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On Symmetric Conformal K-contact Riemannian Manifolds

Dhruwa Narain and Om Prakash Chaudhary
Department of Mathematics & Statistics.
D. D. U. Gorakhpur University, Gorakhpur

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Abstract : In the present paper we have studied properties of curvature tensors on conformal and symmetric conformal K-contact Riemannian manifolds, C-Sasakian symmetric C-Sasakian manifolds. In which we have obtained that

$$2\rho' K(X, Y, Z, W) = 4\rho g(X, W) g(Y, Z) - 4\rho g(X, Z) g(Y, W) + g(Y, W)(XZ\rho)$$
$$- g(X, W)(YZ\rho) - g(Y, z)(XW\rho) + g(X, Z)(YW\rho)$$

1. Introduction

Let M_{2n+1} be an almost Sasakian manifold with structure $\{ F, A, T, g \}$, where F is a tensor field of type (1, 1), and T is a vector field, A is a 1-form and g is a metric tensor, such that

(1.1) $$\bar{\bar{X}} = -X + A(X)T, \quad \bar{X} \overset{\text{def}}{===} FX, \quad A(T) = 1, \quad \bar{T} = 0, \quad A(\bar{X}) = 0$$

(1.2) $$g(\bar{X}, \bar{Y}) = g(X, Y) - A(X)A(Y), \quad g(T, X) \overset{\text{def}}{===} A(X),$$

(1.3) $$2'F(X, Y) = (D_X A)(Y) - (D_Y A)(X), \quad 'F(X, Y) \overset{\text{def}}{===} g(\bar{X}, Y)$$

for arbitrary vector fields X, Y and D is a Riemannian connections.

If the vector field T is conformal killing in an almost Sasakian manifold, i.e.,

(1.4) $$(L_T g) = 2\rho g \quad \text{implies} \quad (D_X A)(Y) + (D_Y A)(X) = 2\rho g$$

then it is called conformal K-contact Riemannian manifolds, where ρ is a scalar function. In particular if $\rho = 0$, the manifold is K-contact Riemannian manifold.

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On a conformal K-contact Riemannian Manifold[3], we have

(1.5) $'F(X, Y) = \rho g(X, Y) - (D_X A)(X) = (D_Y A)(X) - \rho g(X, Y),$

(1.6) $D_X T = \overline{X} + \rho X,$

(1.7) $(D_Z 'F)(\overline{X}, \overline{Y}) + (D_Z 'F)(X, Y) - A(Y) A(D_Z \overline{X}) + A(X) A(D_Z \overline{Y}) = 0$

(1.8) $'K(X, Y, Z, T) + 'K(\overline{X}, \overline{Y}, Z, T) = 'F(X, Z)(\overline{Y}\rho) - 'F(Y, Z)(\overline{X}\rho)$
$+ g(X, Z)(Y\rho) - g(Y, Z)(X\rho) + A(Y) A(D_Z \overline{X}) - A(X) A(D_Z \overline{Y})$

(1.9) $g(X, Y) + \rho\, 'F(X, Y) + A(D_Y \overline{X}) = A(X) A(Y).$

where $'K$ is the curvature tensor of the type (0, 4), such that

$$'K(X, Y, Z, W) \overset{\text{def}}{=\!=\!=} g(K(X, Y, Z), W)$$

Taking Lie derivative of $A(X) = g(T, X)$, with respect to T, we get

(1.10) $(L_T A)(A) = (L_T g)(T, X) = 2\rho A(X),$

If we consider 1-form $A = d(\log \rho) = \dfrac{d\rho}{\rho}$, then we consider

(1.11) $X\rho = \rho A(X), \quad \overline{X}\rho = 0, \quad T\rho = \rho,$

taking covarient derivative w.r.t. X, we get

$$XT\rho = X\rho = \rho A(X), \qquad \overline{X}T\rho = 0.$$

Note : In the present paper we consider

(1.12) $h = L_T F,$

and if T is contravariant almost analytic then $h = 0$.

2. Conformal K-contact Riemannian Manifolds

Theorem (2.1) : *On a conformal K-contact Riemannian Manifold, we have*

(2.1) $(L_T 'K)(X, Y, Z, T) + (L_T 'K)(\overline{X}, \overline{Y}, Z, T) + 'K(hX, \overline{Y}, Z, T) + 'K(\overline{X}, hY, Z, T)$
$= 2\rho \{ 'K(X, Y, Z, T) + 'K(\overline{X}, \overline{Y}, Z, T) + A(X) g(Y, Z) - A(Y) g(X, Z) \}$

$$+ 2\rho^2 \{ A(X)\, g(\overline{Y}, Z) - A(Y)\, g(\overline{X}, z) \} + g(\overline{X}, z)\, \overline{(Y T\rho)}$$

$$- g(\overline{Y}, Z)\, \overline{(X T\rho)} + g(X, Z)\,(X T\rho) - g(Y, Z)\,(X T\rho)$$

$$+ g(hX, Z)\,(\overline{Y}\rho) - g(hY, Z)\,(\overline{X}\rho) + g(\overline{X}, Z)\,(hY\rho)$$

$$- g(\overline{Y}, Z)\,(hX\rho) + \rho A(X)\, g(hY, Z) - \rho A(Y)\, g(hX, Z)$$

Proof : From the equation (1.8) and (1.9), we get

(2.2)
$$'K(X, Y, Z, T) + {}'K(\overline{X}, \overline{Y}, Z, T) = g(\overline{X}, Z)\,\overline{(Y\rho)} - g(\overline{Y}, Z)\,\overline{(X\rho)}$$
$$+ g(X, Z)\,(Y\rho) - g(Y, Z)\,(X\rho) + A(X)\, g(Y, Z)$$
$$- A(Y)\, g(X, Z) + \rho A(X)\, g(\overline{Y}, Z) - \rho A(Y)\, g(\overline{X}, Z)$$

Taking Lie derivative of equation (2.2) w. r. t. T and using (1.12), we get

(2.3)
$$(L_T{}'K)(X, Y, Z, T) + (L_T{}'K)(\overline{X}, \overline{Y}, Z, T) + {}'K(hX, \overline{Y}, Z, T) + {}'K(\overline{X}, hY, Z, T)$$
$$= (L_T g)(\overline{X}, Z)\,(\overline{Y}\rho) + g(hX, Z)\,(\overline{Y}\rho) + g(\overline{X}, Z)\,(hY\rho)$$
$$+ g(\overline{X}, Z)\,(\overline{Y}T\rho) - (L_T g)(\overline{Y}, Z)\,(\overline{X}\rho) - g(hY, Z)\,(\overline{X}\rho)$$
$$- g(\overline{Y}, Z)\,(hX\rho) - g(\overline{Y}, Z)\,(\overline{X}T\rho) + (L_T g)(X, Z)\,(Y\rho)$$
$$+ g(X, Z)\,(XT\rho) - (L_T g)(Y, Z)\,(X\rho) - g(Y, Z)\,(XT\rho)$$
$$+ (L_T A)(X)\, g(Y, Z) + A(X)\,(L_T g)\, g(Y, Z) - (L_T A)(Y)\, g(X, Z)$$
$$- A(Y)\,(L_T g)\, g(X, Z) + (T\rho)\, A(X)\, g(\overline{Y}, Z) + \rho (L_T A)(X)\, g(\overline{Y}, Z)$$
$$- \rho A(x) \{(L_T g)(\overline{Y}, Z) + g(hY, Z)\} - (T\rho) A(Y)\, g(\overline{X}, Z)$$
$$- \rho (L_T A)(Y) \{(L_T g)(\overline{X}, Z + g(hX, Z)\}.$$

From the equation (2.3), (1.4) and (1.10), we get (2.1).

Theorem (2.2) : If $A = d(\log \rho)$ *on a conformal K-contact Riemannian Manifold, we have*

(2.4)
$$(L_T{}'K)(X, Y, Z, T) + (L_T{}'K)(\overline{X}, \overline{Y}, Z, T) + {}'K(hX, \overline{Y}, Z, T) + {}'K(\overline{X}, hY, Z, T)$$
$$= 2\rho\{'K(X, Y, Z, T) + {}'K(\overline{X}, \overline{Y}, Z, T)\} + \rho\{A(X)\, g(Y, Z) - A(Y)\, g(X, Z)\}$$

$$+\ \rho(2\rho + 1)\{A(X)\, g((\bar{Y}, Z) - A(Y)\, g(\bar{X}, Z)\} + g(\bar{X}, Z)\,(hY\rho)$$

$$-\ g(\bar{Y}, Z)\,(hX\rho) + \rho A(X)\, g(hY, Z) - \rho A(Y)\, g(hX, Z)\,.$$

Proof : From the equation (1.2) and (2.1), we get (2.4).

3. Symmetric Conformal K-contact Riemannian Manifolds

A manifold is said to be symmetric, if

(3.1) $$(D_T\, 'K)\,(X, Y, Z, W) = 0$$

Theorem (3.1) : *On a symmetric conformal K-contact Riemannian manifold, we have*

(3.2) $$(L_T\, 'K)\,(\bar{X}, Y, Z, W) + (L_T\, 'K)\,(X, \bar{Y}, Z, W) + 'K\,(X, Y, \bar{Z}, W)$$

$$+\ 'K\,(X, Y, Z, \bar{W}) + 'K\,(hX, Y, Z, W) + 'K\,(X, hY, Z, W)$$

$$+\ 'K\,(X, Y, hZ, W) + 'K\,(X, Y, Z, hW) + 2(T\rho)\, 'K\,(X, Y, Z, W)$$

$$= g(Y, W)\{2\rho(ZX\rho) + (ZXT\rho)\} - g(Y, Z)\{2\rho(WX\rho) + (WXT\rho)\}$$

$$+\ g(X, Z)\,\{2\rho(WX\rho) + (WYT\rho)\} - g(X, W)\{2\rho(ZY\rho) + (ZYT\rho)\}$$

Proof : On a conformal K-contact Riemannian manifold[5], we have

(3.3) $$'K\,(\bar{X}, Y, Z, W) + 'K\,(X, \bar{Y}, Z, W) + 'K\,(X, Y, \bar{Z}, W) + 'K\,(X, Y, Z, \bar{W})$$

$$+\ 2\rho\, 'K\,(X, Y, Z, W) + (D_T\, 'K)\,(X, Y, Z, W)$$

$$= g(Y, W)\,(ZX\rho) - g(Y, Z)\,(WX\rho) + g(X, Z)\,(WY\rho) - g(X, W)\,(ZY\rho)\,.$$

From the equation (3.1) and (3.3), we get

(3.4) $$'K\,(\bar{X}, Y, Z, W) + 'K\,(X, \bar{Y}, Z, W) + 'K\,(X, Y, \bar{Z}, W) + 'K\,(X, Y, Z, \bar{W})$$

$$+\ 2\rho\, 'K\,(X, Y, Z, W) = g(Y, W)\,(ZX\rho) - g(X, Z)\,(WY\rho) - g(X, W)\,(ZY\rho)\,.$$

Taking Lie derivative of equation (3.4) w. r. t. T and using (1.4) and (1.12) we get equation (3.2).

Theorem (3.2) : If $A = d(\log \rho)$ *on a symmetric conformal K-contact Riemannian manifold, we have*

$$(3.5)\qquad (L_T{}'K)(\bar{X}, Y, Z, W) + (L_T{}'K)(X, \bar{Y}, Z, W) + (L_T{}'K)(X, Y, \bar{Z}, W)$$
$$+ (L_T{}'K)(X, Y, Z, \bar{W}) + {}'K(hX, Y, Z, W) + {}'K(X, hY, Z, W)$$
$$+ {}'K(X, Y, hZ, W) + {}'K(X, Y, Z, hW)$$
$$= 2(\rho + 1)\{{}'K(\bar{X}, Y, Z, W) + {}'K(X, \bar{Y}, Z, W) + {}'K(X, Y, \bar{Z}, W)$$
$$+ {}'K(X, Y, Z, \bar{W})\} + 4\rho^2\,{}'K(X, Y, Z, W)$$

Proof : From the equation (1.11) and (3.2), we get (3.5).

4. Recurrent Properties of K-contact Riemannian Manifolds

A manifold is said to be Recurrent, if

$$(D_T{}'K)(X, Y, Z, W) = \alpha(T)\,{}'K(X, Y, Z, W)$$

where $\alpha(T)$ is a scalar function. Now taking $\alpha(T) = -2\rho$ conformal K-contact Riemannian Manifold, we get

$$(4.1)\qquad (D_T{}'K)(X, Y, Z, W) = -2\rho\,{}'K(X, Y, Z, W)$$

Theorem (4.1) : *On a Recurrent conformal K-contact Riemannian manifold, we get*

$$(4.2)\qquad (L_T{}'K)(\bar{X}, Y, Z, W) + (L_T{}'K)(X, \bar{Y}, Z, W) + (L_T{}'K)(X, Y, \bar{Z}, W)$$
$$+ (L_T{}'K)(X, Y, Z, \bar{W}) + {}'K(hX, Y, Z, W) + {}'K(X, hY, Z, W)$$
$$+ {}'K(X, Y, hZ, W) + {}'K(X, Y, Z, hW)$$
$$= 2\rho\{{}'K(\bar{X}, Y, Z, W) + {}'K(X, \bar{Y}, Z, W) + {}'K(X, Y, \bar{Z}, W) + {}'K(X, Y, Z, \bar{W})\}$$
$$+ g(X, Z)(WYT\rho) + g(Y, W)(ZXT\rho) - g(Y, Z)(WXT\rho) - g(X, W)(ZYT\rho)\,.$$

Proof : From the equation (3.3) and (4.1), we get.

$$(4.3)\qquad {}'K(\bar{X}, Y, Z, W) + {}'K(X, \bar{Y}, Z, W) + {}'K(X, Y, \bar{Z}, W) + {}'K(X, Y, Z, \bar{W})$$
$$= g(X, Z)(WY\rho) + g(Y, W)(ZX\rho) - g(Y, Z)(XW\rho) - g(X, W)(ZY\rho)\,.$$

Taking Lie derivative of equation (4.3) w. r. t. T and using (1.4) and (1.12) we get equation (4.2).

Theorem (4.2) : If $A = d(\log \rho)$ *on a recurrent conformal K-contact Riemannian Manifold, we have :*

(4.4)
$$(L_T\,'K)(\overline{X}, Y, Z, W) + (L_T\,'K)(X, \overline{Y}, Z, W) + (L_T\,'K)(X, Y, \overline{Z}, W)$$
$$+ (L_T\,'K)(X, Y, Z, \overline{W}) + 'K(hX, Y, Z, W) + 'K(X, hY, Z, W)$$
$$+ 'K(X, Y, hZ, W) + 'K(X, Y, Z, hW)$$
$$= 2(\rho + 1)\{'K(\overline{X}, Y, Z, W) + 'K(X, \overline{Y}, Z, W) + 'K(X, Y, \overline{Z}, W) + 'K(X, Y, Z, \overline{W})\}$$

Proof : From the equation (1.11) and (4.2), we get (4.4).

5. C-Sasakian Manifolds

If in a conformal K-contact Riemannian Manifold

(5.1)
$$(D_Z\,'F)(X, Z) = A(X)\,g(Y, Z) - A(Y)\,g(X, Z),$$

holds, then, the manifold is called conformal Sasakian or C-Sasakian manifold. Since in a conformal K-contact Riemannian Manifold, we get

(5.2)
$$(D_Z\,'F)(X, Z) = 'K(X, Y, Z, T) - g(X, Z)(Y\rho) + g(Y, Z)(X\rho)$$

Thus, in a C-Sasakian manifold from (5.1) and (5.2), we obtain

(5.3)
$$'K(X, Y, Z, T) = A(X)\,g(Y, Z) - A(Y)\,g(X, Z) - g(X, Z)(Y\rho) - g(Y, Z)(X\rho)$$

Theorem (5.1) : *On a C-Sasakian manifold, we have*

(5.4)
$$(L_T\,'K)(X, Y, \overline{Z}, W) + (L_T\,'K)(X, Y, Z, \overline{W}) + 'K(X, Y, hZ, W) + 'K(X, Y, Z, hW$$
$$= 4\rho\{'K(X, Y, \overline{Z}, W) + 'K(X, Y, Z, \overline{W})\} - 2(T\rho)\,'P(X, Y, Z, W)$$
$$+ 'K(X, Y, Z, hW) - 'P(hX, Y, Z, W) - 'P(X, hY, Z, W),$$

(5.5)
$$(L_T\,'K)(X, Y, \overline{Z}, \overline{W}) + (L_T\,'K)(X, Y, Z, W) + 'K(X, Y, hZ, \overline{W})$$
$$+ 'K(X, Y, \overline{Z}, hW) = 4\rho\{('K(X, Y, \overline{Z}, \overline{W}) - 'K(X, Y, Z, W)\}$$
$$- 2(T\rho)\,'P(X, Y, Z, \overline{W}) + 'P(X, hY, Z, W) + 'P(hX, \overline{Y}, Z, W)$$
$$- 2\rho\,'P(X, Y, Z, hW) + A(W)\{g(Y, Z)(XT\rho) - g(X, Z)(YT\rho)\}$$

$$(L_T{}'K)(X, Y, \bar{Z}, \bar{W}) + (L_T{}'K)(\bar{X}, \bar{Y}, Z, W) + {}'K(X, Y, hZ, \bar{W}) \tag{5.6}$$
$$+ {}'K(X, Y, \bar{Z}, hW) - {}'K(hX, \bar{Y}, Z, W) - {}'K(\bar{X}, hY, Z, W)$$
$$= 4\rho\{{}'P(X, Y, \bar{Z}, \bar{W}) - {}'P(\bar{X}, \bar{Y}, Z, W)\} + 2(T\rho)\{({}'P(X, \bar{Y}, Z, W)$$
$$- {}'P(X, Y, Z, \bar{W})\} + 2\rho\{g(hY, Z)\, g(X, W) + g(hW, Y)\, g(X, Z)\}$$
$$- {}'P(X, Y, Z, hW)\} + A(W)\{g(X, Z)(WT\rho) - g(X, W)(ZT\rho)\}$$
$$+ A(W)\{g(Y, Z)(XT\rho) - g(X, Z)(YT\rho)\},$$

$$(L_T{}'K)(\bar{X}, Y, Z, W) + (L_T{}'K)(X, \bar{Y}, Z, W) + (L_T{}'K)\,{}'K(X, Y, \bar{Z}, W) \tag{5.7}$$
$$+ (L_T{}'K)(X, Y, Z, \bar{W}) + {}'K(hX, Y, Z, W) + {}'K\,{}'K(X, hY, Z, W)$$
$$+ {}'K(X, Y, hZ, W) + {}'K(X, Y, Z, hW) = -4(T\rho + 4\rho^2)\,{}'\rho(X, Y, Z, W),$$

where

$${}'P(X, Y, Z, W) \overset{\text{def}}{=\!=\!=} g(Y, Z)\, g(X, W) - g(Y, W)\, g(X, Z). \tag{5.8}$$

Proof : On a C-Sasakian manifold[5], we have

$${}'K(X, Y, \bar{Z}, W) + {}'K(X, Y, Z, \bar{W}) = g(\bar{X}, Z)\, g(Y, W) \tag{5.9}$$
$$- g(\bar{Y}, Z)\, g(X, W) - g(\bar{X}, W)\, g(Y, Z) + g(\bar{Y}, W)\, g(X, Z)$$
$$+ 2\rho\{g(X, Z)\, g(Y, W) - g(X, W)\, g(Y, Z)\}$$

Taking Lie derivative of equation (5.9) w. r. t. T and using (1.4) and (1.12) we get equation (5.4). Now barring W in equation (5.9) and using (1.1) and (5.3), we get

$${}'K(X, Y, \bar{Z}, \bar{W}) - {}'K(X, Y, Z, W) = g(\bar{Y}, Z)\, g(\bar{X}, W) - g(\bar{X}, Z)\, g(\bar{Y}, W) \tag{5.10}$$
$$- g(X, W)\, g(Y, Z) + g(Y, W)\, g(X, Z) + 2\rho\{g(\bar{W}, Y)\, g(X, Z)$$
$$- g(\bar{W}, X)\, g(Y, W)\} + A(W)\{g(Y, Z)(X\rho) - g(X, Z)(Y\rho)\}$$

Taking Lie derivative of equation (5.10) w. r. t. T and using (1.4), (1.12) and (5.8) we get equation (5.5).

Interchanging $X \leftrightarrow Z$ and $Y \leftrightarrow W$, in equation (5.10) and further subtracting, we get

(5.11) $$'K(X, Y, \bar{Z}, \bar{W}) - 'K(\bar{X}, \bar{Y}, Z, W)$$

$$= 2\rho\{g(\bar{Y}, Z)\,g(X, W) - g(\bar{W}, X)\,g(Y, Z) + 2g(\bar{W}, Y)\,g(X, Z)\}$$

$$+ A(W)\{g(Y, Z)(X\rho) - g(X, Z)(Y\rho)\}$$

$$+ A(Y)\{g(X, Z)(W\rho) - g(X, W)(Z\rho)\},$$

Taking Lie derivative of equation (5.11) and using (1.4), (1.12) and (5.8) we get equation (5.6). Now interchanging $X \leftrightarrow Z$ and $Y \leftrightarrow W$, in equation (5.9) and further adding, we get

(5.12) $$'K(\bar{X}, Y, Z, W) + 'K(X, \bar{Y}, Z, W) + 'K(X, Y, \bar{Z}, W)$$

$$+ 'K(X, Y, Z, \bar{W}) = 4\rho\{g(X, Z)\,g(Y, W) - g(X, W)\,g(Y, Z)\}$$

Taking Lie derivative of equation (6.12) and using (1.4), (1.12) and (5.8) we get equation (5.7).

6. Symmetric C-Sasakian manifolds

Theorem (6.1) : On a Symmetric C-Sasakian manifold, we get

(6.1) $$2\rho\,'K(X, Y, Z, W) = 4\rho\,g(X, W)\,g(Y, Z) - 4\rho\,g(X, Z)\,g(Y, W)$$

$$+ g(Y, W)(XZ\rho) - g(X, W)(YZ\rho) - g(Y, Z)(XW\rho) + g(X, Z)(YW\rho)$$

Proof : Equation (5.3) can be written as

(6.2) $$K(X, Y, T) = XA(Y) - YA(X) + Y(X\rho) - X(Y\rho)$$

Differentiating equation (6.2), w. r. t. Z, we get

(6.3) $$(D_Z K)(X, Y, T) + K(X, Y, D_Z T) = X(D_Z A)(Y) - Y(D_Z A)(X)$$

$$+ Y(XZ\rho) - X(YZ\rho)$$

Now, from equation (1.5), (1.6), (3.1) and (6.3), we get

(6.4) $$K(X, Y, \bar{Z}) + \rho K(X, Y, Z) = X\{g(\bar{Z}, Y) + \rho g(Z, Y)\}$$

$$- Y\{g(\bar{Z}, X) + \rho g(Z, X)\} + Y(XZ\rho) - X(YZ\rho)$$

equation (6.4) can be written as

(6.5) $$'K(X, Y, \bar{Z}, W) + \rho\, 'K(X, Y, Z, W) = g(X, W)\{g(\bar{Z}, Y) + \rho\, g(Z, Y)\} - g(Y, W)\{g(\bar{Z}, X) + \rho\, g(Z, X)\} + g(Y, W)(XZ\rho) - g(X, W)(YZ\rho)$$

Interchanging $Z \leftrightarrow W$, in equation (6.5), we get

(6.6) $$-'K(X, Y, Z, \bar{W}) - \rho\, 'K(X, Y, Z, W) = g(X, Z)\{g(\bar{W}, Y) + \rho\, g(W, Y)\} - g(Y, Z)\{g(\bar{W}, X) + \rho\, g(W, X)\} + g(Y, Z)(XW\rho) - g(X, Z)(YW\rho)$$

Subtracting equation (6.6) from (6.5), we get

(6.7) $$\begin{aligned} 'K(X, Y, \bar{Z}, W) + 'K(X, Y, Z, \bar{W}) + 2\rho\, 'K(X, Y, Z, W) &= g(\bar{Z}, Y) g(W, X) - g(\bar{Z}, X) g(Y, W) - g(\bar{W}, Y) g(X, Z) \\ &+ g(\bar{W}, X) g(Y, Z) + 2\rho\, g(X, W) g(Y, Z) \\ &- 2\rho\, g(X, Z) g(Y, W) + g(Y, W) g(XZ\rho) \\ &- g(X, W) g(YZ\rho) - g(Y, Z) g(XW\rho) + g(X, Z) g(YW\rho) \end{aligned}$$

From equation (5.9) and (6.7), we get (6.1).

Theorem (6.2) : If $A = d(\log \rho)$ *on a symmetric C-Sasakian Manifold, we have :*

(6.8) $$'K(X, Y, Z, T) = \frac{1 - \rho}{(1 + 2\rho)\rho}\{(X\rho)(YZ\rho) - (Y\rho)(XZ\rho)\}$$

Proof : Putting W = T in equation (6.1) and using (1.11), we get

(6.9) $$2\rho\, 'K(X, Y, Z, T) = 3\rho\, A(X)\, g(Y, Z) - 3\rho\, A(Y)\, g(X, Z) + A(Y)(XZ\rho) - A(X)(YZ\rho)$$

Now, from the equation (1.11) and (5.3), we get

(6.10) $$'K(X, Y, Z, T) = (1 - \rho)\{A(X) g(Y, Z) - A(Y) g(X, Z)\}$$

From the equation (6.9) and (6.10), we get (6.8).

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