ISSN 0974 - 9373

Vol. 15(2011) Special Issue 1 **Journal of International Academy of Physical Sciences** pp. 213-230

Effects of Hall Current and Rotation on MHD Couette Flow of Class-II*

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(Received June 14, 2011)

Abstract: Effects of Hall current and rotation on steady MHD Couette flow of Class-II of a viscous, incompressible and electrically conducting fluid in the presence of a uniform transverse magnetic field are studied. Exact solution of the governing equations is obtained in closed form. Expressions for shear stress at the lower and upper plates due to primary and secondary flows and mass flow rates in the primary and secondary flow directions are derived. Asymptotic behavior of the solution for velocity and induced magnetic field is analyzed for large values of rotation parameter K^2 and magnetic parameter M^2 to gain some physical insight into flow pattern. Heat transfer characteristics of the fluid are considered taking viscous and Joule dissipations into account. Numerical solution of energy equation and numerical values of rate of heat transfer at lower and upper plates are computed with the help of MATLAB software. The numerical values of velocity, induced magnetic field and fluid temperature are displayed graphically versus channel width variable η for various values of pertinent flow parameters whereas numerical values of shear stress at the lower and upper plates due to primary and secondary flows, mass flow rates in the primary and secondary flow directions and rate of heat transfer at the lower and upper plates are presented in tabular form for various values of pertinent flow parameters.

Keywords: Hall current, magnetic field, Coriolis force, modified Ekman boundary layer, modified Hartmann boundary layer.
Mathematics Subject Classification: 76U05, 76W05.

*Paper presented in CONIAPS XIII at UPES, Dehradun during June 14-16, 2011.

1. Introduction

Theoretical/experimental investigation of hydromagnetic flow in a rotating environment has received significant attention of several researchers due to occurrence of various natural phenomena, which are directly governed by the action of Coriolis force, and its application in various technological situations. An order of magnitude analysis shows that in the basic field equations the effect of Coriolis force is more significant as compared to that of inertial and viscous forces. Furthermore, it is worthy to note that Coriolis and magnetohydrodynamic forces are comparable in magnitude and Coriolis force induces secondary flow in the fluid. Keeping in view the importance of such investigation, Jana et al[1], Seth and Maiti[2], Seth et al[3-6], Chandran et al[7], Singh et al[8], Singh[9], Das et al[10] and Seth and Singh[11] studied MHD Couette flow of a viscous, incompressible and electrically conducting fluid in a rotating system considering different aspects of the problem. Taking into account the above investigations made on MHD Couette flow in a rotating system, we are of opinion that MHD Couette flow may be induced in two ways and it may be recognized as (i) MHD Couette flow of class-I and (ii) MHD Couette flow of class-II. The fluid flow induced due to movement of a plate, when the fluid is bounded by a stationary plate placed at a finite distance from the moving plate, may be regarded as MHD Couette flow of class-I. This fluid flow is similar to the flow generated due to movement of a plate when the free stream is stationary. The fluid flow past a stationary plate, which is induced due to movement of a plate placed at a finite distance from the stationary plate, may be recognized as MHD Couette flow of class-II. This fluid flow is similar to the flow past a stationary plate due to moving free stream. Investigations carried out by Jana et al[1], Seth and Maiti[2], Seth et al[3-5], Chandran et al[7], Singh et al[8] and Das et al[10] belong to MHD Couette flow of class-I whereas research studies made by Seth et al[6], Singh[9] and Seth and Singh[11] belong to MHD Couette flow of class-II. Unfortunately, the results of above research studies cannot be applied to the flow of an ionized gas. This is due to the fact that in an ionized gas where density is low and/or the applied magnetic field is strong, the effects of Hall current become significant. It may also be noted that Hall current induces secondary flow similar to the flow induced by Coriolis force. Keeping in view these facts Jana and Dutta[12], Seth and Ahmad[13] and Jha and Apere[14] discussed MHD Couette flow of class-I in a rotating system taking Hall current into account.

The aim of the present paper is to study steady MHD Couette flow of class-II of a viscous, incompressible and electrically conducting fluid in a rotating system in the presence of a uniform transverse magnetic field applied parallel to the axis of rotation taking Hall current into account.

2. Formulation of the Problem and its Solution

Consider steady Couette flow of a viscous, incompressible and electrically conducting fluid between two parallel plates $z=0$ and $z=L$ in the presence of a uniform transverse magnetic field B_0 applied in a direction parallel to z-axis. Fluid and channel are in a state of rigid body rotation with uniform angular velocity Ω about z-axis. The flow within the channel is induced due to movement of upper plate $z=L$ in x-direction with uniform velocity U_0 whereas lower plate $z=0$ is kept fixed. Since plates of the channel are of infinite extent in x and y directions and fluid flow is steady so all physical quantities, except pressure, depend on z only. The fluid velocity $\vec{q}$ and magnetic induction vector $\vec{B}$ are assumed in the following form

$$\vec{q} \equiv (u_x, u_y, 0)\,;\ \vec{B} \equiv (B_x', B_y', B_0)\,, \tag{2.1}$$

which are compatible with fundamental equations of Magnetohydrodynamics in a rotating frame of reference. Under the above assumptions the equations of motion and induction equation for magnetic field in rotating frame of reference become

$$-2\Omega u_y = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{d^2 u_x}{dz^2} + \frac{B_0}{\rho\mu_e}\frac{dB_x'}{dz}\,, \tag{2.2}$$

$$2\Omega u_x = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\frac{d^2 u_y}{dz^2} + \frac{B_0}{\rho\mu_e}\frac{dB_y'}{dz}\,, \tag{2.3}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z}\,, \tag{2.4}$$

(2.5) $$0 = B_0 \frac{du_x}{dz} + \upsilon_m \frac{d^2 B_x'}{dz^2} + m\upsilon_m \frac{d^2 B_y'}{dz^2},$$

(2.6) $$0 = B_0 \frac{du_y}{dz} + \upsilon_m \frac{d^2 B_y'}{dz^2} - m\upsilon_m \frac{d^2 B_x'}{dz^2},$$

where $p = (p' + B^2 / 2\mu_e)$, σ, μ_e, ρ, υ, $\upsilon_m = 1/\sigma\mu_e$, $m = \omega_e\tau_e$, ω_e and τ_e are modified pressure including fluid pressure, centrifugal force and magneto-hydrodynamic pressure, electrical conductivity, magnetic permeability, density, kinematic coefficient of viscosity, magnetic viscosity, Hall current parameter, cyclotron frequency and electron collision time respectively.

Boundary conditions for fluid velocity are no-slip conditions. Upper plate of the channel is perfectly conducting and is moving with uniform velocity U_0 in *x*-direction while lower plate is kept fixed and is non-conducting. Thus the boundary conditions for fluid velocity and induced magnetic field are given by

(2.7) $$u_x = 0,\ u_y = 0 \text{ at } z = 0\ ;\ \ B_x' = 0,\ B_y' = 0 \text{ at } z = 0,$$

(2.8) $$u_x = U_0,\ u_y = 0 \text{ at } z = L\ ;\ \ \frac{dB_x'}{dz} = 0,\ \frac{dB_y'}{dz} = 0 \text{ at } z = L.$$

Equation (2.4) shows the constancy of modified pressure along *z*-axis. For MHD Couette flow of class-I the pressure gradient terms $-\frac{1}{\rho}\frac{\partial p}{\partial x}$ and $-\frac{1}{\rho}\frac{\partial p}{\partial y}$, which are present in equations (2.2) and (2.3) respectively, are not considered by researchers[1-5,7,8,10]. This assumption is valid and it is clearly evident from conditions (2.7). For MHD Couette flow of class-II, values of the pressure gradient terms in equations (2.2) and (2.3) are obtained with the help of boundary conditions (2.8) which are given by

(2.9) $$-\frac{1}{\rho}\frac{\partial p}{\partial x}=0\ ; \quad -\frac{1}{\rho}\frac{\partial p}{\partial y}=2\Omega U_0.$$

Using (2.9) in equations (2.2) and (2.3), we obtain

(2.10) $$-2\Omega u_y=\upsilon\frac{\partial^2 u_x}{\partial z^2}+\frac{B_0}{\rho\mu_e}\frac{\partial B_x'}{\partial z},$$

(2.11) $$2\Omega(u_x-U_0)=\upsilon\frac{\partial^2 u_y}{\partial z^2}+\frac{B_0}{\rho\mu_e}\frac{\partial B_y'}{\partial z}.$$

Combining equations (2.5) and (2.10) with the equations (2.6) and (2.11) respectively and representing them in non-dimensional form, we obtain

(2.12) $$0=\frac{dP}{d\eta}+\frac{d^2Q}{d\eta^2}-mi\frac{d^2Q}{d\eta^2},$$

(2.13) $$2iK^2(P-1)=\frac{d^2P}{d\eta^2}+M^2\frac{dQ}{d\eta},$$

where

(2.14) $P=u+iv$, $Q=b_x+ib_y$, $b_x=B_x/R_m$, $b_y=B_y/R_m$, $\eta=z/L$, $u=u_x/U_0$, $v=u_y/U_0$, $B_x=B_x'/B_0$ and $B_y=B_y'/B_0$.

In equations (2.12) and (2.13), $R_m=\sigma\mu_e U_0 L$ is magnetic Reynolds number, $K^2=\Omega L^2/\upsilon$ is rotation parameter which is reciprocal of Ekman number and $M^2=\sigma B_0^2L^2/\rho\upsilon$ is magnetic parameter which is square of Hartmann number.

Boundary conditions (2.7) and (2.8), in non-dimensional form, become

(2.15) $$P=0,\ Q=0\ \text{at}\ \eta=0\ ;\quad P=1,\ \frac{dQ}{d\eta}=0\ \text{at}\ \eta=1.$$

Solution of equations (2.12) and (2.13) subject to the boundary conditions (2.15) is given by

(2.16) $$P(\eta)=\frac{\cosh\lambda}{\sinh\lambda}\sinh\lambda\eta+(1-\cosh\lambda\eta),$$

(2.17) $$Q(\eta)=\frac{1}{M^2}\left(\frac{2iK^2-\lambda^2}{\lambda}\right)\left[\frac{\cosh\lambda}{\sinh\lambda}(\cosh\lambda\eta-1)-\sinh\lambda\eta\right],$$

where

(2.18a) $\lambda=\alpha+i\beta$, and

(2.18b) $$\alpha,\beta=\frac{1}{\sqrt{2\left(1+m^2\right)}}\left[\left\{M^4+\left(2K^2\left(1+m^2\right)+mM^2\right)^2\right\}^{\frac{1}{2}}\pm M^2\right]^{1/2}.$$

Shear Stress at the Plates

Non-dimensional shear stress components τ_x and τ_y, due to primary and secondary flow respectively, at stationary and moving plates of the channel are given by

(2.19) $$\left(\tau_x+i\tau_y\right)\Big|_{\eta=0}=\lambda\frac{\cosh\lambda}{\sinh\lambda}\ ;\qquad \left(\tau_x+i\tau_y\right)\Big|_{\eta=1}=\frac{\lambda}{\sinh\lambda}\ .$$

Mass flow rates

Non-dimensional mass flow rates Q_x and Q_y in the primary and secondary flow direction respectively are given by

(2.20) $$Q_x+iQ_y=\left[\frac{1-\cosh\lambda}{\lambda\sinh\lambda}+1\right].$$

3. Asymptotic Solutions

We shall now analyze the asymptotic behavior of the solution for large values of K^2 and M^2 to gain some physical insight into the flow pattern.

Case-I: When $K^2>>1$ *and* $M^2\sim O(1)$

When K^2 is large, boundary layer type flow is expected near the plates of channel. For the boundary layer flow near stationary plate $\eta=0$, the expressions for fluid velocity and induced magnetic field are obtained from equations (2.16) to (2.18) and are presented in the following form

(3.1) $$u(\eta)=1-e^{-\alpha_1\eta}\cos\beta_1\eta\ ;\qquad v(\eta)=e^{-\alpha_1\eta}\sin\beta_1\eta\ ,$$

$$(3.2)\qquad b_x = \frac{1}{2K(1+m^2)}\left[(1+m)(1-e^{-\alpha_1\eta}\cos\beta_1\eta)+(1-m)(e^{-\alpha_1\eta}\sin\beta_1\eta)\right],$$

$$(3.3)\qquad b_y = \frac{-1}{2K(1+m^2)}\left[(1-m)(1-e^{-\alpha_1\eta}\cos\beta_1\eta)-(1+m)(e^{-\alpha_1\eta}\sin\beta_1\eta)\right],$$

where

$$(3.4)\qquad \alpha_1,\ \beta_1 = K\left\{1+\frac{M^2\left(m\pm1\right)}{4K^2\left(1+m^2\right)}\right\}.$$

It is evident from expressions (3.1) to (3.4) that there arises a thin boundary layer of thickness $O(\alpha_1)^{-1}$ near stationary plate of the channel. This boundary layer may be recognized as modified Ekman boundary layer and may be viewed as classical Ekman boundary layer modified by Hall current and magnetic field. The thickness of this boundary layer decreases with the increase in either M^2 or K^2. Similar type of boundary layer also appears near moving plate of the channel. Exponential terms in the expressions (3.1) to (3.3) damp out quickly as η increases. When $\eta \ge 1/\alpha_1$ i.e. outside the boundary layer region, (3.1) to (3.3) assume the form

$$(3.5)\qquad u_1 \approx 1,\ v_1 \approx 0;\quad b_x \approx \frac{(1+m)}{2K(1+m^2)},\ b_y \approx \frac{-(1-m)}{2K(1+m^2)}.$$

It is revealed from the expressions in (3.5) that, in a region outside the boundary layer region, i.e. in the central core, fluid flows in primary flow direction only. This is due to the reason that fluid flow within the channel is induced due to the movement of upper plate of the channel. The primary and secondary induced magnetic fields b_x and b_y persist. These magnetic fields have considerable effects of Hall current and rotation and are unaffected by applied magnetic field.

Case-II: When $M^2 >> 1$ *and* $K^2 \sim O(1)$

This case also corresponds to boundary layer type flow. For the boundary layer flow near stationary plate $\eta = 0$, the expressions for fluid velocity and induced magnetic field are obtained from equations (2.16) to (2.18) and are given by

(3.6) $$u(\eta)=1-e^{-\alpha_2\eta}\cos\beta_2\eta\ ; \qquad v(\eta)=e^{-\alpha_2\eta}\sin\beta_2\eta,$$

(3.7) $$b_x=\frac{1}{M^2\sqrt{1+m^2}}\left[a(1-e^{-\alpha_2\eta}\cos\beta_2\eta)-be^{-\alpha_2\eta}\sin\beta_2\eta\right],$$

(3.8) $$b_y=\frac{1}{M^2\sqrt{1+m^2}}\left[b(1-e^{-\alpha_2\eta}\cos\beta_2\eta)+ae^{-\alpha_2\eta}\sin\beta_2\eta\right],$$

where

(3.9a) $$\alpha_2=\frac{1}{M\sqrt{(1+m^2)}}\left\{a(M^2+mK^2)-bK^2\right\},$$

(3.9b) $$\beta_2=\frac{1}{M\sqrt{(1+m^2)}}\left\{b(M^2+mK^2)+aK^2\right\},$$

(3.9c) $$a,\, b=\frac{1}{\sqrt{2}}\left\{\sqrt{1+m^2}\pm1\right\}^{1/2}.$$

The expressions (3.6) to (3.9) reveal that there appears a thin boundary layer of thickness $O(\alpha_2)^{-1}$ near stationary plate of the channel. This boundary layer may be identified as modified Hartmann boundary layer and may be viewed as classical Hartmann boundary layer modified by Hall current and rotation. The thickness of this boundary layer decreases with the increase in either M^2 or K^2. In the absence of Hall current (i.e. $m=0$) there appears pure Hartmann boundary layer near stationary plate. Similar type of boundary layer also appears near moving plate. When $\eta\geq 1/\alpha_2$ i.e. outside the boundary layer region, (3.6) to (3.8) take the form

(3.10) $$u_1\approx 1,\ v_1\approx 0\ ; \quad b_x\approx\frac{a}{M\sqrt{(1+m^2)}},\ b_y\approx\frac{b}{M\sqrt{(1+m^2)}}\ .$$

It is evident from the expressions in (3.10) that, in a region outside boundary layer region, i.e. in the central core, fluid flows in primary flow direction only while both the primary and secondary induced magnetic fields b_x and b_y persist. These induced magnetic fields have considerable effects of Hall current and magnetic field and are unaffected by rotation.

4. Heat transfer characteristics

We shall now discuss heat transfer characteristics of the fluid flow, when moving and stationary plates of the channel are maintained at uniform temperature T_1 and T_0 respectively, where $T_0 < T' < T_1$, T' being the fluid temperature. Energy equation taking viscous and Joule dissipations into account is given by

$$(4.1)\quad 0=\alpha^* \frac{d^2T'}{dz^2}+\frac{\upsilon}{C_p}\left[\left(\frac{du_x}{dz}\right)^2+\left(\frac{du_y}{dz}\right)^2\right]+\frac{1}{\mu_e{}^2\sigma\rho C_p}\left[\left(\frac{dB_x{}'}{dz}\right)^2+\left(\frac{dB_y{}'}{dz}\right)^2\right],$$

where $\alpha^* = k/\rho C_p$ is thermal diffusivity of fluid. k and C_p are, respectively, thermal conductivity of fluid and specific heat at constant pressure.
The boundary conditions for temperature field are

$$(4.2)\quad T' = T_0 \text{ at } z = 0 \qquad \text{and} \quad T' = T_1 \text{ at } z = L.$$

Representing equation (4.1), in non-dimensional form with the help of (2.14), we obtain

$$(4.3)\quad 0=\frac{d^2T}{d\eta^2}+P_rE_c\left[\frac{dP}{d\eta}.\frac{d\overline{P}}{d\eta}\right]+M^2\left[\frac{dQ}{d\eta}.\frac{d\overline{Q}}{d\eta}\right],$$

where $T=(T'-T_0)/(T_1-T_0)$, $P_r=\upsilon/\alpha^*$ and $E_r=U_0{}^2/C_p(T_1-T_0)$ are non-dimensional fluid temperature, Prandtl number and Eckert number respectively. $\overline{P}$ and $\overline{Q}$ are complex conjugate of P and Q respectively.
The boundary conditions (4.2), in non-dimensional form, become

$$(4.4)\qquad T(0)=0 \qquad \text{and} \qquad T(1)=1.$$

Making use of (2.16) to (2.18) in equation (4.3) and solving the resulting equation subject to the boundary conditions (4.4) by MATLAB software we have obtained numerical solution for fluid temperature T and numerical values of rate of heat transfer at the stationary and moving plates of the channel for various values of m, K^2, P_r and E_c taking $M^2 = 10$.

5. Results and Discussion

To study the effects of Hall current and rotation on the fluid velocity and induced magnetic field, the numerical values of the fluid velocity and induced magnetic field are depicted graphically versus channel width variable η in figures 1 to 4 for various values of Hall current parameter m and rotation parameter K^2 taking $M^2 = 10$. It is evident from figures 1 and 2 that primary velocity u decreases whereas secondary velocity v increases on increasing m. On increasing K^2, primary velocity u increases throughout the channel whereas secondary velocity v increases in the lower half of the channel.

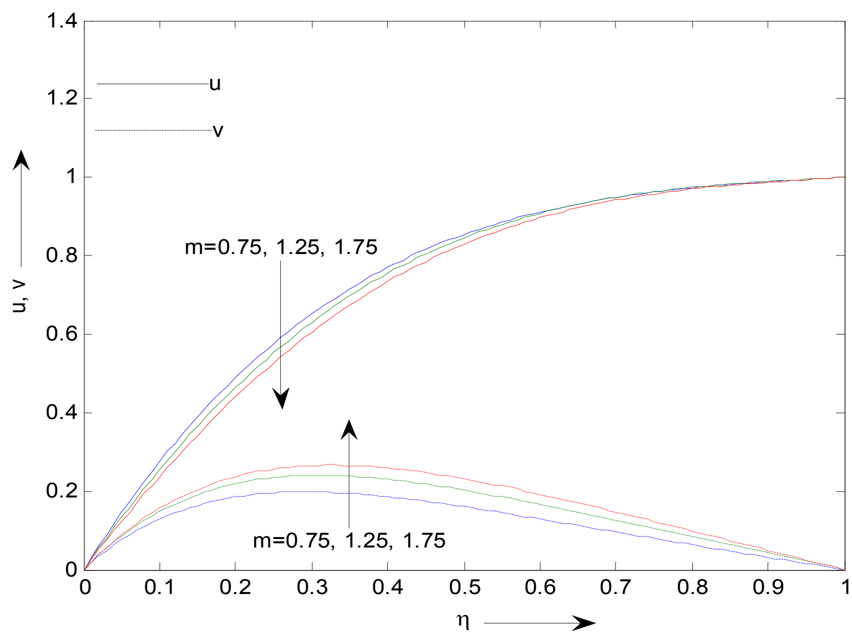

Fig. 1. Velocity profiles when $K^2 = 3$

This implies that Hall current retards fluid flow in primary flow direction whereas it has reverse effect on the fluid flow in secondary flow direction. Rotation has tendency to accelerate fluid flow in primary flow direction throughout the channel whereas it tends to accelerate fluid flow in secondary flow direction in the lower half of the channel.

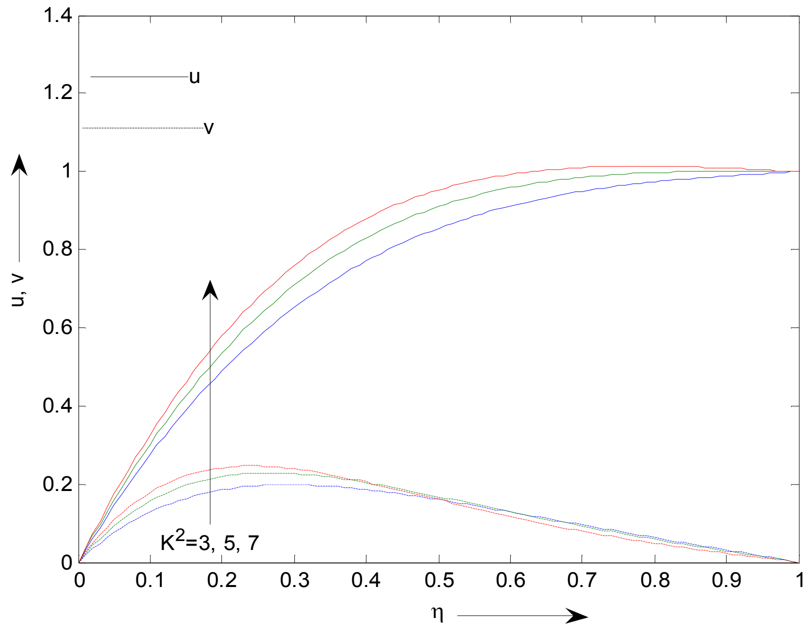

Fig. 2. Velocity profiles when $m = 0.75$

It is noticed from figures 3 and 4 that primary induced magnetic field b_x decreases whereas secondary induced magnetic field b_y increases on increasing m . b_x and b_y increase on increasing K^2.

This implies that Hall current tends to reduce primary induced magnetic field whereas it has reverse effect on the secondary induced magnetic field. Rotation tends to enhance both the primary and secondary induced magnetic fields.

The numerical values of fluid temperature, computed from energy equation (4.3) with the help of MATLAB software, are displayed graphically versus channel width variable η in figures 5 to 8 for various values of m, K^2, P_r and E_c taking $M^2 = 10$. Figures 5 to 8 reveal that fluid temperature T decreases on increasing m whereas it increases on increasing either K^2 or P_r or E_c.

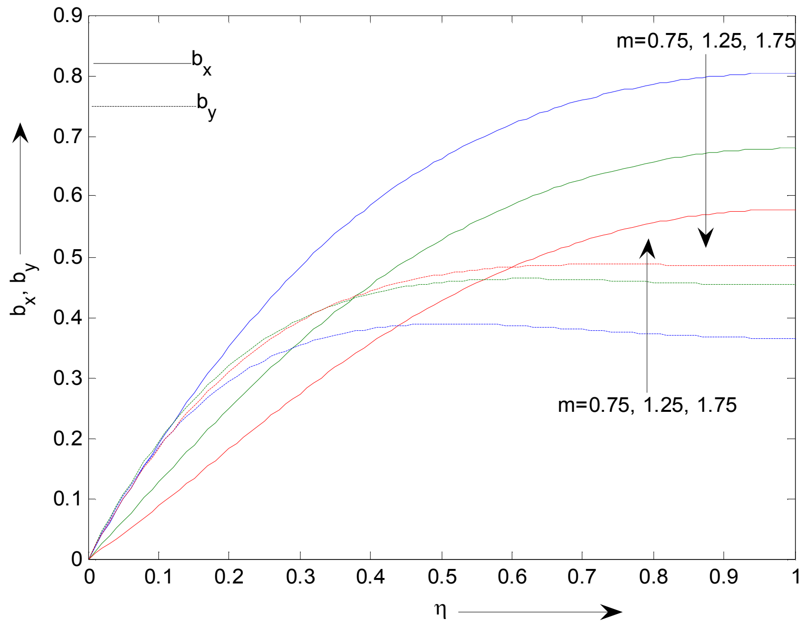

Fig. 3. Induced magnetic field profiles when $K^2 = 3$

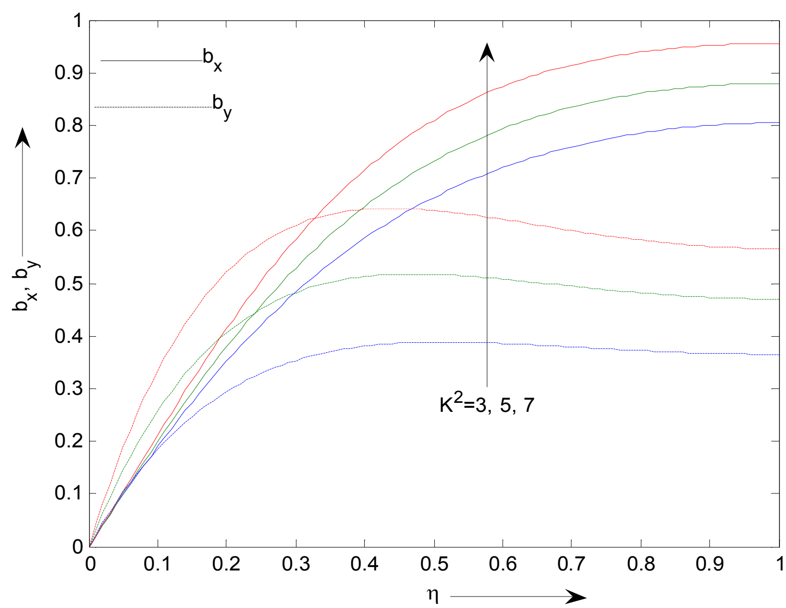

Fig. 4. Induced magnetic field profiles when $m = 0.75$

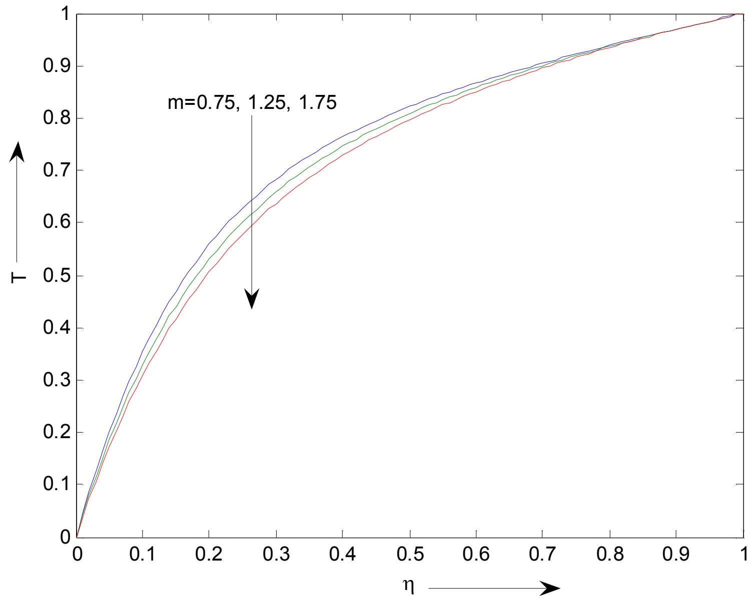

Fig. 5. Temperature profiles when $K^2 = 3, P_r = 0.71$ and $E_c = 2$

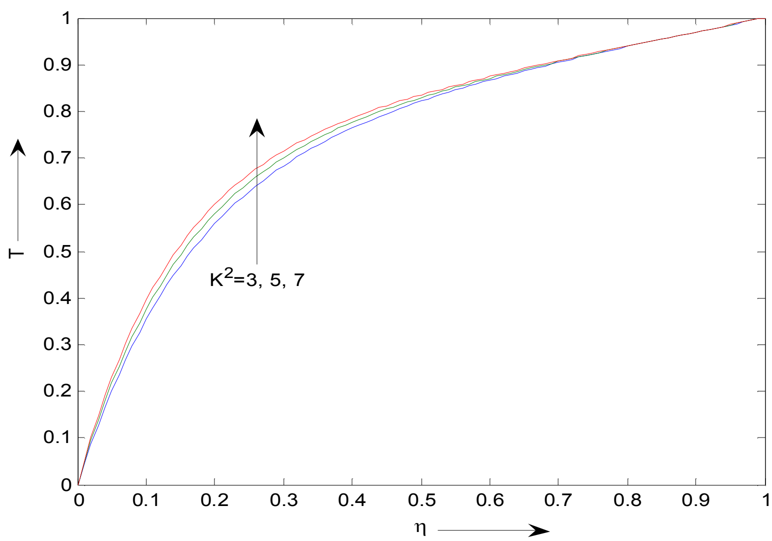

Fig. 6. Temperature profiles when $m = 0.75, P_r = 0.71$ and $E_c = 2$

This implies that Hall current tends to reduce fluid temperature whereas rotation has reverse effect on it.

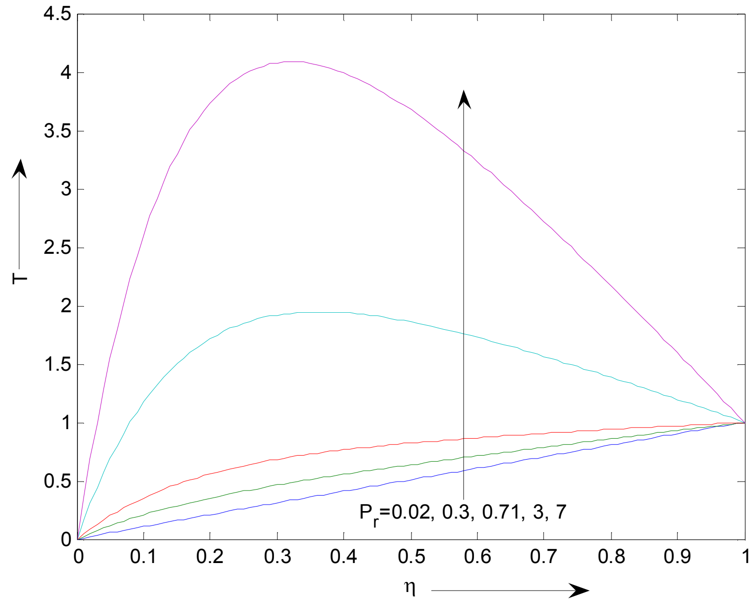

Fig. 7. Temperature profiles when $K^2 = 3, m = 0.75$ and $E_c = 2$

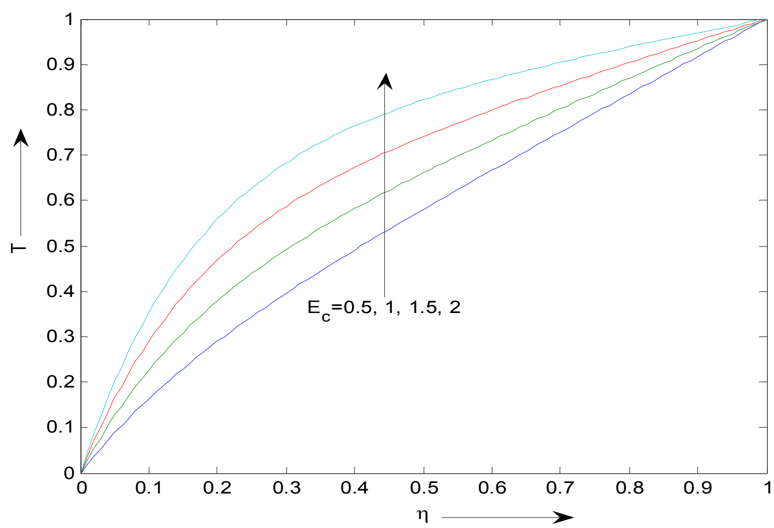

Fig. 8. Temperature profiles when $K^2 = 3, m = 0.75$ and $P_r = 0.71$

It may be noted that E_c and P_r represent the effects of viscous dissipation and thermal diffusion respectively. P_r decreases when thermal diffusivity of the fluid increases. Thus we conclude that viscous dissipation tends to enhance fluid temperature whereas thermal diffusion has reverse effect on it.

The numerical values of the shear stress at both the plates due to primary and secondary flows are presented in tabular form in tables 1 and 2 whereas that of mass flow rates in the primary and secondary flow directions are displayed in table 3 for various values of m and K^2 taking $M^2 = 10$. It is noticed from table 1 that primary shear stress at the stationary plate i.e. $\tau_x|_{\eta=0}$ decreases whereas secondary shear stress at the stationary plate i.e. $\tau_y|_{\eta=0}$ increases on increasing m. $\tau_x|_{\eta=0}$ and $\tau_y|_{\eta=0}$ increase on increasing K^2.

Table 1: Shear stress at the stationary plate

$K^2\downarrow$ $m\rightarrow$	$\tau_x\|_{\eta=0}$			$\tau_y\|_{\eta=0}$		
	0.75	1.25	1.75	0.75	1.25	1.75
3	3.0637	2.7545	2.5171	1.7519	1.9604	2.0254
5	3.3484	3.0965	2.9028	2.2112	2.4089	2.4749
7	3.6226	3.4058	3.2383	2.6005	2.7822	2.8424

This implies that Hall current tends to reduce primary shear stress at the stationary plate whereas it has reverse effect on the secondary shear stress at the stationary plate. Rotation tends to enhance both primary and secondary shear stress at the stationary plate. It is found from table 2 that, on increasing m, primary shear stress at the moving plate i.e. $\tau_x|_{\eta=1}$ increases when $K^2 \geq 5$ and secondary shear stress at the moving plate i.e. $\tau_y|_{\eta=1}$ increases for every values of K^2 considered. $\tau_x|_{\eta=1}$ increases whereas $\tau_y|_{\eta=1}$ decreases on increasing K^2.

Table 2: Shear stress at the moving plate

$K^2\downarrow$ $m\rightarrow$	$-\tau_x\|_{\eta=1}$			$-\tau_y\|_{\eta=1}$		
	0.75	1.25	1.75	0.75	1.25	1.75
3	-0.1071	-0.0960	-0.1132	0.3074	0.4094	0.4909
5	0.0145	0.0580	0.0746	0.2796	0.3468	0.4070
7	0.0930	0.1445	0.1739	0.2190	0.2538	0.2906

This implies that Hall current tends to enhance secondary shear stress at the moving plate and it tends to enhance primary shear stress at the moving plate when $K^2 \geq 5$. Rotation tends to enhance primary shear stress at the moving plate whereas it has reverse effect on secondary shear stress at the moving plate. It is noticed from table 2 that there exists flow separation at the moving plate in the primary flow direction on increasing K^2.

It is observed from table 3 that primary mass flow rate Q_x decreases whereas secondary mass flow rate Q_y increases on increasing m. Q_x and Q_y increase on increasing K^2.

Table 3: Mass flow rates

$K^2\downarrow$ $m\rightarrow$	Q_x			Q_y		
	0.75	1.25	1.75	0.75	1.25	1.75
3	0.7388	0.7293	0.7164	0.1190	0.1473	0.1655
5	0.7754	0.7747	0.7696	0.1301	0.1529	0.1685
7	0.8053	0.8085	0.8073	0.1314	0.1485	0.1605

This implies that Hall current tends to reduce primary mass flow rate whereas it has reverse effect on secondary mass flow rate. Rotation tends to enhance both the primary and secondary mass flow rates.

The numerical values of rate of heat transfer at the stationary and moving plates, computed directly from the energy equation (4.3) with the help of MATLAB software, are presented in tabular form in tables 4 and 5 for various values of m, K^2, P_r and E_c taking $M^2 = 10$. It is found from table 4 that rate of heat transfer at the lower plate i.e. $\left(\frac{d\theta}{d\eta}\right)_{\eta=0}$ decreases on increasing m whereas it increases on increasing K^2. This implies that Hall current tends to reduce rate of heat transfer at the stationary plate whereas rotation has reverse effect on it. It is interesting to note from table 4 that the numerical values of rate of heat transfer at the moving plate i.e. $\left(\frac{d\theta}{d\eta}\right)_{\eta=1}$ do not vary on increasing either m or K^2 which implies that rate of heat transfer at the moving plate is unaffected by Hall current and rotation. It is found from table 5 that $\left(\frac{d\theta}{d\eta}\right)_{\eta=0}$ increases on increasing either P_r or E_c.

$\left(\dfrac{d\theta}{d\eta}\right)_{\eta=1}$ decreases on increasing either P_r or E_c when $P_r \le 1$ whereas it increases in magnitude with the increase in either P_r or E_c when $P_r > 1$.

Table 4: Rate of heat transfer at stationary and moving plates when $P_r = 0.71$ and $E_c = 2$

$K^2 \downarrow$ $m \rightarrow$	$\left(d\theta/d\eta\right)_{\eta=0}$			$\left(d\theta/d\eta\right)_{\eta=1}$		
	0.75	1.25	1.75	0.75	1.25	1.75
3	4.6404	4.2013	3.8643	0.2900	0.2900	0.2900
5	5.0447	4.6870	4.4120	0.2900	0.2900	0.2900
7	5.4341	5.1263	4.8883	0.2900	0.2900	0.2900

Table 5: Rate of heat transfer at stationary and moving plates when $m = 0.75$ and $K^2 = 3$

$P_r \downarrow$ $E_r \rightarrow$	$\left(d\theta/d\eta\right)_{\eta=0}$			$\left(d\theta/d\eta\right)_{\eta=1}$		
	1	1.5	2	1	1.5	2
0.02	1.0513	1.0769	1.1025	0.9900	0.9850	0.9800
0.05	1.1282	1.1923	1.2564	0.9750	0.9625	0.9500
.3	1.7691	2.1536	2.5382	0.8500	0.7750	0.7000
.71	2.8202	3.7303	4.6404	0.6450	0.4675	0.2900
1	3.5637	4.8455	6.1273	0.5000	0.2500	0
3	8.6910	12.5364	16.3819	-0.5000	-1.2500	-2
7	18.9456	27.9184	36.8911	-2.5000	-4.2500	-6

This implies that viscous dissipation tends to enhance rate of heat transfer at the stationary plate whereas thermal diffusion has reverse effect on it. Viscous dissipation tends to reduce rate of heat transfer at the moving plate and thermal diffusion has reverse effect on it when $P_r \le 1$ and viscous dissipation tends to enhance rate of heat transfer at the moving plate whereas thermal diffusion has reverse effect on it when $P_r > 1$. It is worthy to note that there exists flow reversal of heat near the moving plate due to thermal diffusion. Also the value of $\left(\dfrac{d\theta}{d\eta}\right)_{\eta=1}$ is zero when $E_c = 2$ and $P_r = 1$ i.e. there is no flow of heat either from moving plate to the fluid or from fluid to the moving plate when $E_c = 2$ and $P_r = 1$. This situation arises when the thicknesses of viscous and thermal boundary layers are of same order of magnitude. $E_c = 2$ for $P_r = 1$ is called critical Eckert number corresponding to the moving plate.

Acknowledgement: Authors are grateful to University Grant Commission, New Delhi for providing financial assistance to carry out this research.

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