Vol 2 (1998) **Journal of International Academy of Physical Sciences** pp.35-41

Performance Evaluation for Response Time Analysis

Avanish Kumar
Department of Mathematical Sciences & Computer Applications, Bundelkhand University, Jhansi 284128.

M. P. Singh
Department of Mathematics & Statistics, G. K. University, Hardwar- 249404.

and

Vinod Kumar
Department of Computer Science, G. K. University, Hardwar-249 404.

(Received November 17, 1998)

Abstract : A model for mean response time is analyzed based on queuing theoretical approach. First the path sets of the given network are identified. $M/M^{Y}/1$ queue model is applied for response time analysis. The suggested method would be helpful for the network designer in deciding data routing strategies.

1. Introduction

Queuing theory has been recognized as one of the most powerful mathematical tools for making quantitative analysis of Computer Communication Networks. In the early 1960s it was realised that queuing theory would prove to be an effective tool for studying the throughout, response time, and other measures of performance of computer systems. Since that time the literature of queuing theory and computer applications have virtually exploded with analytical models for computer systems. For successful transmission, several protocols came into being. The stop and wait, and continuous error detection and re-transmission schemes are widely used in Computer Communication Networks. The researchers primary focus was the development of an appropriate model for analyzing the delays encountered in establishing a virtual circuit through a switch based LAN.

The methods developed were applied to many related problems i.e. multiple link telephone calls, receiver-transmitter interaction between switches and shared peripherals in computer systems. One of such approache is developed by Neil C. Wilhelm[1] to analyze a disk subsystem in which all of the disks had the same load. W. Whitt[2] studied a similar method applied to markovian queuing networks as a way of extending the approximations developed by W. Whitt[3], F. P. Kelly[4] and W. Whitt[5] considered an approximation technique for a general class of circuit-switched teletraffic models.

Key Words : Mean Response Time ; Queue Theory; Path Sets.

In 1986, A. A. Fredericks[6] developed an approximation method for analyzing the performance of a virtual circuit-switch based LAN. The basic system consists of N input and N output ports, each with its bandwidth divided into L equal segments. For the virtual circuit from a given input to output port, it is necessary to obtain a time slot in each port. He suggested a simple analytical approximation method for the delays encountered in setting up such a circuit. The main aspect of the present model is service of the tansmitter, which is assumed to be bulk. We obtain the path sets of the network, corresponding to each path, we then get the mean response time. This method can be applied for the distributed system network of any size. By knowing the service rate of each path and therby getting its response time, one can appropriately choose the most suitable path.

2. Notations

The notations used in this paper are as follows :

K	: Size of data packet
N	: Number of node in the Path Set
PN	: Number of Path Sets
$T_{MRT}()$	: An array to store Mean Response Time
$T_p()$	: An array to store Mean Service Time of Receiver
$T_s()$	: An array to store Mean Service Time of Transmitter
$T_W()$	: An array to store Waiting Time
λ	: Mean Arrival Rate
μ	: Mean Service Rate.

3. Problem Statement

The model presented in this paper consists of a Computer Communication Network, which is based on the Bulk service $(M/M^y/1)$. Let us have a network having "n" heterogeneous processors, $P = \{p_1, p_2, p_3, \ldots, p_n\}$ which are interconnected by links. One of the "n" processors is called a source node and one of the remaining '$n - 1$' processors is labeled as terminal node. At the source node, the tasks are arriving in the form of data packets and follow the poisson process. If the source node is busy, these are stored in buffer and processed. The tasks propagate over the link from source towards terminal node till they reach to the terminal node. Once a task reaches to terminal node, the terminal node sends an acknowledgement of correct processing of the task to the source node. The main aspect of the model is service mechanism. Each arriving data packet of tasks has variable number of frames which are served by bulk service. In this model, we have obtained the mean response time. For mean response time analysis we have calculated the paths of the networks. The tasks follow any one path for processing for source to terminal node depending upon the allocation of path by the source node. It is assumed that arrivals of tasks occur to a single channel facility as an ordinary poisson process with mean λ, then they are served on the basis of first come first served discipline. Each task of data packet has variable number of frames and these are served K at a time except when less than K are in the data packet. The amount of time required for the service of any batch is an exponentially distributed random variable, whether or not the batch is of full size K.

4. The Proposed Method

We consider a bridge computer communication network. In the network, there are four processors namely p_1, p_2, p_3, p_4, which are interconnected by links l_1, l_2, l_3, l_4 and l_5 as shown in figure 1. The processor p_1 is defined as source node while the processor p_3 is labeled as terminal node. The tasks enter from the source node and get released from the terminal node after getting processed.

The path sets of the network are listed below, along with the number of nodes in a path set.

$PN-1 : p_1 \rightarrow p_3$ no. of node (N) : 2

$PN-2 : p_1 \rightarrow p_2 \rightarrow p_3$ no. of node (N) : 3

$PN-3 : p_1 \rightarrow p_4 \rightarrow p_3$ no. of node (N) : 3

4.1 Mean Waiting Time : The basic model is of course, a non birth death markovian problem. The Kolmogorov equations are given below :

$$\text{(4.1.1)} \qquad \begin{bmatrix} \mu P_{n+k} - (\lambda + \mu) P_n + \lambda P_{n-1} = 0 \ (n > 1) \\ \mu P_k + \mu P_{k-1} + \ldots + \mu P_1 - \lambda P_0 = 0 \end{bmatrix}$$

The above equations may be rewritten in operator notation as :

$$\text{(4.1.2)} \qquad \left[\mu D^{k+1} - (\lambda + \mu) D + \lambda\right] P_n = 0$$

The mean waiting time of data packet[7] for each path is as

$$\text{(4.1.3)} \qquad T_w(i) = \sum_{j=1}^{N-1} \left[\frac{r_0(i, j)}{\lambda\left(1 - r_0(i, j)\right)} - \frac{1}{\mu_s(i, j)} \right], \ i = 1, \ldots, PN$$

where $r_0(i, j)$ is the root of [4.1.2]. One can obtain the value of $r_0(i, j)$ from the following relation :

$$\text{(4.1.4)} \qquad \mu r_0^{k+1}(i, j) - (\lambda + \mu) r_0(i, j) + \lambda = 0$$

4.2 Mean Service Time : Service of the processor, which bulkly transmits the task from one node to another depends upon a batch of size K which is exponentially distributed with mean $\mu_s(i, j)$ Therefore, the mean service time for each path, out going from the transmitting node is given as :

(4.2.1)
$$T_s(i) = \sum_{j=1}^{N-1} \frac{1}{\mu_s(i, j)}, \quad i = 1, \ldots, PN$$

Similarly, for the service of the processor, which receives the task, it is assumed that the service time of this node, obeys exponential law and is given by

(4.2.2)
$$T_R(i) = \sum_{j=1}^{N-1} \frac{1}{\mu_R(i, j)}, \quad i = 1, \ldots, PN$$

4.3 Mean Response Time : Mean Response Time (T_{MRT}) is defined as the sum of the mean waiting time and mean service time of processor which transmits, plus the mean service time of the processor which receives the task i.e.

(4.3.1)
$$T_{MRT}(i) = \left[T_w(i) + T_s(i) + T_R(i) \right]$$

where $T_w(i)$, $T_s(i)$ and $T_R(i)$ are given in the equations 4.1.3, 4.2.1 and 4.2.2 respectively.

5. Computation of Numerical Results

Mean response time as well as mean service time and mean receiving time have been calculated for different value of K i.e. different data packet size which are shown through table 1 to 4. Mean response time for the different arrival rates has been obtained, which is given in table 5.

Service times for p_1, p_2, p_4 are 1, 1.5, 2.5 respectively and receiving times for p_2, p_4, p_3 are 0.5, 1.5, 1 respectively.

For $K = 1$, $r_0 = 0.100$; for $K = 2$, $r_0 = 0.092$; for $K = 3$, $r_0 = 0.091$

Table 1

$T_W(i)$	Mean Waiting Time		
	K = 1	K = 2	K = 3
$T_W(1)$	0.110	0.013	0.001
$T_W(2)$	0.554	0.360	0.336
$T_W(3)$	0.820	0.626	0.602

Table 2

$T_s(i)$	Mean Service Time
$T_s(1)$	1.000
$T_s(2)$	1.666
$T_s(3)$	1.400

Table 3

$T_R(i)$	Mean Receiving Time
$T_R(1)$	1.000
$T_R(2)$	3.000
$T_R(3)$	1.666

Table 4

$T_{MRT(i)}$	Mean Response Time		
	K = 1	K = 2	K = 3
$T_{MRT(1)}$	2.110	2.013	2.001
$T_{MRT(2)}$	5.220	5.026	5.002
$T_{MRT(3)}$	3.886	3.692	3.668

Table 5

λ	$T_{MRT(1)}$	TMRT (2)	T_{MRT} (3)
0.1	2.110	5.220	3.886
0.2	3.250	7.166	5.566
0.3	3.430	7.526	5.926
0.4	3.660	7.986	6.386
0.5	4.000	8.666	7.066
0.6	4.500	9.666	8.066
0.7	5.330	11.326	9.726
0.8	7.000	14.666	13.066
0.9	12.000	24.666	23.066

6. Conclusion

Graphical representation of the values given in table 4 and 5 have been shown in figure 2 and 3.

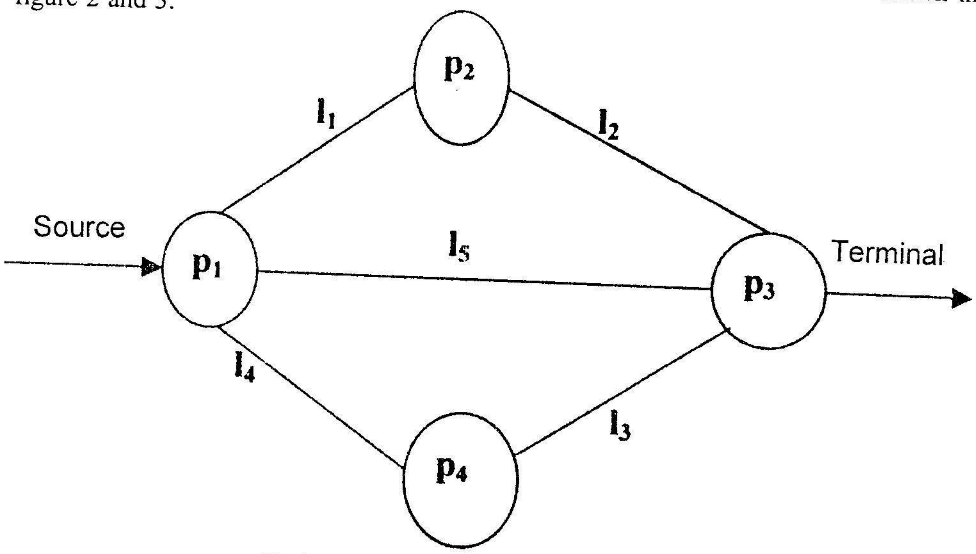

Fig. 1 : Bridge Computer Communication Network

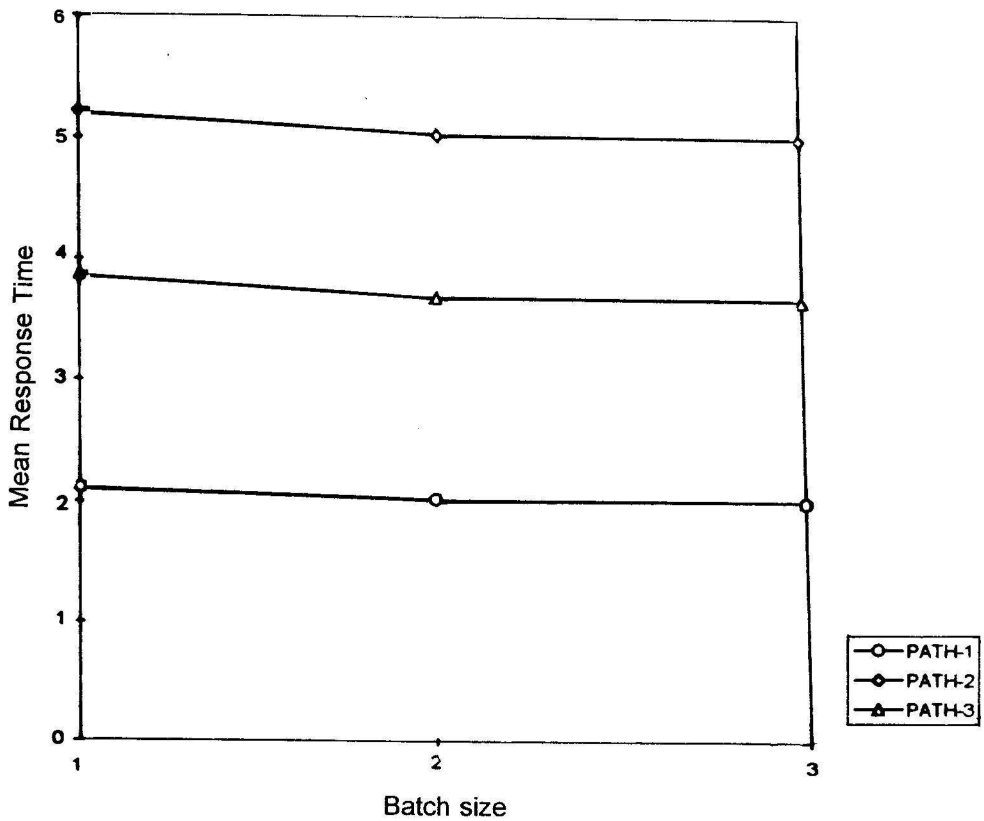

Fig. 2 : Batch Size vs. Mean Response Time

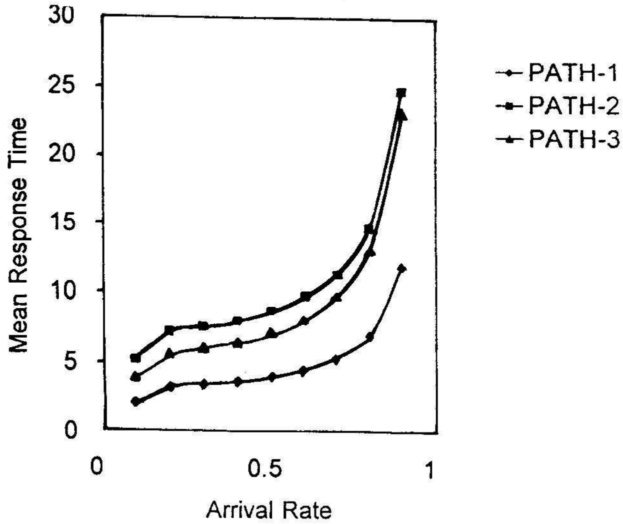

Fig. 3 : Arrival Rate vs. Mean Response Time

It can be seen from the figure 2, that the Mean response time is lesser for the path-1 i.e. $p_1 \rightarrow p_3$. Further for the path-2 i.e. $p_1 \rightarrow p_2 \rightarrow p_3$, the mean response time is higher as the service rate is poor. In case of Path-3 i.e. $p_1 \rightarrow p_4 \rightarrow p_3$, it is between the path-1 and path-2 shows that the mean response time decrease with the increase service rate.

The effect of arrival rate on the Mean response time has also been studied in the graph shown in figure 3. It has been observed that with the increase in the arrival rate, the mean response time increases.

This will be very useful to the network system designer working in the field of distributed computing systems. The decision of routing strategy in the design of computer communication network may also be facilitated by the present techniques.

References

1. Neil C. Wilhelm : A general model for the performance of disk systems, *J. ACM,* **24** (1977) 14-31.
2. W. Whitt : The Queuing Network Analyzer, *AT & T BLTJ,* **63** (1983) 1911-1979.
3. W. Whitt : The open and closed models for networks of queues, *AT & T BLTJ,* **63** (1983) 2779-2815.
4. F. P. Kelly : *Blocking probabilities in large Circuit-switched networks,* Wiley, New York, 1985.
5. W. Whitt : Blocking when service is required from several facilities simultaneously, *A.T.& T. Tech. J.,* 1987.
6. A. A. Fredericks : *An approximation method for analysing a virtual circuit-switch based LAN-solving the simultaneous resource possession problem,* Teletraffic Analysis and Computer Performance Evaluation, Science Publishers B.V., North-Holland, 1986.
7. C. M. Donalad and D. Gross : *Fundamentals of queuing theory,* John Wiley & Sons, 1985.