Vol. 1 (1997), Nos. 3-4 **Journal of International Academy of Physical Sciences** pp. 171-182

Two-unit System with Degradation and Degraded Failure

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(Received November 5, 1997)

Abstract. This paper presents reliability of a two unit three state system in which efficiency of an operative unit reduces with time. A unit may suffer two types of failure i.e., normal and degraded failure. Supplementry variable technique has been used to obtain expressions for various reliability characteristics of interest. Efforts have also been made to perform numerical computation and plot the graphs to give a clear picture of the results.

1. Introduction

Most of the studies in reliability have been directed towards the analysis of complex systems with two states only i.e., good and failed. In actual practice the two-state assumption may many times lead to over-estimation of reliability, due to the fact that with increase in time some components/units of the system get degraded and cannot be assumed to function normally. Thus a third state between the good and failed states must be included for a real estimation of reliability characteristics. In some studies[1-4] a third state has been included as a degraded state but these studies viewed the degraded state to be the one in which less number of components remain operative than the desired. Under this assumption every operative unit has to pass through the degraded state before its complete failure. These studies therefore exploited the degraded state only to the extent of increase in failure rate from this state.

In fact it is necessary that a unit may always reach to a degraded state before its failure. Any unit/component of the system may fail directly to lead the system to a failed state. In the present work we have taken into account the following situations that occur in real practice:

(1) The efficiency of an operative unit reduces with time and it gets degraded in its functioning .

(2) A degraded unit is more prone to failure than a new unit.

(3) A new unit may fail even without getting degraded.

Further to reach closer to real situations, we have also included the possibilities of common cause failure and human failure in this work. Supplementry variable technique has been used to obtain various reliability characteristics of interest. Efforts have also been made to perform numerical computations and plot the graphs to give a clear picture of the results.

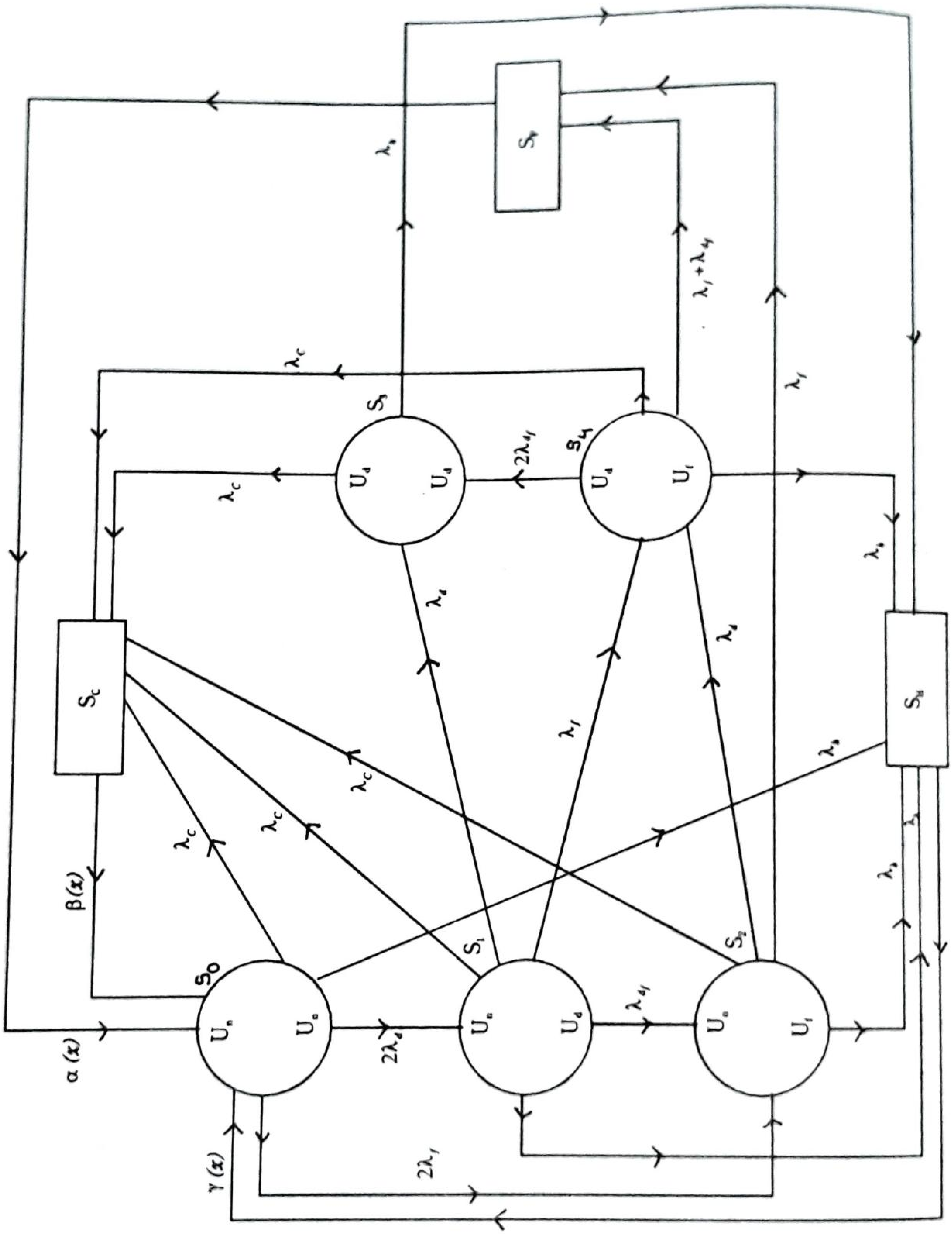

Fig. -1

2. Assumptions

The following assumptions are associated with the system under study.

(a) Failures are statistically independent.

(b) A unit failure distribution is negative exponential.

(c) A common cause failure of human failure lead to system failure.

(d) Failed system repair distribution are general.

(e) Both units are active and identified.

(f) The stage where only one unit is operative in degraded state and the other failed, the failure rate becomes higher and is sum of the failure rates of a new unit and degraded unit.

3. Model Description

The state transition diagram of the system is given in Fig. 1. In State S_0, both units are considered to be in perfect conditions. When any of the unit in S_0 gets degraded/failed, the system goes to state S_1/S_2. Upon the failure of degraded unit in S_1, the system reaches the up state S_2. The state S_3 having both units in degraded state is arrived at from the state S_1 upon the degradation of the normal unit. The State S_4 is critically operative state having one unit in degraded state and the other failed. This state can be reached from states $S_1/S_2/S_3$ upon the failure of a unit/the degradation of the normal unit/the degraded failure of either unit respectively. State S_F is obtained from State S_4 when the only working unit there gets failed. States S_H and S_c respectively denote the states reached due to human failure and common cause failure. The human failure and common cause failure can occur from any of the working states of the systems.

3. Notations

$P_j(t)$:Probability that the system is in state S_j at time (t = 0 1 2, 3, 4, F, C, H)

$P_j(x, t)$:Probability density (w.r.t. repair time) that the failed system is in state S_j and has elapsed repair time x ($j = F, C, H$).

P_j :Steady state probability that the system is in state S_j (j = 0, 1, 2, 3, 4, F, C, H)

λ_f :Constant failure rate of a unit including standing unit

λ_d :Constant rate of degradation of a unit.

λ_{df} :Constant failure rate of degraded unit.

λ_c/λ_h :Constant failure rate from the states S_j, (j = 0, 1, 2, 3, 4) to the state S_C/S_H.

$\alpha(x)/\beta(x)/\upsilon(x)$:Repair rates from states $S_F/S_C/S_H$ to state S_0

$f(S)$:Laplace transforem of $f(t)$

$$S_k(s) = \int K(x) \exp\left[-s x - \int_0^x K(x)\, d x\right] d x,\ k = \alpha, \beta\, \gamma$$

The $\int$ definite integral fro.n 0 to ∞.

$U_N/U_d/U_f$:Operating unit in normal/degraded/failed state.

4. Differential equations of the model

Using the supplementry variable method, the folloing system of differential equations, have been obtained:

$$\frac{d P_0(t)}{d t} + [2 \lambda_f + 2 \lambda_d + \lambda_c + \lambda_h] P_0(t) = \int P_F(x, t)\, \alpha(x)\, d x + \int P_c(x, t)\, \beta(x)\, d x$$

$$+ \int P_H(x, t)\, \upsilon(x)\, d x \quad (1)$$

$$\frac{d P_1(t)}{d t} + [\lambda_f + \lambda_d + \lambda_{df} + \lambda_c + \lambda_h] P_1(t) = 2 \lambda_d P_0(t) \quad (2)$$

$$\frac{d P_2(t)}{d t} + [\lambda_f + \lambda_d + \lambda_c + \lambda_h] P_2(t) = 2 \lambda_f P_0(t) + \lambda\, d_f P_1(t) \quad (3)$$

$$\frac{d P_3(t)}{d t} + [\, 2\lambda d_f + \lambda_c + \lambda_h] P_3(t) = \lambda_d P_1(t) \quad (4)$$

$$\frac{d P_4(t)}{d t} + [\lambda_f + \lambda_{d_f} + \lambda_c + \lambda_h] P_4(t) = \lambda_f P_1(t) + \lambda_d P_2(t) + 2 \lambda_{d_f} P_3(t) \quad (5)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + k(x)\right] P_j(x, t) = 0;\ k = \alpha/\beta/\upsilon,\ j = F, C, H \quad (6)$$

$$P_F(0, t) = \lambda_f P_2(t) + (\lambda_f + \lambda_{d_f}) P_4(t) \quad (7)$$

$$P_C(0, t) = \lambda_c [P_0(t) + P_1(t) + P_2(t) + P_3(t) + P_4(t)] \quad (8)$$

$$P_F(0, t) = \lambda_H [P_0(t) + P_1(t) + P_2(t) + P_4(t)] \quad (9)$$

(10) $$P_i(0) = \begin{cases} 1 & \text{where } i = 0 \\ 0 & \text{otherwise} \end{cases}$$

Solving the above equation using Laplace transforms leads to the following Laplace trasnforms of the state probabilities:

(11) $$P_0(s) = \frac{1}{K(s)}$$

where

$$K(s) = [s + 2\lambda_f + 2\lambda_d + \lambda_c + \lambda_h - \{\lambda_f X_2 + (\lambda_f + \lambda_{d_f}) X_4\} S_\alpha(s)$$

(12) $$-\{\lambda_c\ S_\beta(s) + \lambda_H S_\upsilon(s)\}\ \{1 + X_1 + X_2 + X_3 + X_4\}]$$

where $A_j = S + I_j\ (j = 1, 2, 3, 4)$

$$X_1 = 2\lambda d / A_1,\ X_2 = (2\lambda_f + \lambda_{d_f} X_1)/A_2,\ X_3 = \lambda d\, X_1/A_3$$

$$X_4 = (\lambda_f X_1 + \lambda_d X_2 + \lambda_{d_f} X_3)/A_4$$

(13) $$P_j(s) = X_j P_0(s),\ (j = 1, 2, 3, 4)$$

(14) $$P_F(S) = \frac{1}{s}[\lambda_f X_2 + (\lambda_f + \lambda_{d_f}) X_4]\ (1 - S_\alpha(s))\ P_0(s)$$

(15) $$P_c(S) = \frac{\lambda_c}{s}\{1 + X_1 + X_2 + X_3 + X_4\}\ (1 - S_\beta(s))\ P_0(s)$$

(16) $$P_H(S) = \frac{\lambda_H}{s}\{1 + X_1 + X_2 + X_3 + X_4\}\ (1 - S_\upsilon(s))\ P_0(s)$$

The Laplace transforms of the up and down state probabilities are given by

(17) $$P_{up}(s) = \sum_{j=0}^{4} P_j(s)$$

(18) $$P_{down}(s) = P_F(S) + P_c(s) + P_H(s)$$

It has been verified from (17) and (18) that

$$P_{up}(s) + P_{down}(s) = \frac{1}{s}$$

5. Solution for constant repair rates

Now taking the failure and repair rates as constant and solving the differential equations by taking their Laplace transforms, we get

$$\overline{P}_0(s) = \frac{1}{K_1(s)} \tag{19}$$

where

$$K_1(s) = \Big[S + 2\lambda_f + 2\lambda_d + \lambda_c + \lambda_h - \left(\frac{\alpha}{S+\alpha}\right)(\lambda_f X_2 + (\lambda_f + \lambda_{d_f}) X_4)$$

$$- \left(\frac{\beta}{S+\beta}\lambda_c + \frac{\upsilon}{S+\upsilon}\lambda_H\right)(1 + X_1 + X_2 + X_3 + X_4)\Big] \tag{20}$$

$$P_j(s) = X_j \overline{P}_0(s), \ (j = 1, 2, 3, 4) \tag{21}$$

$$P_F(s) = \frac{1}{(\beta+\alpha)}[\lambda_f X_2 + (\lambda_f + \lambda_{d_f}) X_4] \overline{P}_0(s) \tag{22}$$

$$P_c(s) = \frac{\lambda_c}{(s+\beta)}[1 + X_1 + X_2 + X_3 + X_4] \overline{P}_0(s) \tag{23}$$

$$P_H(s) = \frac{\lambda_H}{(s+\upsilon)}[1 + X_1 + X_2 + X_3 + X_4] \overline{P}_0(s) \tag{24}$$

Using (19) to (24) we can obtain Laplace transforms of the up and down state probabilities as below

$$\overline{P}_{up}(s) = \sum_{j=0}^{4} \overline{P}_j(s) = [1 + X_1 + X_2 + X_3 + X_4] \overline{P}_0(s)$$

$$= [1 + X_1 + X_2 + X_3 + X_4](K_1(s)^{-1})$$

$$\overline{P}_{down}(s) = 1 - \overline{P}_{up}(s)$$

6. Availability Analysis

Let us assume that the repairs follow exponential time distribution. Letting therefore

$$S_\alpha(s) = \frac{\alpha}{s+\alpha};\ S_\beta(s) = \frac{\alpha}{s+\beta};\ S_\gamma(s) = \frac{\gamma}{s+\gamma}.$$

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Further setting $\alpha = \beta = \gamma = \phi$, say, Laplace transforms of the state probilities reduce to the following:

$$\overline{P}_0(s) = \frac{1}{\overline{K}(s)}$$

where

(25) $$\overline{K}(s) = \Big[s + 2\lambda_f + 2\lambda_d + \lambda_c + \lambda_h - \frac{\phi}{s+\phi}\{\lambda_f X_2 + (\lambda_f + \lambda_{d_f}) X_4$$

$$+ (\lambda_c + \lambda_h)(1 + X_1 + X_2 + X_3 + X_4)\} \Big]$$

(26) $$\overline{P}_j(s) = X_j \overline{P}_0(s); (j = 1, 2, 3, 4)$$

From (26)

(27) $$P_{up}(s) = \overline{P}_j(s)\ j = 0, 1, 2, 3, 4$$

$$= (1 + X_1 + X_2 + X_3 + X_4)\,\overline{P}_0(s)$$

$$= \frac{s^5 + c_1 s^4 + c_2 s^3 + c_3 s^2 + c_4 s + c_5}{s^6 + d_1 s^4 + d_3 s^3 + d_4 s^2 + d_5 s + d_6}$$

where

(28) $$c_1 = \sum_{i=1}^{4} I_i + 2(\lambda_f + \lambda_d) + \phi$$

$$c_2 = \sum I_1 I_2 + 2\lambda_d(I_2 + I_3 + I_4) + 2\lambda_f(I_1 + I_3 + I_4) + 4\lambda_f\lambda_d$$

(29) $$+ 2\lambda_d^2 + 2\lambda_d\lambda_{d_f} + \phi(c_1 - \phi)$$

$$c_3 = \sum I_1 I_2 I_3 + 2\lambda_f\lambda_d(U_1 + I_2 + 2I_3) + 2\lambda_d^2(I_2 + I_4) + 2\lambda_d\lambda_{d_f}(I_3 + I_4) + 6\lambda_d^2\lambda_{d_f}$$

(30) $$+ 2\lambda_d(I_2 I_3 + I_3 I_4 + I_4 I_2) + 2\lambda_f(I_1 I_3 + I_3 I_4 + I_4 I_1) + \phi(c_2 - (c_1 - \phi)\phi)$$

$$c_4 = \prod_{i=1}^{4} I_i + 2\,\lambda_f \lambda_d (I_2 I_3 + I_1 I_3) + 2\,\lambda_d^2 I_2 I_4 + 2\,\lambda_d \lambda_{d_f} I_3 I_4 + 2\,\lambda_d^2 \lambda_{d_f} (2\,I_2 + I_3)$$

$$+ 2\,\lambda_f I_1 I_3 I_4 + 2\,\lambda_d I_2 I_3 I_4 + \phi\,(c_3 - \phi\,c_2 - \phi^2\,(c_1 - \phi)) \tag{31}$$

$$c_5 = \phi \Bigg[\prod_{i=1}^{4} I_i + 2\,\lambda_f \lambda_d (I_2 I_3 + I_1 I_3) + 2\,\lambda_d^2 I_2 I_4 + 2\,\lambda_d \lambda_{d_f} I_3 I_4$$

$$+ 2\,\lambda_d^2 \lambda_{d_f} (2\,I_2 + I_3) + 2\,\lambda_f I_1 I_3 I_4 + 2\,\lambda_f I_1 I_3 I_4 + 2\,\lambda_4 I_2 I_3 I_4 \Bigg] \tag{32}$$

$$d_1 = c_1 + \lambda_c + \lambda_h \tag{33}$$

$$d_2 = (I_2 + \lambda_f + \lambda_d + \phi) \sum_{i=1}^{4} I_i + \sum I_1 I_2 + \phi\,(I_2 + \lambda_f + \lambda_d) - \phi\,(\lambda_c + \lambda_h) \tag{34}$$

$$d_3 = \sum_{i=1}^{4} I_i\,\phi\,(I_2 + \lambda_f + \lambda_d) + \sum I_1 I_2 (I_2 + \lambda_f + \lambda_d + \phi) + \sum I_1 I_2 I_3$$

$$- \phi \Bigg[(\lambda_c + \lambda_h) \sum_{i=1}^{4} I_i + 2\,\lambda_d (\lambda_c + \lambda_h) + 2\,\lambda_f (\lambda_c + \lambda_h + \lambda_f) \Bigg] \tag{35}$$

$$d_4 = \sum I_1 I_2\,\phi\,(I_2 + \lambda_f + \lambda_d) + \sum I_1 I_2 I_3 (I_2 + \lambda_f + \lambda_d + \phi) + \prod_{i=1}^{4} I_i - \phi\,[(\lambda_c + \lambda_h) \sum I_1 I_2$$

$$+ 2\,\lambda_d (\lambda_c + \lambda_h)(I_2 + I_3 + I_4) + 2\,\lambda_f (\lambda_c + \lambda_h + \lambda_f)(I_1 + I_2 + I_4)$$

$$+ 2\,\lambda_d \lambda_f I_4 + 2\,\lambda_d^2 (\lambda_c + \lambda_h) + 2\,\lambda_d \lambda_{d_f} (\lambda_c + \lambda_h + \lambda_f) + 2\,\lambda_d \lambda_f I_4] \tag{36}$$

$$d_5 = \sum I_1 I_2 I_3\,\phi\,(I_2 + \lambda_f + \lambda_d) + \prod_{i=1}^{4} I_i (I_2 + \lambda_f + \lambda_d + \phi) - \phi\,[(\lambda_c + \lambda_h) \sum I_1 I_2 I_3$$

$$+ 2\lambda_d(\lambda_c + \lambda_h)(l_2 l_3 + l_3 l_4 + l_4 l_2) + 2\lambda_f(\lambda_c + \lambda_h + \lambda_f)(l_1 l_3 + l_3 l_4 + l_4 l_1)$$

$$+ 2\lambda_d \lambda_f l_4 (l_2 + l_3) + 2\lambda_d^2(\lambda_c + \lambda_h)(l_2 + l_4) + 2\lambda_d \lambda_{d_f}(\lambda_c + \lambda_h + \lambda_f)(l_3 + l_4)$$

(37)
$$+ 2\lambda_d \lambda_f l_4 (l_1 + l_3) + 6\lambda_d^2 \lambda_{d_f} l_4]$$

$$d_6 = \phi\Bigg[(l_2 + \lambda_2 + \lambda_f \lambda_d)\prod_{i=1}^{4} l_i - \Bigg\{(\lambda_c + \lambda_h)\prod_{i=1}^{4} l_i + 2\lambda_d(\lambda_c + \lambda_h)\, l_2 l_3 l_4$$

$$+ 2\lambda_f(\lambda_c + \lambda_h + \lambda_f)\, l_1 l_3 l_4 + 2\lambda_d \lambda_f l_2 l_3 l_4 + 2\lambda_d^2(\lambda_c + \lambda_h)\, l_2 l_4$$

(38)
$$+ 2\lambda_d \lambda_{d_f}(\lambda_c + \lambda_h + \lambda_f)\, l_3 l_4 + 2\lambda_d l_1 l_2 l_4 + 2\lambda_d^2 \lambda_{d_f} l_4 (2 l_2 + l_3)\Bigg\}\Bigg]$$

To obtain the exression for $P_{up}(s)$, we substitute $\phi = 1$ in equation (28) to (38)

(39)
$$P_{up}(s) = \frac{s^5 + \bar{c}_1 s^4 + \bar{c}_2 s^3 + \bar{c}_3 s^2 + \bar{c}_4 s + \bar{c}_5}{s^6 + \bar{d}_1 s^5 + \bar{d}_2 s^4 + \bar{d}_3 s^3 + \bar{d}_4 s^2 + \bar{d}_5 s + \bar{d}_6}$$

where $\bar{c}_1, \bar{c}_2, \bar{c}_3, \bar{c}_4, \bar{c}_5$, and $\bar{d}_1, \bar{d}_2, \bar{d}_3, \bar{d}_4, \bar{d}_5, \bar{d}_6$ are obtained by putting $\phi = 1$.

MTTF: To obtain expression for MTTF, one needs an expression for $R(S)$. It may be obtained by substituting $\phi = 0$ in (28) to (38). Therefore

(40)
$$R(s) = \frac{s^5 + c_1^\bullet s^4 + c_2^\bullet s^3 + c_3^\bullet s^2 + c_4^\bullet s + c_5^\bullet}{s^6 + d_1^\bullet s^5 + d_2^\bullet s^4 + d_3^\bullet s^3 + d_4^\bullet s^2 + d_5^\bullet s + d_6^\bullet}$$

where

$$c_1^\bullet = \sum_{i=1}^{4} l_i + 2(\lambda_f + \lambda_d),$$

$$c_2^\bullet = \sum l_1 l_2 + 2\lambda_d(l_2 + l_3 + l_4) + 2\lambda_f(l_1 + l_3 + l_4) + 4\lambda_f \lambda_d + 2\lambda_d^2 + 2\lambda_d \lambda_{d_f};$$

$$c_3^\bullet = \sum l_1 l_2 l_3 + 2\lambda_f \lambda_d (l_1 + l_2 + 2 l_3) + 2\lambda_d^2(l_2 + l_4) + 2\lambda_d \lambda_{d_f}(l_3 + l_4)$$

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$$+ 6\,\lambda_d^2\,\lambda_{d_f} + 2\,\lambda_d\,(I_2\,I_3 + I_3\,I_4 + I_4\,I_2) + 2\,\lambda_f\,(I_1\,I_3 + I_3\,I_4 + I_4\,I_1).$$

$$c_4^* = \prod_{i=1}^{4} I_i + 2\,\lambda_f\lambda_d\,(I_2\,I_3 + I_1\,I_3) + 2\,\lambda_d^2\,I_2\,I_4 + 2\,\lambda_d\,\lambda_{d_f}\,I_3\,I_4$$

$$+ 2\,\lambda_d^2\,\lambda_{d_f}\,(2\,I_2 + I_3) + 2\,\lambda_f\,I_1\,I_3\,I_4 + 2\,\lambda_d\,I_2\,I_3\,I_4$$

$$c_5^* = 0.$$

$$d_1^* = c_1^* + \lambda_c + \lambda_H,$$

$$d_2^* = (I_2 + \lambda_f + \lambda_d + \phi)\sum_{i=1}^{4} I_i + \sum I_1\,I_2,$$

$$d_3^* = \sum I_1\,I_2\,(I_2 + \lambda_f + \lambda_d) + \sum I_1\,I_2\,I_3,$$

$$d_4^* = \sum I_1\,I_2\,I_3\,(I_2 + \lambda_f + \lambda_d) + \prod_{i=1}^{4} I_i\,,$$

$$d_5^* = \prod_{i=1}^{4} I_i\,(I_2 + \lambda_f + \lambda_d)$$

$$d_6^* = 0$$

$$MTTF = \lim_{s\to 0} R\,(s) = c_4^*/d_5^*$$

7. Numerical Computations

For obatining values of pointwise availability for different values of time. We use, the following numerical values.

$\lambda_c = 0.001$, $\lambda_H = 0.002$, $\lambda_f = 0.02$, $\lambda_d = 0.03$, $\lambda_{d_f} = 0.04$

Using the above values we get the following expression for $P_{up}\,(s)$

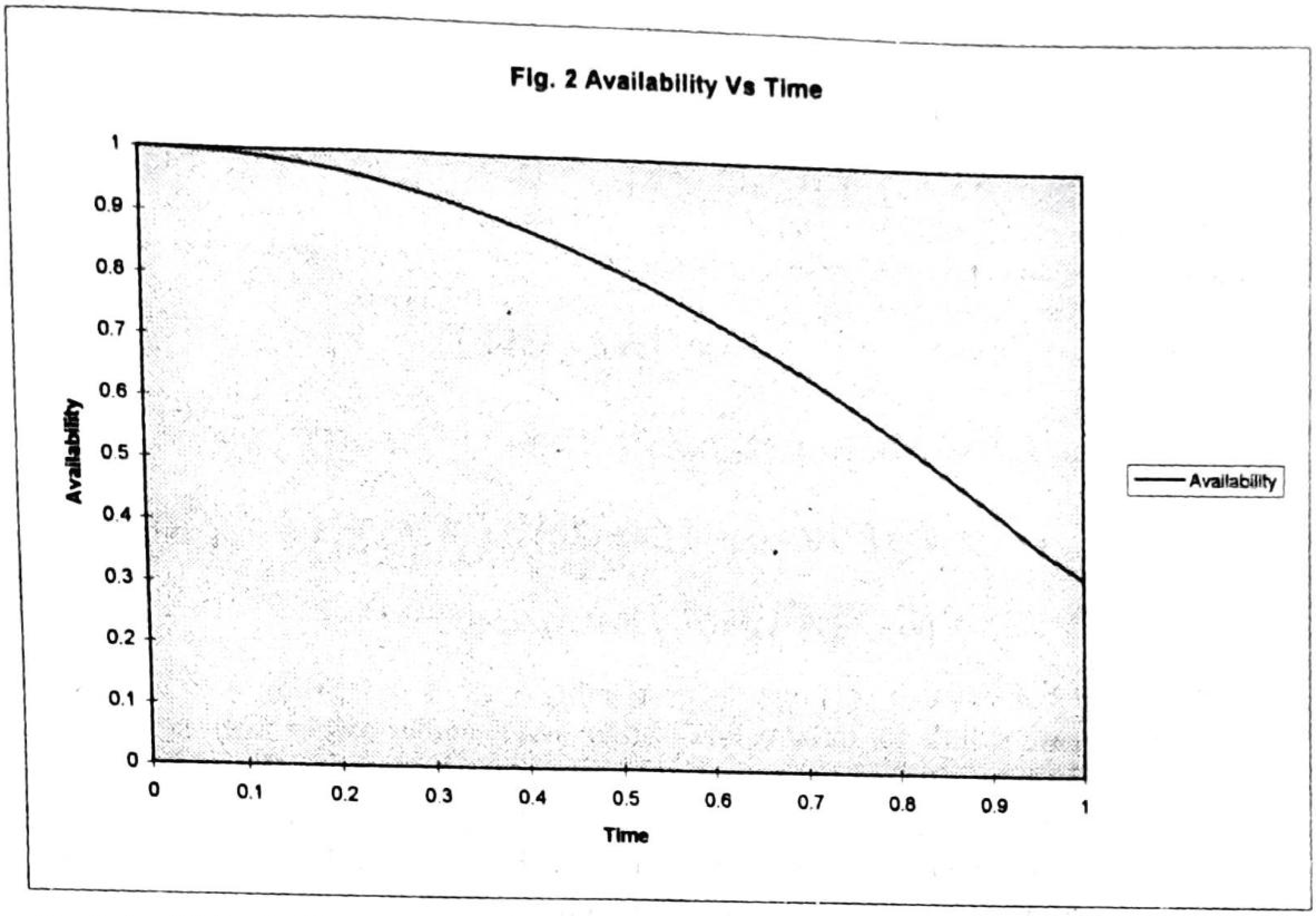

Fig. 2. Availability Vs Time

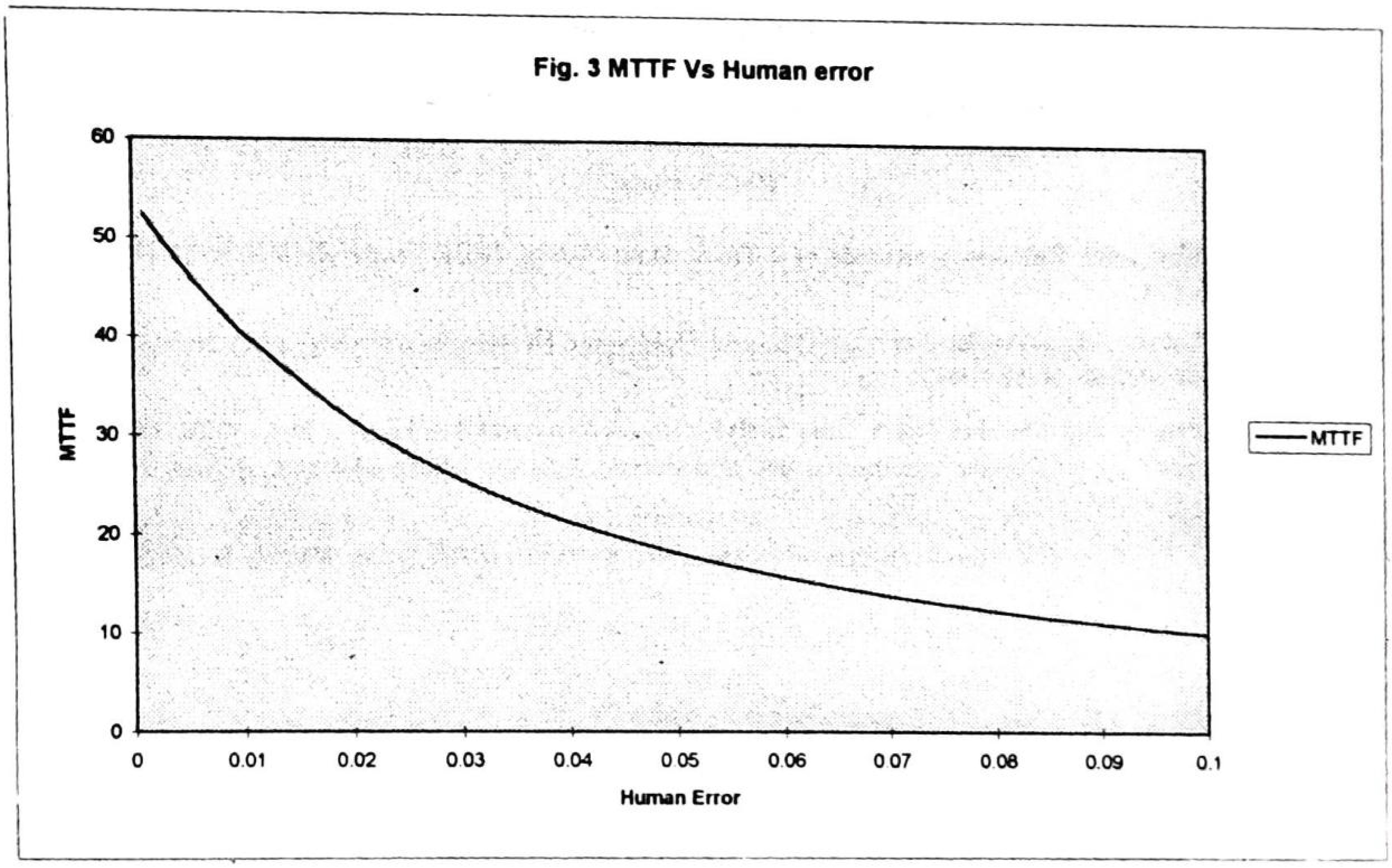

Fig. 3. MTTF Vs Human error

$$P_{up}(s) = \frac{s^5 + (1.392)\,s^4 + (0.451574)\,s^3 + (4.162968 \times 10^{-3})\,s^2 + \{-(0.8393024)\,s + 1.085632 \times 10^{-4}\}}{s\,(s^5 + (1.395)\,s^4 + 0.45355\,s^3 + (6.429889 \times 10^{-2})\,s^2 + 5.276498 \times 10^{-3}\,s + 2.029795 \times 10^{-4})} \quad (41)$$

Taking inverse Laplace trasform of the function given in (41) one may get the following expressions for point-wise availability of the system over time t as given below

$$P_{up}(t) = 1.630384 \times 10^{-5} + (-1.1403836\,E-02)\exp(-9.39976\,E-02\,t)$$

$$+ (-- 0.927365)\exp(-1.001023t)$$

$$+ (1.785499\,E-02)\exp(0.2306121t)\cos(0.2663671t)$$

$$+ (-5.007618286)\exp(0.230612t)\sin(0.266371t)$$

The graph 1 shows that although the availability goes on decreasing with time, yet the rate of decrease is little for intial values of time and becomes higher with increase of time.

The graph of MTTF Vs humman error show that MTTF falls with increase in human error. An interesting point of observation in this graph is that for λ_H having its value $0 < \lambda_H < 0.04$, MTTF falls sharply but for $\lambda_H > 0.04$ its rate of fall decreases.

Acknowledgment

One of the authors (Rachna Kumari) is greatful to CSIR for financial support.

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