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On a Generalized H-Recurrent Finsler Space

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Abstract: In the present paper, a Finsler space whose curvature tensor H^i_{jkh} satisfies $\mathscr{B}_m H^i_{jkh} = \lambda_m H^i_{jkh} + \mu_m \left(\delta^i_j g_{kh} - \delta^i_k g_{jh}\right)$, where $\mathscr{B}_m$ is Berwald covariant differential operator, λ_m and μ_m are non-null covariant vectors, is introduced and such space is called as a generalized H-recurrent Finsler space. The Ricci tensor H_{kh}, the vector H_k and the scalar curvature H of such space are non-vanishing. Under certain conditions a generalized H-recurrent Finsler space becomes a Landsberg space. Some conditions have been pointed out which reduce a generalized H-recurrent Finsler space $F^n (n > 2)$ into a Riemannian space of constant Riemannian curvature. If the covariant vector λ_m is independent of $\dot{x}^i$ and the dimension of the space is greater than two, the space is necessarily Riemannian.

Keywords: Finsler space, Generalized H-recurrent Finsler space, Ricci tensor, Landsberg space, Riemannian space of constant Riemannian curvature

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1. Introduction

A 3-dimensional Riemannian space of recurrent curvature was introduced and studied by H. S. Ruse[1]. This study was extended to an n-dimensional Riemannian space by A. G. Walker[2]. Several significant contributions towards such spaces were made by a large number of geometers including E. M. Patterson[3], Y. C. Wong[4], Y. C. Wong and K. Yano[5]. Such theory was extended to Finsler spaces by A. Móor[6,7], R. S.

Mishra and H. D. Pande[8], R. N. Sen[9], R. B. Misra[10], P. N. Pandey[11-18] and many others. U. C. De and N. Guha[19] introduced a generalized recurrent Riemannian manifold. U. C. De and D. Kamilya[20], Y. B. Maralabhavi and M. Rathnamma[21] also contributed towards a generalized recurrent and generalized concircular recurrent Riemannian manifolds. The aim of the present paper is to introduce and study a generalized H-recurrent Finsler space.

Let F^n be an n-dimensional Finsler space equipped with the metric function F satisfying the requisite conditions[22]. Let the components of the corresponding metric tensor and the connection coefficients of Berwald be denoted by g_{ij} and G^i_{jk} respectively. These are positively homogeneous of degree zero in $\dot{x}^i$. Due to their homogeneity in $\dot{x}^i$, we have

(1.1) (a) $C_{jkh}\dot{x}^h = 0$ and (b) $G^i_{jkh}\dot{x}^h = 0$,

where $C_{jkh} = \dot{\partial}_h g_{jk}$, $G^i_{jkh} = \dot{\partial}_h G^i_{jk}$ and $\dot{\partial}_h \equiv \frac{\partial}{\partial \dot{x}^h}$. C_{jkh} and G^i_{jkh} are components of tensors and are symmetric in their lower indices. The Berwald covariant derivative of an arbitrary tensor T^i_j with respect to x^k is given by

$$\mathscr{B}_k T^i_j = \partial_k T^i_j - (\dot{\partial}_r T^i_j) G^r_k + T^r_j G^i_{rk} - T^i_r G^r_{jk}, \tag{1.2}$$

where $G^r_k = G^r_{sk}\dot{x}^s$.

The Berwald covariant derivative gives rise to the commutation formula

$$\mathscr{B}_j \mathscr{B}_k T^i_h - \mathscr{B}_k \mathscr{B}_j T^i_h = T^r_h H^i_{jkr} - T^i_r H^r_{jkh} - (\dot{\partial}_r T^i_h) H^r_{jk}, \tag{1.3}$$

where H^i_{jkh} defined by[1]

$$H^i_{jkh} = 2\left\{\partial_{[j} G^i_{k]h} + G^i_{rh[j} G^r_{k]} + G^i_{r[j} G^r_{k]h}\right\}, \tag{1.4}$$

are components of Berwald curvature tensor and

(1.5) (a) $H^i_{jk} = H^i_{jkh}\dot{x}^h$.

In (1.4), the square brackets denote the skew-symmetric part of the tensor with respect to the indices enclosed therein and $\partial_j \equiv \frac{\partial}{\partial x^j}$. It is clear from the

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[1] In Rund's book, H^i_{jkh} defined here, is denoted by H^i_{hkj}. This difference must be noted.

definition that the Berwald curvature tensor H^i_{jkh} is skew-symmetric in its first two lower indices and positively homogeneous of degree zero in $\dot{x}^i$. Berwald deviation tensor H^i_j is defind as

$$(1.5) \quad (b) \quad H^i_j = H^i_{jk}\dot{x}^k.$$

From the contraction of the indices i and j in H^i_{jkh}, H^i_{jk} and H^i_j, we have

$$(1.5) \quad (c)\ H_{kh} = H^i_{ikh},\ (d)\ H_k = H^i_{ik},\ (e)\ H_k\dot{x}^k = (n-1)H.$$

The Berwald curvature tensor H^i_{jkh} satisfies the following Bianchi identities

$$(1.6) \quad \mathscr{B}_m H^i_{jkh} + \mathscr{B}_k H^i_{mjh} + \mathscr{B}_j H^i_{kmh} + H^r_{jk}G^i_{hmr} + H^r_{kh}G^i_{jmr} + H^r_{hj}G^i_{kmr} = 0.$$

The commutation formulae for the operators $\dot{\partial}_j$ and $\mathscr{B}_k$ are given by

$$(1.7) \qquad \dot{\partial}_j \mathscr{B}_k T^i_h - \mathscr{B}_k \dot{\partial}_j T^i_h = T^r_h G^i_{jkr} - T^i_r G^r_{jkh}.$$

A Finsler space F^n is called recurrent if its curvature tensor H^i_{jkh} satisfies

$$(1.8) \qquad \mathscr{B}_m H^i_{jkh} = \lambda_m H^i_{jkh},$$

where λ_m is a non-null covariant vector[10]. The vector λ_m appearing in (1.8) is called the recurrence vector. P. N. Pandey[11] proved that the recurrence vector λ_m is independent of $\dot{x}^i$'s and showed that the Bianchi identities (1.6) for a recurrent Finsler space split into the following identities

$$(1.9) \quad \text{(a)} \quad \lambda_m H^i_{jkh} + \lambda_k H^i_{mjh} + \lambda_j H^i_{kmh} = 0,$$

$$\text{(b)} \quad H^r_{jk}G^i_{hmr} + H^i_{rk}G^r_{hmj} + H^i_{jr}G^r_{hmk} = 0.$$

The Riemannian curvature R of a Finsler Space F^n at a point x^i with respect to 2-directions $\left(\dot{x}^i, X^i\right)$ is defined as[22]

$$(1.10) \qquad R = \frac{K_{ijhk}(x^i,\dot{x}^i)\dot{x}^i\dot{x}^h X^j X^k}{(g_{ih}g_{jk} - g_{ij}g_{hk})\dot{x}^i\dot{x}^h X^j X^k},$$

where K^i_{jkh} are components of Cartan curvature tensor and $K_{ijhk} = g_{rj}K^r_{ihk}$.

A point x^i of a Finsler space F^n is said to be an isotropic point if the Riemannian curvature at x^i is independent of the choice of the direction X^i.

A Finsler space F^n is called isotropic Finsler space or a Finsler space of scalar curvature if every point of F^n is isotropic. A two dimensional Finsler space is necessarily isotropic.

The necessary and sufficient condition for a Finsler space $F^n (n > 2)$ to be a Finsler space of scalar curvature is given by

(1.11) $$H^i_h = F^2 R\left(\delta^i_h - l^i l_h\right).$$

If the Riemannian curvature R is constant, the space is said to be a space of constant curvature. The necessary and sufficient condition for a Finsler space $F^n (n > 2)$ to be of constant curvature is given by

(1.12) $$H_{likj} = R\left(g_{ij} g_{lk} - g_{ik} g_{lj}\right).$$

A Finsler space F^n is said to be a Landsberg space if it satisfies

(1.13) $$y_i G^i_{jkh} = 0,$$

where $y_i = g_{ij}\dot{x}^j$.

2. Generalized H-Recurrent Finsler Space

Let us consider a Finsler space F^n whose Berwald curvature tensor H^i_{jkh} satisfies

(2.1) $$\mathscr{B}_m H^i_{jkh} = \lambda_m H^i_{jkh} + \mu_m\left(\delta^i_j g_{kh} - \delta^i_k g_{jh}\right),$$

where λ_m and μ_m are non-null covariant vector fields. We shall call such Finsler space as a generalized H-recurrent Finsler space.

Let us consider a generalized H-recurrent Finsler space characterized by (2.1). Transvecting (2.1) by $\dot{x}^h$, we get

(2.2) $$\mathscr{B}_m H^i_{jk} = \lambda_m H^i_{jk} + \mu_m\left(\delta^i_j y_k - \delta^i_k y_j\right).$$

Further transvecting (2.2) by $\dot{x}^k$, we have

(2.3) $$\mathscr{B}_m H^i_j = \lambda_m H^i_j + \mu_m\left(\delta^i_j F^2 - \dot{x}^i y_j\right).$$

Contracting the indices i and j in (2.1), (2.2) and (2.3) and using (1.5 c), we get

(2.4) $$\mathscr{B}_m H_{kh} = \lambda_m H_{kh} + (n-1)\mu_m g_{kh},$$

(2.5) $$\mathscr{B}_m H_k = \lambda_m H_k + (n-1)\mu_m y_k \quad \text{and}$$

(2.6) $$\mathscr{B}_m H = \lambda_m H + \mu_m F^2.$$

The last three equations show that the tensor H_{kh}, the vector H_k and the scalar curvature H cannot vanish because the vanishing of any one of these would imply $\mu_m = 0,$ a contradiction.

Therefore, we conclude

Theorem 2.1. *The Ricci tensor* H_{kh}, *the curvature vector* H_k *and the scalar curvature H of a generalized H-recurrent Finsler space are non-vanishing.*

Differentiating (2.5) partially with respect to $\dot{x}^h$, we get

(2.7) $$\dot{\partial}_h \mathscr{B}_m H_k = \left(\dot{\partial}_h \lambda_m\right) H_k + \lambda_m H_{kh} + (n-1)\left(\dot{\partial}_h \mu_m\right) y_k + (n-1)\mu_m g_{kh}.$$

Using the commutation formula (1.7), $\dot{\partial}_h H_k = H_{kh}$, and $\dot{\partial}_h y_k = g_{kh}$, we have

(2.8) $$\mathscr{B}_m H_{kh} - H_r G^r_{hmk} = \left(\dot{\partial}_h \lambda_m\right) H_k + \lambda_m H_{kh} + (n-1)\left(\dot{\partial}_h \mu_m\right) y_k + (n-1)\mu_m g_{kh}.$$

Using (2.4) in (2.8), we get

(2.9) $$-H_r G^r_{hmk} = \left(\dot{\partial}_h \lambda_m\right) H_k + (n-1)\left(\dot{\partial}_h \mu_m\right) y_k.$$

Transvecting (2.9) by $\dot{x}^k$ and using (1.1 b) and (1.5 e), we have

(2.10) $$0 = \left(\dot{\partial}_h \lambda_m\right) H + \left(\dot{\partial}_h \mu_m\right) F^2,$$

which implies

(2.11) $$\dot{\partial}_h \mu_m = -\frac{\left(\dot{\partial}_h \lambda_m\right)}{F^2} H.$$

If the vector λ_m is independent of $\dot{x}^i$, the equation (2.11) shows that the vector μ_m is also independent of $\dot{x}^i$. Conversly, if the vector μ_m is independent of $\dot{x}^i$, we get $H \dot{\partial}_h \lambda_m = 0.$ In view of theorem 3.1, the condition

$H\dot{\partial}_h\lambda_m = 0$ implies $\dot{\partial}_h\lambda_m = 0$, i.e. the covariant vector λ_m is also independent of $\dot{x}^i$. This leads to

Theorem 2.2. *The covariant vector* μ_m *is independent of* $\dot{x}^i$ *if and only if the covariant vector* λ_m *is independent of* $\dot{x}^i$.

Suppose the vector λ_m is not independent of $\dot{x}^i$, then (2.9) and (2.11) together imply

$$(2.12) \qquad -H_r G^r_{hmk} = \left(\dot{\partial}_h\lambda_m\right)\left(H_k - \frac{(n-1)H}{F^2}y_k\right).$$

Transvecting (2.12) by $\dot{x}^m$, we get

$$(2.13) \qquad \left(\dot{\partial}_h\lambda_m\right)\dot{x}^m\left(H_k - \frac{(n-1)H}{F^2}y_k\right) = 0,$$

which implies

$$(2.14) \qquad \left(\lambda_h - \dot{\partial}_h\lambda\right)\left(H_k - \frac{(n-1)H}{F^2}y_k\right) = 0,$$

where $\lambda = \lambda_h\dot{x}^h$.

The equation (2.14) implies at least one of the following conditions

$$(2.15) \qquad (a)\ \ \lambda_h = \dot{\partial}_h\lambda, \qquad (b)\ \ H_k = \frac{(n-1)H}{F^2}y_k.$$

Thus, we have

Theorem 2.3. *In a generalized H-recurrent Finsler space for which the covariant vector* λ_m *is not independent of* $\dot{x}^i$*, at least one of the conditions (2.15 a) and (2.15 b) holds.*

Suppose (2.15 b) holds. Then (2.12) implies

$$(2.16) \qquad \frac{(n-1)H}{F^2}y_r G^r_{hmk} = 0.$$

Since $n \neq 1$ and $H \neq 0$, we have $y_r G^r_{hmk} = 0$. Therefore the space is a Landsberg space. Thus, we have

Theorem 2.4. *A generalized H-recurrent Finsler space is a Landsberg space if condition (2.15 b) holds.*

If the covariant vector $\lambda_m \neq \dot{\partial}_m \lambda$, in view of Theorem 3.3, (2.15 b) holds good. In view of this fact, we may rewrite Theorem 3.4 in the following form

Theorem 2.5. *A generalized H-recurrent Finsler space is necessarily a Landsberg space provided* $\lambda_m \neq \dot{\partial}_m \lambda$.

Differentiating (2.2) partially with respect to $\dot{x}^h$, we get

$$\dot{\partial}_h\left(\mathscr{B}_m H^i_{jk}\right) = \left(\dot{\partial}_h \lambda_m\right) H^i_{jk} + \lambda_m H^i_{jkh} + \left(\dot{\partial}_h \mu_m\right)\left(\delta^i_j y_k - \delta^i_k y_j\right) + \mu_m\left(\delta^i_j g_{kh} - \delta^i_k g_{jh}\right).$$

Utilizing the commutation formula exhibited by (1.7) and using $\dot{\partial}_h H^i_{jk} = H^i_{jkh}$ and (2.1), we have

$$\text{(2.17)} \quad H^r_{jk} G^i_{hmr} - H^i_{rk} G^r_{jhm} - H^i_{jr} G^r_{khm} = \left(\dot{\partial}_h \lambda_m\right) H^i_{jk} + \left(\dot{\partial}_h \mu_m\right)\left(\delta^i_j y_k - \delta^i_k y_j\right).$$

Transvecting (2.17) by y_i and using the identity $y_i H^i_{jk} = 0$ established by the first author[11], we get

$$\text{(2.18)} \qquad H^r_{jk} y_i G^i_{hmr} = 0.$$

Transvection of (2.18) by $\dot{x}^k$ gives

$$\text{(2.19)} \qquad H^r_j y_i G^i_{hmr} = 0.$$

If $\det H^i_j \neq 0$, (2.19) implies $y_i G^i_{hmr} = 0$, i.e. the space is a Landsberg space. Thus, we have

Theorem 2.6. *A generalized H-recurrent Finsler space is a Landsberg space provided* $\det H^i_j \neq 0$.

Transvecting (2.17) by $\dot{x}^k$ and using $\dot{x}^k y_k = F^2$, we have

$$\text{(2.20)} \qquad H^r_j G^i_{hmr} - H^i_r G^r_{jhm} = \left(\dot{\partial}_h \lambda_m\right) H^i_j + \left(\dot{\partial}_h \mu_m\right)\left(\delta^i_j F^2 - \dot{x}^i y_j\right).$$

Substituting the value of $\dot{\partial}_h \mu_m$ from (2.11) in (2.20), we get

$$\text{(2.21)} \quad H^r_j G^i_{hmr} - H^i_r G^r_{jhm} = \left(\dot{\partial}_h \lambda_m\right)\left[H^i_j - H\left(\delta^i_j - l^i l_j\right)\right],$$

where $l^i = \dfrac{\dot{x}^i}{F}$ and $l_i = \dfrac{y_i}{F}$.

If

(2.22) $H^r_j G^i_{hmr} - H^i_r G^r_{jhm} = 0$.

We have at least one of the following conditions

(2.23) (a) $\dot{\partial}_h \lambda_m = 0,$ (b) $H^i_j = H\left(\delta^i_j - l^i l_j\right).$

Putting $H = F^2 R$, (2.23 b) may be written as

(2.24) $H^i_j = F^2 R\left(\delta^i_j - l^i l_j\right).$

Therefore, the space is a Finsler space of scalar curvature.
Using (2.23 b) in (2.22), we get $H l_r G^r_{hmj} = 0,$ which, in view of Theorem 3.1 and $l_r = \dfrac{y_r}{F}$, implies $y_r G^r_{hmj} = 0,$ i.e. the space is a Landsberg space. Thus, we see that if (2.23 a) does not hold, the space is a Landsberg space of scalar curvature. But in view of Numata's theorem[21], a Landsberg space $F^n\,(n > 2)$ of scalar curvature is a Riemannian space of constant Riemannian curvature provided $R \neq 0$. This leads to

Theorem 2.7. *A generalized H-recurrent Finsler space* $F^n\,(n > 2)$ *admitting* $H^r_j G^i_{hmr} - H^i_r G^r_{jhm} = 0$ *is a Riemannian space of constant Riemannian curvature provided the covariant vector field* λ_m *is not independent of* $\dot{x}^i$.

3. A Generalized H-recurrent Finsler Space with the Vector λ_m Independent of the Directional Arguments

Let us consider a generalized H-recurrent Finsler space with the covariant vector λ_m independent of $\dot{x}^i$. In view of Theorem 3.1, the covariant vector μ_m is also independent of $\dot{x}^i$. Thus, for the space considered, we have

(3.1) (a) $\dot{\partial}_h \lambda_m = 0,$ (b) $\dot{\partial}_h \mu_m = 0.$

In view of (3.1), (2.9) and (2.17) reduce to

(3.2) $H_r G^r_{hmk} = 0$

and

(3.3) $H^r_{jk} G^i_{hmr} - H^i_{rk} G^r_{hmj} - H^i_{jr} G^r_{hmk} = 0.$

Transvecting (3.3) with $\dot{x}^k$ and using (1.1 b) and (1.5 b), we get

(3.4) $$H^r_j G^i_{hmr} = H^i_r G^r_{hmj}.$$

Taking skew-symmetric part of (3.3) with respect to the indices *j, k* and *h*, we get

(3.5) $$H^r_{jk} G^i_{hmr} + H^r_{kh} G^i_{jmr} + H^r_{hj} G^i_{kmr} = 0.$$

Transvecting (3.5) with $\dot{x}^k$, we have

(3.6) $$H^r_j G^i_{hmr} = H^r_h G^i_{jmr}.$$

In view of (3.5), Bianchi identities (1.6) reduces to

(3.7) $$\mathscr{B}_m H^i_{jkh} + \mathscr{B}_k H^i_{mjh} + \mathscr{B}_j H^i_{kmh} = 0,$$

which in view of (2.1), further reduce to

(3.8) $$\lambda_m H^i_{jkh} + \lambda_k H^i_{mjh} + \lambda_j H^i_{kmh} + \mu_m \left(\delta^i_j g_{kh} - \delta^i_k g_{jh}\right) + \mu_k \left(\delta^i_m g_{jh} - \delta^i_j g_{mh}\right) + \mu_j \left(\delta^i_k g_{mh} - \delta^i_m g_{kh}\right) = 0.$$

In view of (2.1), the identities

$$\mathscr{B}_m H^i_{jk} + \mathscr{B}_k H^i_{mj} + \mathscr{B}_j H^i_{km} = 0,$$

reduces to

(3.9) $$\lambda_m H^i_{jk} + \lambda_k H^i_{mj} + \lambda_j H^i_{km} = 0.$$

Differentiating (3.9) partially with respect to $\dot{x}^h$ and using (3.1 a), we get

(3.10) $$\lambda_m H^i_{jkh} + \lambda_k H^i_{mjh} + \lambda_j H^i_{kmh} = 0.$$

Using (3.10) in (3.8), we have

(3.11) $$\mu_m \left(\delta^i_j g_{kh} - \delta^i_k g_{jh}\right) + \mu_k \left(\delta^i_m g_{jh} - \delta^i_j g_{mh}\right) + \mu_j \left(\delta^i_k g_{mh} - \delta^i_m g_{kh}\right) = 0.$$

Contracting the indices *i* and *j* in (3.11), we find

(3.12) $$(n-2)\left(\mu_m g_{kh} - \mu_k g_{mh}\right) = 0.$$

If $n > 2$, we have

(3.13) $$\mu_m g_{kh} - \mu_k g_{mh} = 0.$$

Differentiating partially with respect to $\dot{x}^j$, and using $\dot{\partial}_j g_{kh} = 2C_{jkh}$ and (3.1b), we obtain

(3.14) $$\mu_m C_{jkh} - \mu_k C_{jmh} = 0.$$

Transvecting (3.14) with $\dot{x}^m$, we have $\mu_m \dot{x}^m C_{jkh} = 0$, which implies at least one of the following

(3.15) (a) $\mu_m \dot{x}^m = 0$, (b) $C_{jkh} = 0$.

Since the vector μ_m is independent of $\dot{x}^i$, $\mu_m \dot{x}^m = 0$ implies $\mu_m = 0$[13]. Therefore (3.15 a) is not true. Hence (3.15 b) is true, i.e. the Finsler space is necessarily Riemannian. This leads to

Theorem 3.1. *A generalized H-recurrent Finsler space* $F^n (n > 2)$ *with the covariant vector field* λ_m *independent of* $\dot{x}^i$*, is necessarily Riemannian.*

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