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Reliability Characterisitcs of Ring Type Local Area Networking System

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Abstract : The present work studies the availability analysis of ring type local area networking system working in either direction. In ring network topology, the failure of a single ring terminal breaks down the complete network. To increase the availability of ring network, one may work on the ring having data transmission in both directions. The regenerative point technique has been used to derive expressions for various reliability characteristics and numerical computations have also been done to present the results graphically.

1. Introduction

In the ring type network topology, the failure of a single ring breaks down the complete network. To increase the availability of the ring network, it is the simplest and the best proposal to configure the ring to have data transmission in both the directions. In such cases the failure of a single terminal or for that matter several terminals (depending upon their positions in the ring) does not affect the functioning of the ring network. If the ring includes k terminals than the failure of m terminals would not let the network fail, where $m = (k + 2)/2$, if k is even and $(k + 1)/2$, when k is odd. Failure of more than m terminals may either allow the system to continue to function partially or may have a complete failure, depending upon the position of the failed terminal in the ring.

2. System Description and Underlying Assumptions

The system contains n computer terminals out of which k terminals are linked in the form of a ring and $r = (n - k)$ terminals are present as cold standbys. Upon the failure of any terminal, the standby terminals start functioning instantaneously through a perfect switching mechanism. It is also assumed that the ring network has data transmission flow in both the directions. The system is supposed to work until two terminals work in the network, however its functioning would be considered degraded when less than k-terminals work in the ring.

The state transition diagram with flow of states is given in Fig. 1. The state S_0 represents the stage when all k-terminals of the ring are operative and all r standby terminals are in good state. Upon the failure of any terminal, the standby replaces it and

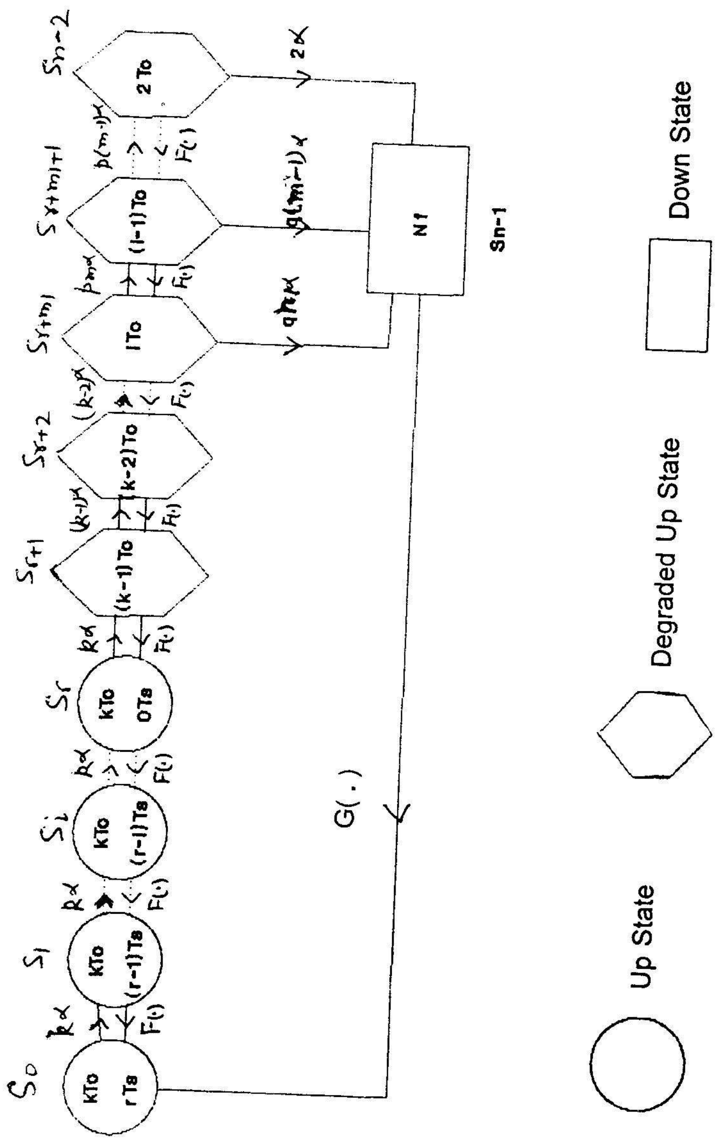

Fig. 1. State Transition Diagram

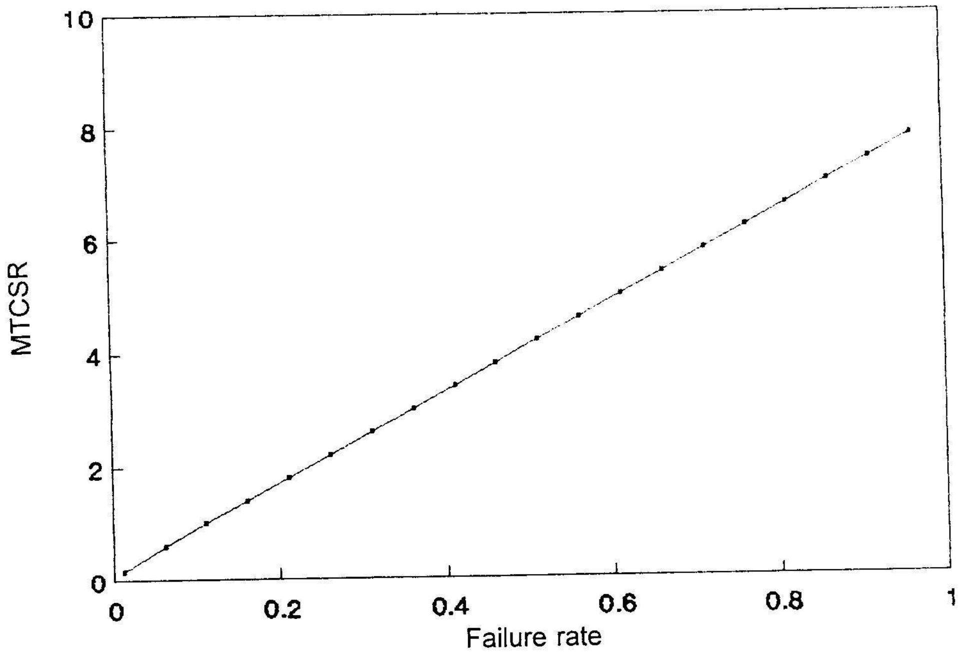

Fig. 2 : Failure Rate vs MTCSR

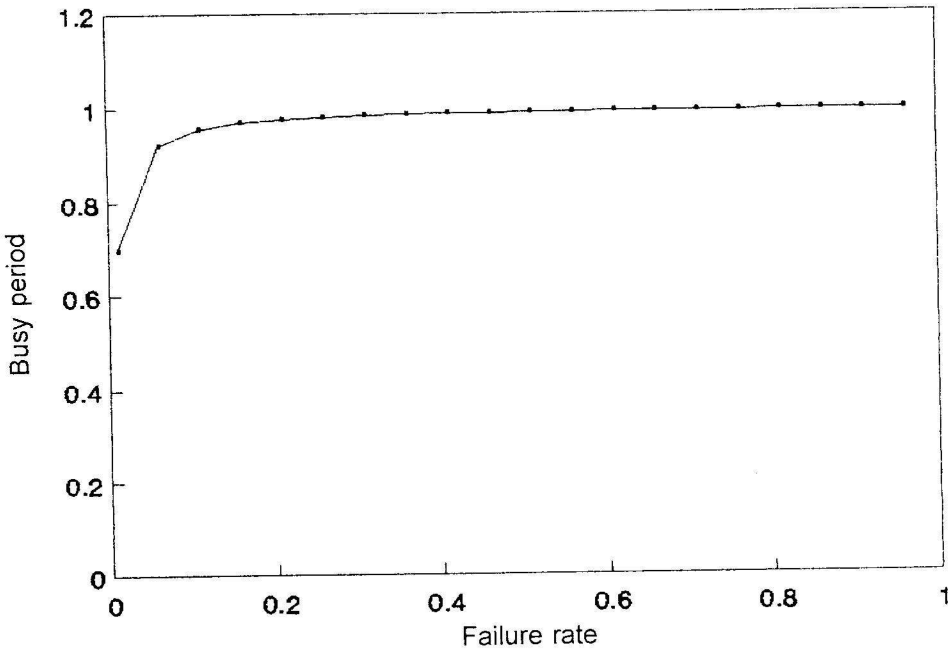

Fig. 3 : Failure Rate vs Busy Period

system remains in the up state denoted by S_1. Since there are r-standbys, the system remains in an up state till r terminals fail and these states are represented by S_1 to Sr in Fig. 1. When more than r-terminals fail, the system works in degraded up states represented by $Sr + 1$ to $Sr + m1$. Upon the failure of any terminal in state $Sr + m1$, the system has two possible states $Sr + m1 + 1$ with probability p, when it continues to work or state N_f with probability $q = (1 - p)$, where it breaks down completely. This complete breakdown depends on the position of the failed terminal. This situation is assumed to be the one where a terminal is in a good condition but fails to get the link with other terminals of ring. Lastly from the state Sn-2 where only two terminals are working, failure of any terminal leads to the complete failure of the network. It is also assumed that for the repair more than one repairmen are required.

The failure rates are assumed to be constant whereas the repair rates of terminals are considered to be general. The component is assumed to be as good as new after repair. The regenerative point technique has been used to obtain several reliability characteristics of the system.

3. Notations and the States of the System

$q_{ij}(\,.\,)$, $Q_{ij}(\,.\,)$ — The p.d.f. and c.d.f. of the first transition times from a regenerative state S_i, to a regenerative state S_j or to a failed state, S_j without visiting any other regenerative state in $(0, t\,]$.

P_{ij} — $\lim\limits_{t \to \infty} Q_{ij}\,(t)$.

E — Set of regenerative states.

* — Symbol for a Laplace transform, e.g. $f^*(s) = \int_0^\infty e^{-st} f(t)\,dt$.

$\sim$ — Symbol for a Laplace-Stieljes transform, e.g. $\widetilde{F}(s) = \int_0^\infty e^{-st}\,dF(t)$.

Ⓢ — Symbol for a Stieljes convolution, e.g. $A\,(t)$ Ⓢ $B\,(t) = \int_0^t B\,(t - u) dA(u)$.

© — Symbol for ordinary convolution, e.g. $a(t)$ © $b(t) = \int_0^t a(u)\,b(t - u)\,du$.

μ_{ij} — Contribution of mean sojourn time in a state $S_i \in E$ and the non- regenerative state. $\mu_{ij} = -\widetilde{Q}'_{ij}\,(0) = -\frac{d}{ds}\left[p^*_{ij}\,(s)\right]_{s=0}$; $\mu_i = \sum_j \mu_{ij}$

$\overline{F}(\,.\,)$ — Complementary function of any distribution $F(\,.\,)$, e.g. $\overline{F}(\,.\,) = 1 - F(\,.\,)$.

$\varphi_i(t)$ c.d.f. of first passage time from regenerative state S_i to a failed state.

$A_i(t)$ Probability that the system is in an up state at an instant t, given that the system enters the regenerative state.

$f(.), F(.)$ The p.d.f. and c.d.f. of repair time of each intelligent terminal.

$g(.), G(.)$ The p.d.f. and c.d.f. of repair time of the networking from the down state.

α Constant failure rate of each intelligent terminal.

kTo k terminals in operative mode.

rTs r terminals in standby mode.

N_f Networking in failure mode.

$$n = k + r\,;\; m = \begin{cases} \dfrac{k+2}{2} & \text{if } k \text{ is even} \\ \dfrac{k+1}{2} & \text{if } k \text{ is odd} \end{cases} \;;\quad m1 = \begin{cases} \dfrac{k-2}{2} & \text{if } k \text{ is even} \\ \dfrac{k-1}{2} & \text{if } k \text{ is odd} \end{cases}$$

The limit of integration whenever $(0, \infty)$ are not mentioned.

Possible States of the System

Up states : $S_i = (k\,To\,,\,(r - i)\,Ts\,)$; for $i = 0$ to r

Degraded up states : $S_{r+i} = ((k - i)\,To\,)$; for $i = 1$ to $k - 2$

Down states : $S_{n-1} = (N_f)$

Transition Probabilities

All possible outcome transition probabilities of this system are given below :

$P_{i,i+1} = 1 - f^*(k\alpha)$; $P_{i,i-1} = \int e^{-k\alpha t}\,dF(t)$; for $i = 1$ to r

$P_{r+i,r+i+1} = 1 - f^*((k - i)\alpha)$; $P_{r+i,r+i-1}$

$= \int e^{-(k-i)\alpha t}\,dF(t)$; for $i = 1$ to $m1 - 1$

$P_{j,j+1} = p(1 - f^*((r + m + m1 - j)\alpha)$;

$P_{j,j-1} = \int e^{-(r+m+m1-j)\alpha t}\,dF(t)$; for $j = r + m1$ to $n - 3$

$$P_{n-2,n-1} = 1 - f^*(2\alpha);\ P_{n-2,n-3} = \int e^{-2\alpha t}\, dF(t);\ P_{j,n-1}$$

$$= q\left(1 - f^*((r+m+m1-j)\alpha)\right);\quad \text{for } j = r + m1 \text{ to } n-3$$

$$P_{01} = 1 = P_{n-1,0} = P_{i,i-1} + P_{i,i+1};\quad \text{for } i = 1 \text{ to } n-3$$

$$P_{n-2,n-3} + P_{n-2,n-1} = 1 = P_{j,j-1} + P_{j,n-1}$$

$$\text{for } j = r + m1 \text{ to } n-3$$

The Mean sojourn time μ_i in state $S_i \in E$ is given by

$$\mu_0 = \frac{1}{k\alpha};\ \mu_i = \int e^{-k\alpha t}\, \bar{F}(t)\, dt;\quad \text{for } i = 1 \text{ to } r$$

$$\mu_j = \int e^{-(r+k-j)\alpha t}\, \bar{F}(t)\, dt;\quad \text{for } j = r+1 \text{ to } n-3;$$

$$\mu_{n-2} = \int e^{-2\alpha t}\, \bar{F}(t)\, dt;\ \mu_{n-1} = \int \bar{G}(t)\, dt.$$

4. Mean Time to Complete System Recovery

The time to complete system recovery (TCSR) can be regarded as the first passage time to the normal state S_0. To obtain it, we regard the state S_0 as an absorbing state. Let $V_i(t)$ be the complete system recovery time when $E_0 = S_i$, then $U_i(t) = P[V_i < t]$.

By employing the arguments used for regenerative process, we obtain the following recursive relations for $U_i(t)$, the cdf of $V_i(t)$.

$$U_i(t) = Q_{i,i-1}(t) \circledS U_{i-1}(t) + Q_{i,i+1}(t) \circledS U_{i+1}(t);$$

$$\text{for } i = 0 \text{ to } r + m1 - 1, \text{ where } U_{-1}(t) = 0 \text{ and } U_0(t) = 1$$

$$U_j(t) = Q_{j,j-1}(t) \circledS U_{j-1}(t) + Q_{j,j+1}(t) \circledS U_{j+1}(t)$$

$$+ Q_{j,n-1}(t) \circledS U_{n-1}(t);\ \text{for } j = r + m1 \text{ to } n-3$$

$$U_{n-2}(t) = Q_{n-2,n-3}(t) \circledS U_{n-3}(t) + Q_{n-2,n-1}(t) \circledS U_{n-1}(t)$$

$$U_{n-1}(t) = Q_{n-1,0}(t) \circledS U_0(t).$$

Taking Laplace-Stieljes transforms in relations (1) and solving for $\tilde{U}_0(s)$ we obtain

(1) $$\tilde{U}_0(s) = \frac{N1(s)}{D1(s)}$$

where, the determinants of $N1(s)$ and $D1(s)$ are given below with argument s dropped for brevity

$$N1(s) = \begin{vmatrix} 0 & -\tilde{Q}_{0,1} & 0 & 0 & & 0 & 0 \\ \tilde{Q}_{10} & 1 & -\tilde{Q}_{1,2} & 0 & & 0 & 0 \\ 0 & -\tilde{Q}_{2,1} & 1 & -\tilde{Q}_{2,3} & & 0 & 0 \\ 0 & 0 & -\tilde{Q}_{3,2} & 1 & \vdots\vdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots\vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots\vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \vdots & 0 & \vdots & \vdots & & 1 & -\tilde{Q}_{n-3,n-2} \\ 0 & 0 & \vdots & \vdots & & -\tilde{Q}_{n-2,n-3} & 1 \\ 0 & & & & & & \end{vmatrix}$$

and

$$D1(s) = \begin{vmatrix} 1 & -\tilde{Q}_{0,1} & 0 & 0 & & 0 & 0 \\ 0 & 1 & -\tilde{Q}_{1,2} & 0 & & 0 & 0 \\ 0 & -\tilde{Q}_{2,1} & 1 & -\tilde{Q}_{2,3} & & 0 & 0 \\ \vdots & 0 & -\tilde{Q}_{3,2} & 1 & & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots\vdots & \vdots & \vdots \\ -\tilde{Q}_{r+m1,n-1}\,\tilde{Q}_{n-1,0} & \vdots & \vdots & \vdots & \vdots\vdots & \vdots & \vdots \\ -\tilde{Q}_{r+m1+1,n-1}\,\tilde{Q}_{n-1,0} & \vdots & \vdots & \vdots & \vdots\vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \vdots & 0 & \vdots & \vdots & & 1 & -\tilde{Q}_{n-3,n-2} \\ -\tilde{Q}_{n-2,n-1}\,\tilde{Q}_{n-1,0} & 0 & \vdots & \vdots & & -\tilde{Q}_{n-2,n-3} & 1 \end{vmatrix}$$

For a particular case $n = 5$, $k = 3$ and $r = 2$, the expressions for $N1(s)$ and $D1(s)$ are given below :

$$N1(s) = \tilde{Q}_{10}\left(1 - \tilde{Q}_{23}\,\tilde{Q}_{32}\right)$$

(2) $$D1(s) = 1 - \tilde{Q}_{12}\,\tilde{Q}_{21} - \tilde{Q}_{23}\,\tilde{Q}_{32} - \tilde{Q}_{01}\,\tilde{Q}_{12}\,\tilde{Q}_{23}\,\tilde{Q}_{34}\,\tilde{Q}_{40}$$

The mean time to complete system recovery (MTCSR), given $E_0 = S_0$ is

(3) $$MTCSR = -\left\{\frac{d\tilde{U}_0(s)}{ds}\right\}_{s=0} = \frac{D'1(0) - N'1(0)}{D1(0)}$$

where

(4) $$D'1(0) - N'1(0) = P_{12} P_{23} P_{34} \mu_0 + \left(1 - P_{23} P_{32}\right) \mu_1 + p_{12}\left(\mu_2 + p_{23}\left(\mu_3 + p_{34}\mu_4\right)\right)$$

(5) $$D\,1(0) = P_{10}\left(1 - P_{23} P_{32}\right)$$

5. Busy Period Analysis

Let $B_i(t)$ be the probability that the system having started from regenerative state S_i at $t = 0$ is under repair. By probabilistic arguments we have the following recursive relations for $B_i(t)$.

$$B_i(t) = W_i(t) + q_{i,i-1}(t) © B_{i-1}(t) + q_{i,i+1}(t) © B_{i+1}(t);$$

$$\text{for } i = 0 \text{ to } r + m1 - 1, \text{ where } B_{-1}(t) = 0 = W_0(t)$$

$$B_j(t) = W_j(t) + q_{j,j-1}(t) © B_{j-1}(t) + q_{j,j+1}(t) © B_{j+1}(t);$$

$$+ q_{j,n-1}(t) © B_{n-1}(t); \text{ for } j = r + m1 \text{ to } n - 3$$

(6) $$B_{n-2}(t) = W_{n-2}(t) + q_{n-2,n-3}(t) © B_{n-3}(t)$$

$$+ q_{n-2,n-1}(t) © B_{n-1}(t)$$

$$B_{n-1}(t) = W_{n-1}(t) + q_{n-1,0}(t) © B_0(t).$$

Taking Laplace transforms of relations (6) and solving for $B_0^*(s)$

(7) $$B_0^*(s) = \frac{N2(s)}{D2(s)}$$

where, dropping argument s for brevity, the determinant of $N2(s)$ is defined below

$$\mathrm{N2(s)} = \begin{vmatrix} 0 & & & & & & \\ W_1^* & -q_{0,1}^* & 0 & & & & 0 \\ W_2^* & 1 & -q_{1,2}^* & & & & 0 \\ & -q_{2,1} & 1 & & & & 0 \\ W_3^* & 0 & -q_{3,2}^* & & & & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ W_{r+l1}^* + q_{r+l1,n-1}^* W_{n-1}^* & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ W_{r+l1+1}^* + q_{r+l1+1,n-1}^* W_{n-1}^* & \vdots & \vdots & & & & \vdots \\ \vdots & \vdots & \vdots & & & & \vdots \\ \vdots & \vdots & \vdots & & & & -q_{n-3,n-2}^* \\ W_{n-2}^* + q_{n-2,n-1}^* W_{n-1}^* & \vdots & \vdots & & & & 1 \end{vmatrix}$$

and

$$
D2(s) = \begin{vmatrix}
1 & -q^*_{0,1} & 0 & \cdots & 0 \\
0 & 1 & -q^*_{1,2} & \cdots & 0 \\
0 & -q^*_{2,1} & 1 & \cdots & 0 \\
\vdots & 0 & -q^*_{3,2} & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots \\
-q^*_{r+1,n-1}q^*_{n-1,0} & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & & \vdots \\
-q^*_{n-2,n-1}q^*_{n-1,0} & \vdots & \vdots & & -q^*_{n-3,n-2} \\
 & \vdots & \vdots & & 1
\end{vmatrix},
$$

It is not possible to obtain $N\,2(s)$ and $D\,2(s)$ in explicit form, therefore for a particular case $n = 5$, $k = 3$ and $r = 2$, we have

$$N\,2\,(s) = W_1^*\left(1 - q_{23}^* q_{32}^*\right) + q_{12}^*\left(W_2^* + q_{23}^*\left(W_3^* + q_{34}^* W_4^*\right)\right)$$

(8) $$D\,2\,(s) = \left(1 - q_{10}^* q_{01}^*\right)\left(1 - q_{23}^* q_{32}^*\right) - q_{12}^*\left(q_{21}^* + q_{23}^* q_{34}^* q_{40}^* q_{01}^*\right)$$

In steady state, the expected time, that the repairmen are busy, is given by

(9) $$B_0 = \lim_{s \to 0} sB_0^*(s) = \frac{N\,2\,(0)}{D'\,2\,(0)}$$

where

(10) $$N\,2\,(0) = \left(1 - P_{23}P_{32}\right)\mu_1 + P_{12}\left(\mu_2 + P_{23}\left(\mu_3 + P_{34}\mu_4\right)\right)$$

$$D'\,2(0) = \left(P_{12}P_{23} + P_{10}P_{34}\right)\mu_0 + \left(1 - P_{23}P_{32}\right)\mu_1$$
$$+ P_{12}\left(\mu_2 + P_{23}\left(\mu_3 + P_{34}\mu_4\right)\right)$$

6. Numerical Computations and Graphical Representations

Letting $F(t) = 1 - e^{-at}$ and $G(t) = 1 - e^{-bt}$, Fig. 2 and Fig. 3 show the graphs for MTCSR and Busy Period, using the following particular values of the constants $a = 0.025$, $b = 0.015$, $p = 0.5$ and $q = 0.5$.

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