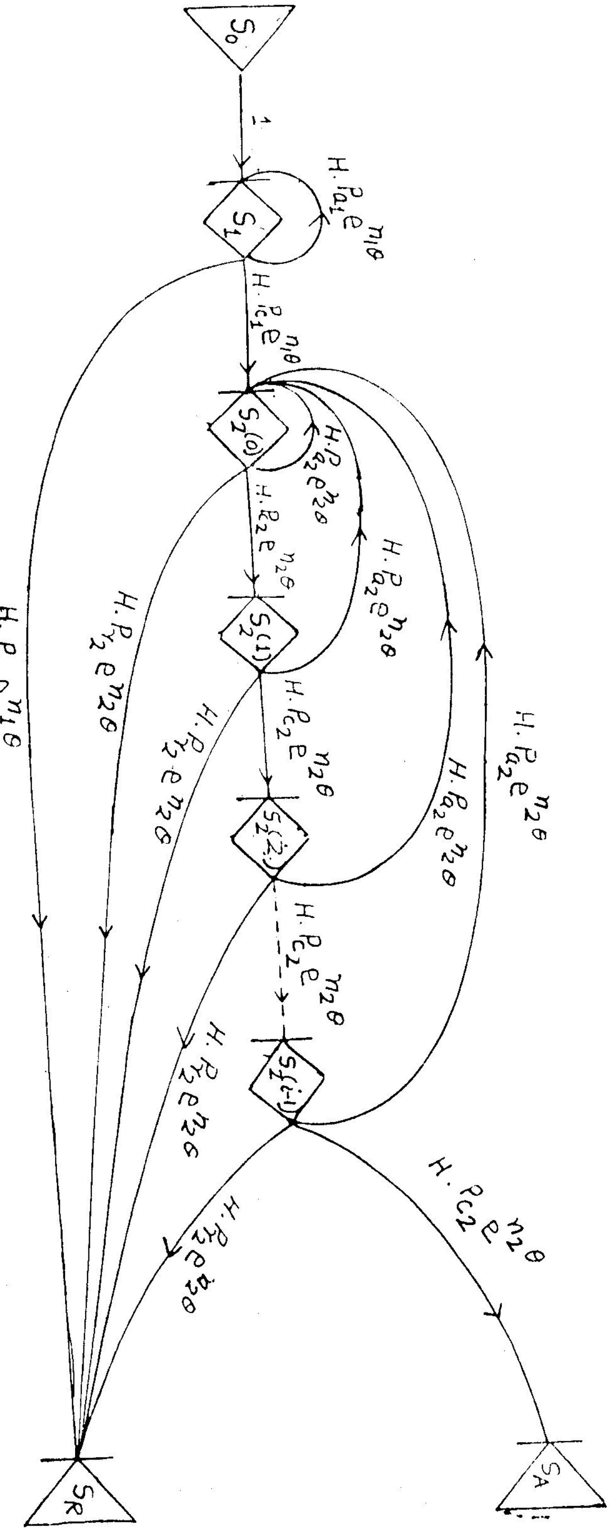

Fig 2. : GERT Network of Two-Phase Inspection Process Control Plan

The moment generating function (M.G.F.) of sample size n is $M_n(\theta) = \exp(n\theta)$, because n is constant and θ is any real variable

The W-function from the initial node S_0 to the corrective action S_{CA} and termination node S_R are respectively obtained by applying Mason's[6] rule on the GERT network representation in Fig. 2 as follows :

$$W_A(\theta, s) = \frac{H^{i+1} P_{c_1} P_{c_2}^{i} e^{(i n_1 + n_2)} (1 - P_{c_2} e^{(n_2 \theta)}}{G}$$

$$W_R(\theta, s) = \left[\frac{HP_r e^{(n_1 \theta)}\{ 1 - HP_{c_2} e^{(n_2 \theta)} - HP_{a_2} e^{(n_2 \theta)} (1 - (P_{c_2} e^{(n_2 \theta)} H)^i)\}}{G}\right]$$

where

$$G = (1 - P_{a_1} e^{(n_1 \theta)})[1 - HP_{c_2} e^{(n_2 \theta)} - HP_{a_2} e^{(n_2 \theta)} \{1 - (HP_{c_2} e^{(n_2 \theta)})^i\}]$$

and $H = \frac{\lambda}{(\lambda - s)}$. Now, from the definition of W-function, we obtain,

$$P_{CA} = \left[W_A(\theta, s) \right]_{\theta = s = 0} .$$

$$= \frac{P_{c_1} \cdot P_{c_2}^i (1 - P_{c_2})}{(1 - P_{a_1})(P_{r_2} + P_{a_2} P_{c_2}^i)} \tag{3.2}$$

$$P_R = \left[W_R(\theta, s) \right]_{\theta = s = 0}$$

$$= \frac{P_{r_1} (P_{r_2} + P_{a_2} \cdot P_{c_2}^i) + P_{c_1} \cdot P_{r_2} (1 - P_{c_2}^i)}{(1 - P_{a_1})(P_{r_2} + P_{a_2} P_{c_2}^i)} \tag{3.3}$$

$$= 1 - P_{CA}$$

Here, probability P_{CA} shows the probability of corrective action to achieve original objective of the process. Similarly, P_R describes the probability that process fails to achieve its objective. The symbols P_{a_1}, P_{r_1} and P_{c_1} etc. are defined as rfollows :

$$P_{a_1} = L(p, n_1, c_1) \quad ; \quad P_{a_2} = L(p, n_2, c_1)$$
$$P_{r_1} = 1 - L(p, n_1, c_2) \quad ; \quad P_{r_2} = 1 - L(p, n_2, c_1)$$
$$P_{c_1} = 1 - P_{a_1} - P_{r_1} \quad ; \quad P_{c_2} = 1 - P_{a_2} - P_{r_2}$$

where $L(p, n, c) = \sum_{x=0}^{c} \frac{e^{-np} (np)^x}{x!}$ is the probability of getting c or less defectives with product quality p and sample size n.

Further characterization of the plan ASN function found to be

$$ASN = E(n) = P_{CA}\left[\frac{d}{d\theta} M_A(\theta)\right]_{\theta=0} + P_R\left[\frac{d}{d\theta} M_R(\theta)\right]_{\theta=0}$$

$$(3.4) \qquad = \frac{n_1(P_r + P_{a_2} P^i_{c_2}) + n_2 P_{c_1}(1 - P^i_{c_2})}{(1 - P_{a_1})(P_r + P_{a_2} P_{c_2})}$$

where $M_A(\theta) = \dfrac{W_A(\theta, 0)}{W_A(0, 0)}$ and $M_R(\theta) = \dfrac{W_R(\theta, 0)}{W_R(0, 0)}$

Here, it may be noted that ASN describes the expected number of itoms inspected for corrective action/termination of the processs for variation in process quality p

Similarly, the average length of inspection cycle, $E(t)$ is found as follows :

$$E(t) = P_A\left[\frac{d}{ds} M_{1A}(s)\right]_{s=0} + P_R\left[\frac{d}{ds} M_{1R}(s)\right]_{s=0}$$

$$(3.5) \qquad = \frac{(P_{r_2} + P_{a_2} P^i_{c_2}) + P_{c_1}(1 - P^i_{c_2})}{\lambda(1 - P_{a_1})(P_{r_2} + P_{a_2} P^i_{c_2})}$$

where $M_{1A}(s) = \dfrac{W_A(0, s)}{W_A(0, 0)}$ and $M_{1R}(s) = \dfrac{W_R(0, s)}{W_R(0, 0)}$

We now consider the special case when i = 1. Now, putting i = 1 in (3.2), (3.4) and (3.5) we have

$$P_{CA} = \frac{P_{c_1} \cdot P_{c_2}}{(1 - P_{a_1}) \cdot (1 - P_{a_2})}, \qquad ASN = \frac{n_2 P_{c_1} + n_1 \cdot (1 - P_{a_2})}{(1 - P_{a_1}) \cdot (1 - P_{a_2})},$$

and $$E(t) = \frac{1 + P_{c_1} - P_{a_2}}{\lambda \cdot (1 - P_{a_1}) \cdot (1 - P_{a_2})},$$

These results agree with Srivastava[12]. Here, it may be noted that in case of normal inspection only, the state $S_2[k]$ is treated as corrective state S_{CA} and resulting plan reduces to Shankar[2]. The respective expressions, thus, come out to be :

$$P_{CA} = \frac{P_{c_1}}{(1 - P_{c_1})}, \qquad ASN = \frac{n_1 \cdot (1 - P_{a_2}) + n_2 P_{c_1}}{(1 - P_{a_1})(1 - P_{a_2})},$$

and $$E(t) = \frac{(1 + P_{c_1} - P_{a_2})}{(1 - P_{a_1}) \cdot (1 - P_{a_2})}$$

These results agree with Shankar[2].

4. Construction of Table

The expression for OC function, P_{CA}, and ASN function of the proposed plan under Poisson model has been given in (3.2) and (3.4), respectively, with

$$P_{a_1} = \sum_{x=0}^{c_1} \frac{e^{-n_1 p}(n_1 p)^x}{x!}$$

$$P_{a_2} = \sum_{x=0}^{c_1} \frac{e^{-n_2 p}(n_2 p)^x}{x!} = \sum_{x=0}^{c_1} \frac{e^{-k n_1 p}(k n_1 p)^x}{x!}$$

$$P_{c_1} = 1 - \sum_{x=0}^{c_2} \frac{e^{-n_1 p}(n_1 p)^x}{x!}, \qquad P_{c_2} = 1 - \sum_{x=0}^{c_2} \frac{e^{-k n_1 p}(k n_1 p)^x}{x!},$$

$$P_{r_1} = 1 - P_{a_1} - P_{c_1} \quad \text{and} \quad P_{r_2} = 1 - P_{a_2} - P_{c_2}$$

where $k = \dfrac{n_2}{n_1}$

Let $P_a(p^i) = L(p^i, n, c)$ Now, for given (p_1, $1 - \alpha$) on the OC curve, one may write :

(4.1) $$P_{CA}(p_1) = 1 - \alpha$$

For given c_1, c_2, i and α equation (4.1) is a function of $n_1 p_1$ only. Therefore, unity values $n_1 p_1$ have been obtained from (4.1) by Newton's method of successive approximation Similarly, for given (p_2, β) on the OC curve, one may write : ·

(4.2) $$P_{CA}(p_2) = \beta$$

The values of $n_1 p_2$ have been obtained from (4.2) by Newton's method of successive Approximation. The operating ratio R = $n_1 p_2 / n_1 p_1$ may be used to generate unity values. Unity values $n_1 p_1$ presented in Table 1 were derived by using the theory of unity values due to Duncan[10] (1976, pp. 187-188). A wide range of c_1 and c_2 have been considered in developing such tables. Table gives the relevent entries for various combination of k, α = .05, β = .10 and i = 2. Tables related to i = 1, 3 and 4 are also available with the author.

5. Construction and Evalution of the Plan

For construction and evalution of the two-phase inspection process control plan, unity values $n_1 p_1$ are presented in Table 1.

Table 1

Values of R = p2/p1 And n1p2 TWP-Phase Inspection Process

Control Plan : $\alpha = .05$, $\beta = .10$, $i = 2$

C1	C2	k = 1.25		k = 1.50		k = 1.75		k =2	
		R	n1p2	R	n1p2	R	n1p2	R	n1p2
0	3	5.193	3.2227	5.217	2.7903	5.378	2.4628	8.072	2.3496
0	4	3.715	4.1244	3.749	3.5650	3.836	3.1361	3.856	2.7762
0	5	3.107	5.0022	3.131	4.3239	3.185	3.7790	3.187	3.3587
0	6	2.743	5.8696	2.778	5.0601	2.812	4.4293	2.852	3.9097
0	7	2.520	6.7315	2.553	5.7917	2.588	5.0659	2.620	4.4650
1	4	6.091	4.0000	6.162	3.4676	6.382	3.0488	6.285	2.7001
1	5	3.696	4.9471	3.740	4.2734	3.816	3.7392	3.806	3.3252
1	6	2.999	5.8449	3.024	5.0502	3.053	4.4117	3.096	3.8945
1	7	2.635	6.7205	2.667	5.7954	2.694	5.0580	2.730	4.4580
1	8	2.425	7.5858	2.462	6.5190	2.490	5.6795	2.507	5.0118
2	6	3.932	5.7538	3.950	4.9611	4.060	4.3491	4.096	3.8526
2	7	2.957	6.6770	2.993	5.7448	3.020	5.0152	3.078	4.4444
2	8	2.578	7.5659	2.602	6.5123	2.620	5.6634	2.667	5.0128
2	9	2.347	8.4488	2.395	7.2550	2.405	6.3132	2.437	5.5679
2	10	2.203	9.2967	2.235	7.9740	2.270	6.9289	2.288	6.1164

Table 1 Cont.

C1	C2	k = 1.25		k = 1.50		k = 1.75		k =2	
		R	n1p2	R	n1p2	R	n1p2	R	n1p2
3	7	4.778	6.5471	4.850	5.6393	4.944	4.9371	4.974	4.3564
3	8	2.992	7.4985	3.036	6.4489	3.103	5.5624	3.125	4.9749
3	9	2.536	8.4016	2.570	7.2147	2.590	6.2776	2.627	5.5347
3	10	2.301	9.2802	2.343	7.9588	2.350	6.9154	2.386	6.1065
3	11	2.158	10.1459	2.186	8.6907	2.220	7.5577	2.228	6.6392
4	9	3.166	8.3137	3.215	7.1502	3.203	6.2161	3.278	5.4999
4	10	2.544	9.2331	2.567	7.9160	2.609	6.8823	2.631	6.0809
4	11	2.272	10.1210	2.302	8.6666	2.324	7.5409	2.354	6.6412
4	12	2.123	10.9918	2.162	9.4167	2.182	8.1728	2.194	7.1908
4	13	2.023	11.8532	2.048	10.1450	2.077	8.7972	2.087	7.7335
5	10	3.539	9.1204	3.556	7.8249	3.589	6.8243	3.686	6.0033
5	11	2.565	10.0667	2.621	8.6339	2.647	7.5013	2.653	6.5922
5	12	2.265	10.9598	2.292	9.3913	2.309	8.1510	2.332	7.1559
5	13	2.094	11.8361	2.137	10.1320	2.138	8.7672	2.165	7.7233
5	14	1.996	12.7011	2.019	10.8618	2.030	9.3891	2.054	8.2662

This table may be used to derive individual plans to meet specific value of probability of corrective action and termination of the process. It requires the specification of type 1 error probability (α), type 2 error probability (β), process levels p_1, p_2 and differing parameter i such that $P_{CA}(p_1) = 0.95$ and $P_{CA}(p_2) = 0.10$ or equivalently P_R $(p_2) = 0.90$. The steps to be follwed are following :

(1) Specify,

p_1 = process level for corrective action ($P_{CA} = 0.95$)

p_2 = process level for termination of system ($P_{CA} = 0.10$ or $P_R = 0.90$) with differing parameter i and k.

(2) Form the operating ratio $R = p_2/p_1$

(3) Choose a plan having c_1, and c_2 with an operating ratio less than or equal to R in the corresponding table of i.

(4) Determine the sample sizes $n_1 = \dfrac{n_1 p_2}{p_2}$ and $n_2 = k\, n_1$.

Round up in determining the sample sizes.

Thus, the plan consists of n_1, n_2, c_1, c_2 and i is chosen.

Now, we give an illustrative example. Suppose a two-phase inspection process control plan is desired having 95% probability of corrective action at $p_1 = 0.010$ and 10% probability of termination of the process $p_2 = 0.0375$.

(1) $p_1 = 0.010$ and $p_2 = 0.0375$ with $i = 2$.

(2) Operating ratio $R = p_2/p_1 = 0.0375/0.10 = 3.75$.

(3) The operating ratio R corresponding to $i = 2$ in the Table 1 gives the value of $n_1 p_2 = 3.565$ and $k = 1.5$ with aceptance constants $c_1 = 0$ and $c_2 = 4$.

(4) Sample sizes $n_1 = \dfrac{n_1 p_2}{p_2} = 3.565/0.375 = 95.09 \approx 95$ and $n_2 = k \times n_1$

$= 1.5 \times 95.09 = 142.65 \approx 143$.

Thus, the desired plan consists of (i, n_1, n_2, c_1, and c_2) = (2, 95, 142, 0 and 4).

The effects of different choises of i on OC and *ASN* have been studied for some selected values of p, say p_1, p^* and p_2. Where $P_A(p^*) = 0.50$. The results are shown in Table 2 for the plan $c_1 = 0$, $c_2 = 3$ and $i = 2$.

It is observed from theTable 2 that for the process level $p = p_1$ the *ASN* of the proposed plan decreases with increasing k, However, the situation is for otherway for poor quality process level i.e. for indifference quality level p^* amd p_2. This fact may be summarised by stating that the average sample number decreases with increasing k when variation in quality occurs mainly due to deterioration of machines and equipemt.

Table 2 : Average Sample Number (ASN)

p	P_A	Average Sample Number (ASN)			
		k = 1.25	k = 1.50	k = 1.75	k = 2.00
p_1 = 0.1	.99	608.53	601.69	597.50	594.79
	.95	533.73	528.69	524.30	521.26
p^* = .05	.50	125.34	126.47	126.91	127.00
p_2 = .10	.10	67.41	69.59	71.37	72.83
	.05	67.52	69.89	72.11	73.93

On the other hand, *ASN* increases with increasing k when variation in quality is produced by any potential assignable causes. Furthermore, the presence of assignable causes may be observed more earlier than that of deterioration state of the process because of the *ASN* in the former case is smaller than the latter.

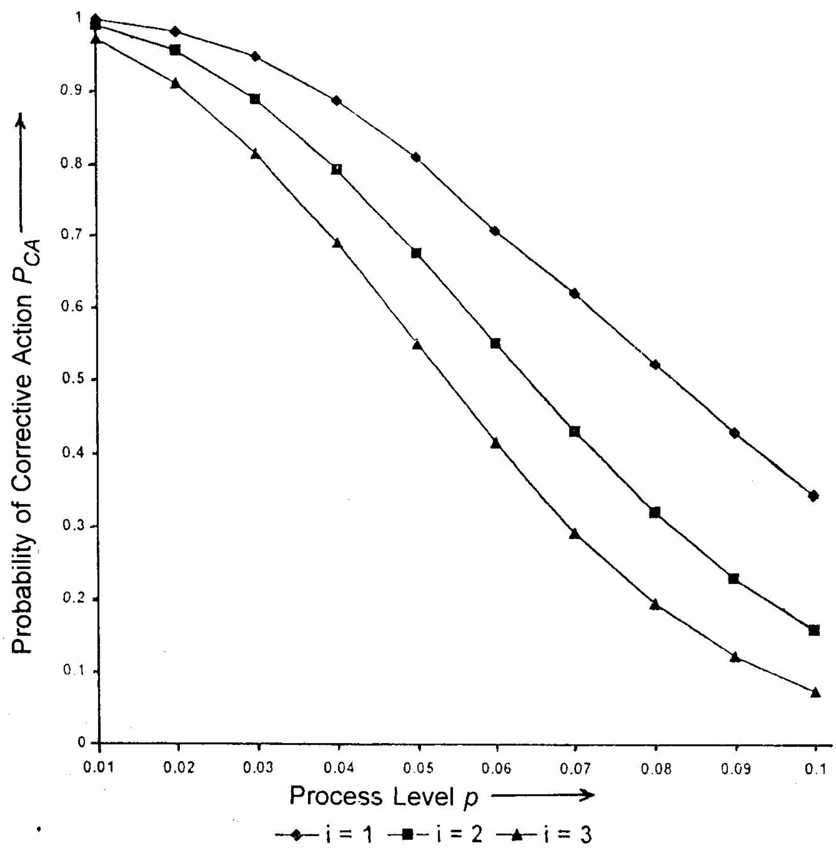

Fig. 3 : OC Curve for different values of i

$n = 20$, c_1 0, $c_2 = 4$, and k = 1.5

Now, in order to study the effect of i on the performance of the plan, OC curve have been drawan in Fig. 3 for $c_1 = 0$, $c_2 = 4$ and $n = 20$. It is observed from the figure that as i increases the plan is tightened up, and the effect is to lower the OC curve (probability of corrective action). As i diminishes, the plan becomes more lax, and the effect is to raise the OC curve.

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