

# **Numerical Investigations of Thermal Convection in Rotating Micropolar Fluid in Hydromagnetics Saturating a Porous Medium\***

**Veena Sharma and Sudrshna Sharma**

Department of Mathematics & statistics  
H. P. University, Summer Hill Shimla-171005, India  
Email: [veena\\_math\\_hpu@yahoo.com](mailto:veena_math_hpu@yahoo.com)

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**Abstract:** The thermal convection in a layer of electrically conducting micropolar fluid heated from in the presence of a uniform magnetic field and rotation in a porous medium is considered. Using Boussinesq approximation, the linearized stability theory and normal mode analysis method for perturbation equations relevant to the problem, the dispersion relation is derived and the exact solutions are obtained for the case of two free boundaries. The presence of coupling between thermal and micropolar effects, uniform rotation, magnetic field and medium permeability may bring overstability in the system. The critical Rayleigh number for the onset of ordinary cellular convection and the onset of overstability are computed numerically using Newton-Raphson method through the software FORTRAN-95. It is found that these critical Rayleigh numbers increase with the increase in magnetic field, rotation and micropolar coefficients (dynamic microrotation viscosity and the coefficient of angular viscosity) implying thereby their stabilizing effect on the system, whereas critical Rayleigh numbers decrease with increase in medium permeability. It is evident from graphs that overstability is dominant parameter accounting for low wave numbers, however reverse occur for large wave numbers.

**Keywords:** Dynamic microrotation viscosity, micropolar coefficients, Newton-Raphson method, magnetic field

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## **1. Introduction**

Micropolar theory was introduced by Eringen<sup>1</sup> in order to describe some physical systems which do not satisfy the Navier Stokes equations. To explain the kinematics of such media, micropolar fluid involves a spin vector, responsible for microrotation and microinertia tensor (gyration parameter) which accounts for the atoms and molecules inside the macroscopic fluid particles in addition to the classical velocity vector field. These fluids are

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able to describe the behaviour of suspension, liquid crystals, animal blood etc, Eringen<sup>2</sup> and Perez-Garcia et al.<sup>3</sup>, Laidlaw<sup>4</sup>, Lekkerkerker<sup>5</sup> and Bradley (in dielectric fluids<sup>6</sup>). The medium has been considered to be non-porous in all the above studies. When a fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result of this microscopic law, the usual viscous term in the equations of micropolar fluid is replaced by  $[-(1/k_1)(\mu + \kappa')\mathbf{q}]$ , where  $\mu$  is the viscosity and  $\kappa'$  is micropolar coefficient of viscosity,  $k_1$  is the medium permeability and  $\mathbf{q}$  is the Darcian(filter) velocity of the fluid. The problem of stability of a fluid layer in a porous medium subjected to a temperature gradient is of importance in geophysics, ground water hydrology, petroleum engineering, chemical engineering etc. The effect of magnetic field on the stability of such a flow is of great interest in geophysics for example in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. Also, the rotation of the Earth distorts the boundaries of hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. The rotating fluid also finds applications in metrophysics and oceanography. Sharma and Kumar<sup>7,8,9</sup> have studied thermal convection of micropolar fluids with different parameters on the system in porous media.

The problem of hydromagnetics of rotating micropolar fluids has relevance and importance in chemical engineering, geophysics and bio-mechanics. Keeping in mind several geophysical situations involving uniform magnetic field, the numerical investigations of the stability of electrically conducting rotating micropolar fluids heated from below is considered in the presence of uniform magnetic field in a porous medium.

## 2. Formulation of the Problem and Disturbance Equations

Consider an infinite horizontal layer of an incompressible, electrically conducting micropolar fluids of thickness  $d$  in a porous medium acted on by a uniform vertical rotation  $\Omega(0,0,\Omega)$  and a uniform vertical magnetic field  $\mathbf{H}(0,0,H)$  and gravity force  $\mathbf{g}(0,0,-g)$ . This fluid layer is heated from below but convection sets in when the temperature gradient ( $\beta = |dT/dZ|$ ) between the lower and upper boundaries exceeds a certain critical value. The critical temperature gradient depends upon the bulk properties and boundary conditions of the fluid.

Let  $\mathbf{q}, \boldsymbol{\omega}, \nu, T, \rho, \eta, g, k, T, \rho, c_v, \varepsilon, p, \hat{e}_z$  and  $j$  denote the velocity, the spin, the temperature, the density, electrical resistivity, the specific heat at constant volume, medium porosity, thermodynamic pressure, the unit vector in  $z$ -direction and microinertia constant, respectively.  $\varepsilon', \beta'', \gamma'$  are the coefficients of angular viscosity and  $r = (x, y, z)$ . Assume that external couples and heat sources are not present.  $k_T$  and  $\delta$  are thermal conductivity and coefficients giving account of coupling between the spin flux and the heat flux. The magnetic permeability is assumed to be unity. Also the equation of state is given by

$$\rho = \rho_0 [1 - \alpha (T - T_0)],$$

where  $\rho_0, T_0$  are reference density, reference temperature at the lower boundary and  $\alpha$  is the coefficient of thermal expansion.

Let us now consider the stability of the system in the usual way by giving small perturbations on the initial state and on seeing the reaction of the perturbations on the system. The steady state solution is

$$\mathbf{q} = 0, \boldsymbol{\omega} = 0, p = p(z), \rho = \rho(z), \text{ and } T = T(z).$$

Let  $\mathbf{u} (u_x, u_y, u_z), \boldsymbol{\omega}, \delta p, \delta \rho, \theta$  and  $\mathbf{h} (h_x, h_y, h_z)$  denote, respectively, the perturbations in fluid velocity  $\mathbf{q}(0, 0, 0)$ , spin  $\boldsymbol{\omega}$ , pressure  $p$ , density  $\rho$ , temperature  $T$  and magnetic field  $\mathbf{H}(0, 0, H)$ . Then the linearized perturbation equations relevant to the problem, using Boussinesq approximation are

$$(2.1) \quad \nabla \cdot \mathbf{u} = 0,$$

$$(2.2) \quad \frac{\rho_0}{\varepsilon} \frac{\partial}{\partial t} \mathbf{u} = -\nabla \delta p - \frac{1}{k_1} (\mu + \kappa''') \mathbf{u} + \kappa'' (\nabla \times \boldsymbol{\omega}) + \alpha \rho_0 g \theta \hat{e}_z + \frac{1}{4\pi} (\nabla \times \mathbf{h}) \times \mathbf{H} + \frac{2\rho_0}{\varepsilon} (\mathbf{u} \times \boldsymbol{\Omega}).$$

$$(2.3) \quad \rho_0 j \frac{\partial}{\partial t} \boldsymbol{\omega} = (\varepsilon' + \beta'') \nabla (\nabla \cdot \boldsymbol{\omega}) + \gamma' \nabla^2 \boldsymbol{\omega} + \frac{\kappa'''}{\varepsilon} \nabla \times \mathbf{u} - 2 \kappa'' \boldsymbol{\omega} [\rho_0 c_v \varepsilon + \rho_s c_s (1 - \varepsilon)],$$

$$(2.4) \quad \frac{\partial}{\partial t} \theta = \kappa_T \nabla^2 \theta - \delta (\nabla \times \boldsymbol{\omega})_z \beta + \delta (\nabla \times \boldsymbol{\omega}) \cdot \nabla \theta + \rho_0 c_v \beta \mathbf{u}_z,$$

$$(2.5) \quad \nabla \cdot \mathbf{h} = 0,$$

$$(2.6) \quad \varepsilon \frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}) + \varepsilon \eta \nabla^2 \mathbf{h},$$

Also the equation of state is given by  $\delta p = \rho_0 [1 - \alpha (T - T_0)]$ , where  $\rho_0, T_0$  are reference density, reference temperature at the lower boundary and  $\alpha$  is the coefficient of thermal expansion . Using the non-dimensional numbers

$$(2.7) \quad z = z^* d, \quad \theta = \beta d \theta^*, \quad t = \frac{\rho_0 d^2}{\mu} t^*, \quad \mathbf{u} = \frac{\kappa}{d} \mathbf{u}^*,$$

$$p = \frac{\mu \kappa}{d^2} p^*, \quad \boldsymbol{\omega} = \frac{\kappa}{d^2} \boldsymbol{\omega}^*, \quad \mathbf{h} = \left( \frac{\mu \kappa}{d^2} \right)^{\frac{1}{2}} \mathbf{h}^*, \quad \Omega = \frac{\mu}{\rho_0 d^2} \Omega^*$$

and then removing the stars for convenience, the non-dimensional forms of equations (2.1)–(2.5) become

$$(2.8) \quad \nabla \cdot \mathbf{u} = 0,$$

$$(2.9) \quad \frac{1}{\varepsilon} \frac{\partial}{\partial t} \mathbf{u} = -\nabla \delta p - \frac{1}{k_1} (1 + K) \mathbf{u} + K (\nabla \times \boldsymbol{\omega}) + R \theta \hat{e}_z + \frac{1}{4\pi} (\nabla \times \mathbf{h}) \times H$$

$$+ \frac{2}{\varepsilon} (\mathbf{u} \times \Omega),$$

$$(2.10) \quad j \frac{\partial}{\partial t} \boldsymbol{\omega} = C_1 \nabla (\nabla \cdot \boldsymbol{\omega}) - C_0 \nabla \times (\nabla \times \boldsymbol{\omega}) + K \left( \frac{1}{\varepsilon} \nabla \times \mathbf{u} - 2 \boldsymbol{\omega} \right),$$

$$(2.11) \quad E p_1 \frac{\partial}{\partial t} \theta = \nabla^2 \theta + \mathbf{u}_z + \bar{\delta} [\nabla \theta \cdot (\nabla \times \boldsymbol{\omega}) - (\nabla \times \boldsymbol{\omega})_z],$$

$$(2.12) \quad \nabla \cdot \mathbf{h} = 0,$$

$$(2.13) \quad \varepsilon \frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}) + \frac{\varepsilon}{p_2} \nabla^2 \mathbf{h},$$

where new dimensionless coefficients are  $\bar{k}_1 = \frac{k_1}{\mu}$ ,  $\bar{j} = \frac{j}{d^2}$ ,  $\bar{\delta} = \frac{\delta}{\rho_0 c_v d^2}$ ,  
 $K = \frac{\kappa''}{\mu}$ ,  $C_0 = \frac{\gamma'}{\mu d^2}$ ,  $C_1 = \frac{\varepsilon' + \beta'' + \gamma'}{\mu d^2}$ ,  $E = \varepsilon + (1 - \varepsilon) \frac{\rho_s c_s}{\rho_0 c_v}$  and the  
 dimensionless Rayleigh number  $R$ , thermal Prandtl number  $p_1$ , the  
 magnetic Prandtl number  $p_2$  are  $R = \frac{g\alpha\beta\rho_0 d^4}{\mu\kappa}$ ,  $p_1 = \frac{\mu}{\rho_0 \kappa}$ ,  $\kappa = \frac{k_T}{\rho_0 c_v}$   
 $p_2 = \frac{\mu}{\rho_0 \eta}$ , is the thermal diffusivity.

Let us assume both the boundaries to be free and perfectly heat  
 conducting. The case of two free boundaries, though little artificial is the  
 most appropriate for stellar atmosphere. Since the surfaces are fixed and are  
 maintained at fixed temperature

$$(2.14) \quad u_z = 0, \frac{\partial^2 u_z}{\partial z^2} = 0, \boldsymbol{\omega} = 0 = \theta = \zeta \quad \text{at } z = 0 \text{ and } z = 1,$$

where  $\zeta = (\nabla \times \mathbf{u}_z)$ , is the  $z$ -component of vorticity

Applying the curl operator twice to equation (2.9) and taking  $z$ -component,  
 we get

$$(2.15) \quad \frac{1}{\varepsilon} \frac{\partial}{\partial t} (\nabla^2 u_z) = R \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \frac{1}{k_1} (1 + K) \nabla^2 u_z + K \nabla^2 \zeta_z + \frac{H}{4\pi} \frac{\partial}{\partial z} (\nabla^2 h_z) - \frac{2}{\varepsilon} \Omega \frac{\partial \zeta}{\partial z},$$

where  $\zeta_z = (\nabla \times \boldsymbol{\omega})_z$ .

Applying the curl operator to equation(2.9), (2.11) and (2.13), and  
 taking  $z$ -component, we get

$$(2.16) \quad \frac{1}{\varepsilon} \frac{\partial \zeta}{\partial t} = -\frac{1}{k_1} (1 + K) \zeta + \frac{H}{4\pi} \frac{\partial \xi}{\partial z} + \frac{2}{\varepsilon} \Omega \frac{\partial u_z}{\partial z},$$

$$(2.17) \quad \bar{j} \frac{\partial \zeta}{\partial t} = c_0 \nabla^2 \zeta_z - K \left( \frac{1}{\varepsilon} \nabla^2 u_z + 2\zeta_z \right),$$

$$(2.18) \quad \varepsilon \frac{\partial \xi}{\partial t} = H \frac{\partial}{\partial z} \zeta + \frac{\varepsilon}{p_2} \nabla^2 \xi,$$

where  $\xi = (\nabla \times h)_z$ , is the  $z$ -components of current density. Taking the  $z$ -component of equation (2.13) and linearized form of equation (2.11) are

$$(2.19) \quad \varepsilon \frac{\partial h_z}{\partial t} = H \frac{\partial u_z}{\partial z} + \frac{\varepsilon}{p_2} \nabla^2 h_z, .$$

$$(2.20) \quad Ep_1 \frac{\partial \theta}{\partial t} = \nabla^2 \theta + u_z - \bar{\delta} \zeta_z.$$

If the medium adjoining the fluid is electrically non conducting ,then bounadry conditions are

$$(2.21) \quad u_z = 0, \frac{\partial^2 u_z}{\partial z^2} = 0, \frac{\partial \zeta}{\partial z} = 0, \xi = \theta = \zeta_z = 0, \frac{\partial h_z}{\partial z} = 0 \quad \text{at } z=0 \text{ and } z=1.$$

### 3. Methodology (Normal Mode Analysis Method)

Analyzing the disturbances into the normal mode, we assume that the solutions of equations (2.15)-(2.20) are given by

$$(3.1) \quad [u_z, \zeta_z, \zeta, \xi, \theta, h_z] = [U(z), G(z), Z(z), X(z), \Theta(z), B(z)] \\ \exp ( ik_x x + ik_y y + nt ) ,$$

where  $k = (k_x^2 + k_y^2)^{1/2}$  is the resultant wave-number,  $k_x$  and  $k_y$  are real constants and  $n$  is the stability parameter which can be, complex, in general. For solution having the dependence of the form (3.1), equations (2.15) – (2.20) take the form

$$(D^2 - k^2) [\varepsilon^{-1} n + \frac{1}{k_1} (1 + K)] U = -Rk^2 \Theta + K (D^2 - k^2) G + \frac{H}{4\pi} (D^2 - k^2) DB - \frac{2\Omega}{\varepsilon} DZ$$

$$(3.2) \quad [\varepsilon^{-1} n + \frac{1}{k_1} (1 + K)] Z = \frac{H}{4\pi} DX + \frac{2}{\varepsilon} \Omega DU ,$$

$$(3.3) \quad \{ \ln + 2A - (D^2 - k^2) \} G = -A \varepsilon^{-1} (D^2 - k^2) U ,$$

$$(3.4) \quad \{ Ep_1 n - (D^2 - k^2) \} \Theta = U - \bar{\delta} G$$

$$(3.5) \quad \left\{ n - \frac{1}{p_2} (D^2 - k^2) \right\} X = \varepsilon^{-1} H DZ,$$

$$(3.6) \quad \left\{ n - \frac{1}{p_2} (D^2 - k^2) \right\} B = \varepsilon^{-1} H DU,$$

where

$$(3.7) \quad A = \frac{K}{C'_0}, \quad \ell = \bar{j} \frac{A}{K}, \quad \text{and } D = \frac{d}{dz}.$$

Eliminating  $\Theta, Z, B, G, X$  from equations (3.2)–(3.7), we get

$$(3.8) \quad \begin{aligned} & (D^2 - k^2) \left[ \varepsilon^{-1} n + \frac{1}{k_1} (1 + K) \right] [Ep_1 n - (D^2 - k^2)] [\ln + 2A - (D^2 - k^2)] \\ & \left[ n - \frac{1}{p_2} (D^2 - k^2) \right] \left[ \left\{ \varepsilon^{-1} n + \frac{1}{k_1} (1 + K) \right\} - \left\{ n - \frac{1}{p_2} (D^2 - k^2) \right\} - \frac{H^2 \varepsilon^{-1}}{4\pi} D^2 \right] U \\ & = -Rk^2 \left[ n - \frac{1}{p_2} (D^2 - k^2) \right] \left[ \left\{ \varepsilon^{-1} n + \frac{1}{k_1} (1 + K) \right\} - \left\{ n - \frac{1}{p_2} (D^2 - k^2) \right\} - \frac{H^2 \varepsilon^{-1}}{4\pi} D^2 \right] \\ & \quad \left[ \{\ln + 2A - (D^2 - k^2)\} + \bar{\delta} A \varepsilon^{-1} (D^2 - k^2) \right] U - KA \varepsilon^{-1} (D^2 - k^2)^2 [Ep_1 n - (D^2 - k^2)] \\ & \quad \left[ n - \frac{1}{p_2} (D^2 - k^2) \right] \left[ \left\{ \varepsilon^{-1} n + \frac{1}{k_1} (1 + K) \right\} - \left\{ n - \frac{1}{p_2} (D^2 - k^2) \right\} - \frac{H^2 \varepsilon^{-1}}{4\pi} D^2 \right] \\ & \quad U + \frac{H^2 \varepsilon^{-1}}{4\pi} (D^2 - k^2) [Ep_1 n - (D^2 - k^2)] [\ln + 2A - (D^2 - k^2)] \\ & \quad \left[ \left\{ \varepsilon^{-1} n + \frac{1}{k_1} (1 + K) \right\} - \left\{ n - \frac{1}{p_2} (D^2 - k^2) \right\} - \frac{H^2 \varepsilon^{-1}}{4\pi} D^2 \right] D^2 U \\ & \quad - 4\varepsilon^{-2} \Omega^2 D^2 \left[ n - \frac{1}{p_2} (D^2 - k^2) \right]^2 [Ep_1 n - (D^2 - k^2)] \\ & \quad [\ln + 2A - (D^2 - k^2)] U. \end{aligned}$$

The boundary conditions (2.21) using equations (3.2)–(3.7) transform to

$$U = 0, \quad D^2 U = 0, \quad DZ = 0, \quad G = 0, \quad X = 0, \quad \Theta = 0, \quad DB = 0$$

$$(3.9) \quad D^2\Theta = 0, D^2G = 0, D^3Z = 0, D^3G = 0, D^2X = 0, D^3B = 0, \text{ at } z=0 \text{ \& } 1.$$

It can be shown from equations (3.2)–(3.7) and boundary conditions (3.9), that all even order derivatives of  $U$  vanish on the boundaries. The proper solution of  $U$  characterizing the lowest mode is

$$(3.10) \quad U = U_0 \sin(\pi z),$$

where  $U_0$  is a constant. Substituting the solution (3.10) in equation (3.8) and putting  $b = \pi^2 + k^2$ , we obtain

$$(3.11) \quad \begin{aligned} & Rk^2 \left\{ n + \frac{b}{p_2} \right\} \left[ \left\{ \varepsilon^{-1}n + \frac{1}{k_1}(1+K) \right\} - \left\{ n + \frac{b}{p_2} \right\} + \frac{H^2\pi\varepsilon^{-1}}{4} \right] \\ & [\ln + 2A + b - b\bar{\delta}A\varepsilon^{-1}] = b \left[ \varepsilon^{-1}n + \frac{1}{k_1}(1+K) \right] [Ep_1n + b] [\ln + 2A + b] \left[ n + \frac{b}{p_2} \right] \\ & \left[ \left\{ \varepsilon^{-1}n + \frac{1}{k_1}(1+K) \right\} \left\{ n + \frac{b}{p_2} \right\} + \frac{H^2\pi\varepsilon^{-1}}{4} \right] - KA\varepsilon^{-1}b^2 [Ep_1n + b] \left[ n + \frac{b}{p_2} \right] \\ & \left[ \left\{ \varepsilon^{-1}n + \frac{1}{k_1}(1+K) \right\} \left\{ n + \frac{b}{p_2} \right\} + \frac{H^2\pi\varepsilon^{-1}}{4} \right] + \frac{H^2\pi\varepsilon^{-1}}{4} [Ep_1bn + b^2] [\ln + 2A + b] \\ & \left[ \left\{ \varepsilon^{-1}n + \frac{1}{k_1}(1+K) \right\} \left\{ n + \frac{b}{p_2} \right\} + \frac{H^2\pi\varepsilon^{-1}}{4} \right] + 4\varepsilon^{-2}\Omega^2 \left[ n + \frac{b}{p_2} \right] \\ & [Ep_1n + b] [\ln + 2A + b] = 0. \end{aligned}$$

In the absence of rotation i.e. ( $\Omega = 0$ ), equation(34) reduces to

$$(3.12) \quad \begin{aligned} & Rk^2 \left\{ n + \frac{b}{p_2} \right\} [\ln + 2A + b - b\bar{\delta}A\varepsilon^{-1}] = b \left[ \varepsilon^{-1}n + \frac{1}{k_1}(1+K) \right] [Ep_1n + b] \\ & [\ln + 2A + b] \left\{ n + \frac{b}{p_2} \right\} - KA\varepsilon^{-1}b^2 [Ep_1n + b] \left( n + \frac{b}{p_2} \right) + \frac{H^2\pi\varepsilon^{-1}}{4} \\ & [Ep_1bn + b^2] [\ln + 2A + b], \end{aligned}$$

### 4. Mathematical Analysis

#### The case of overstability

Since for overstability, we wish to determine critical Rayleigh number for the onset of overstability, it suffices to find conditions for which (3.11) will admit of solution with  $n_i$  real. Substituting  $n = in_i$  in equation (3.11), the real part and eliminating from it and the imaginary part ,yield

$$\begin{aligned}
 R = & [ < -n_i^6 Ep_1 \varepsilon^{-1} lb + n_i^4 [b^3 \varepsilon^{-2} \{ (1 + \frac{Ep_1 l}{p_2}) + \frac{2}{p_2} (Ep_1 + l) \} + b^2 \{ (1 + \frac{2Ep_1}{p_2}) 2A \varepsilon^{-2} \\
 & + \frac{2\varepsilon^{-1}}{k_1} (1 + K) (Ep_1 + l) + \frac{4Ep_1 l \varepsilon^{-1}}{p_2 k_1} (1 + K) \} + b \{ \frac{4Ep_1 A \varepsilon^{-1}}{k_1} (1 + K) + H^2 \pi \varepsilon^{-2} \frac{Ep_1 l}{4} \\
 & + \frac{Ep_1 l}{k_1^2} (1 + K)^2 \} ] - n_i^2 [ b^5 \frac{\varepsilon^{-2}}{p_2^2} + b^4 \{ \frac{4\varepsilon^{-1}}{p_2 k_1} (1 + K) + \frac{2\varepsilon^{-1}}{p_2^2 k_1} (1 + K) (Ep_1 + l) + \frac{2A \varepsilon^{-2}}{p_2^2} \\
 & + b^3 \{ \frac{4A \varepsilon^{-1}}{p_2 k_1} (1 + K) (2 + \frac{Ep_1}{p_2}) + \frac{2}{p_2 k_1^2} (1 + K)^2 (Ep_1 + l) + \frac{(1 + K)^2}{k_1^2} (1 + \frac{Ep_1 l}{p_2^2}) \\
 & + \frac{H^2 \pi \varepsilon^{-2}}{4} (1 + \frac{Ep_1 + l}{p_2}) \} + b^2 \{ \frac{2A}{k_1^2} (1 + K)^2 (1 + \frac{2Ep_1}{p_2}) + \frac{H^2 \pi \varepsilon^{-1}}{4 k_1} (1 + K) \\
 & (Ep_1 + l + \frac{Ep_1 l}{p_2}) + \frac{H^2 \pi \varepsilon^{-2}}{2 p_2} Ep_1 A \} + b \frac{H^2 \pi \varepsilon^{-1} Ep_1 A}{2 k_1} (1 + K) ] + [ b^5 \frac{(1 + K)^2}{p_2^2 k_1^2} \\
 & + b^5 \{ \frac{2A(1 + K)^2}{p_2^2 k_1^2} + \frac{H^2 \pi \varepsilon^{-1}}{4 p_2 k_1} (1 + K) \} + b^3 \frac{H^2 \pi \varepsilon^{-1} A}{2 p_2 k_1} (1 + K) ] \\
 & > -KA \varepsilon^{-1} [ n_i^4 \varepsilon^{-1} Ep_1 b^2 - n_i^2 \{ \frac{b^4 \varepsilon^{-1}}{p_2} (2 + \frac{Ep_1}{p_2}) + b^3 \frac{(1 + K)}{k_1} (1 + \frac{2Ep_1}{p_2}) \\
 & + b^2 \frac{H^2 \pi \varepsilon^{-1} Ep_1}{4} \} + \{ b^5 \frac{(1 + K)}{p_2^2 k_1} + b^4 \frac{H^2 \pi \varepsilon^{-1}}{4 p_2} \} ] + \frac{H^2 \pi \varepsilon^{-1}}{4} [ n_i^4 \varepsilon^{-1} Ep_1 lb - n_i^2 \{ b^3 \varepsilon^{-1} (1 + \frac{Ep_1 + l}{p_2}) \\
 & + b^2 \{ 2A \varepsilon^{-1} (1 + \frac{Ep_1}{p_2}) + \frac{(1 + K)}{k_1} (Ep_1 + l + \frac{Ep_1 l}{p_2}) \} + b \{ \frac{2A Ep_1}{k_1} (1 + K) + \frac{H^2 \pi \varepsilon^{-1}}{4} Ep_1 l \} \\
 & + \{ b^4 \frac{(1 + K)}{p_2 k_1} + b^3 (\frac{2A}{p_1 k_1} (1 + K) + \frac{H^2 \pi \varepsilon^{-1}}{4}) + b^2 \frac{H^2 \pi \varepsilon^{-1} A}{2} \} ] + 4 \varepsilon^{-2} \Omega^2 \pi^2 [ n_i^4 Ep_1 l \\
 & - n_i^2 \{ b^2 (1 + \frac{Ep_1 l}{p_2} + \frac{2}{p_2} (Ep_1 + l)) + 2Ab (1 + \frac{Ep_1 l}{p_2}) \} + \frac{b^3}{p_2^2} (2A + b) ] ] \times k^{-2} [ n_i^4 l \varepsilon^{-1}
 \end{aligned}
 \tag{4.1}$$

$$\begin{aligned}
 & -n_i^2 \left\{ \frac{b^2 \varepsilon^{-1}}{p_2} \left( \frac{l}{p_2} + 2(1 - \bar{\delta} A \varepsilon^{-1}) \right) + b \left( \frac{2l}{p_2 k_1} (1 + K) + \frac{(1 + K)}{k_1} (1 - \bar{\delta} A \varepsilon^{-1}) + \frac{4A \varepsilon^{-1}}{p_2} \right) \right. \\
 & + \left( \frac{2A}{k_1} (1 + K) + \frac{H^2 \pi \varepsilon^{-1} l}{4} \right) \left. \right\} + \left\{ b^3 \frac{(1 + K)}{p_2^2 k_1} (1 - \bar{\delta} A \varepsilon^{-1}) + b^2 \left( \frac{2A(1 + K)}{p_2^2 k_1} \right. \right. \\
 & \left. \left. + (1 - \bar{\delta} A \varepsilon^{-1}) \frac{H^2 \pi \varepsilon^{-1}}{4 p_1} + b \frac{H^2 \pi \varepsilon^{-1} A}{4 p_2} \right) \right\}^{-1}.
 \end{aligned}$$

and

$$(4.2) \quad B_0 n_i^8 + B_1 n_i^6 + B_2 n_i^4 + B_3 n_i^2 + B_4 = 0,$$

where  $B_0, B_1, B_2, B_3, B_4$  are the coefficients, which are quite lengthy, not mentioned.

**The case of stationary convection**

When the instability sets in as stationary convection, the marginal state is characterized by  $n = 0$ . Putting  $n_i = 0$  in equation (4.1), we obtain

$$\begin{aligned}
 (4.3) \quad R = & \left[ \frac{b^4}{k_1 p_2^2} (1 + K) \left\{ \frac{(1 + K)}{k_1} - K A \varepsilon^{-1} \right\} + b^3 \left\{ \frac{(1 + K)}{k_1 p_2} \left( 2A \frac{(1 + K)}{k_1 p_2} + \frac{H^2 \pi \varepsilon^{-1}}{2} \right) + \frac{4 \varepsilon^{-2} \Omega^2 \pi^2}{p_2^2} - \frac{H^2 \pi \varepsilon^{-2} K A}{4 p_2} \right\} \right. \\
 & + b^2 \left\{ \frac{H^2 \pi \varepsilon^{-1}}{2 p_2 k_1} (1 + K) + \frac{H^2 \pi \varepsilon^{-1}}{4} \left( 2A + \frac{H^2 \pi \varepsilon^{-1}}{4} \right) + \frac{8 A \varepsilon^{-2} \Omega^2 \pi^2}{p_2^2} \right\} \\
 & + b \frac{A \varepsilon^{-2} H^4 \pi^2}{8} \left. \right] \left[ \frac{k^2}{p_2} \left\{ b^2 \frac{(1 + K)}{k_1 p_2} (1 - \bar{\delta} A \varepsilon^{-1}) \right. \right. \\
 & \left. \left. + b \left( 2A \frac{(1 + K)}{k_1 p_2} + \frac{H^2 \pi \varepsilon^{-1}}{4} (1 - \bar{\delta} A \varepsilon^{-1}) \right) + \frac{A H^2 \pi \varepsilon^{-1}}{2} \right\} \right]^{-1}.
 \end{aligned}$$

**5. Numerical Results and Conclusion**

We have plotted the variation of the Rayleigh number (R) with wave number (k) using equation (4.1) satisfying (4.2) and (4.3) for both stationary and overstable cases for values of dimensionless parameters  $A = 0.5, \bar{\delta} = 1, K = 1, l = 1, p_1 = 0.1, p_2 = 1, E = 1$  and  $\varepsilon = 0.5$ ,

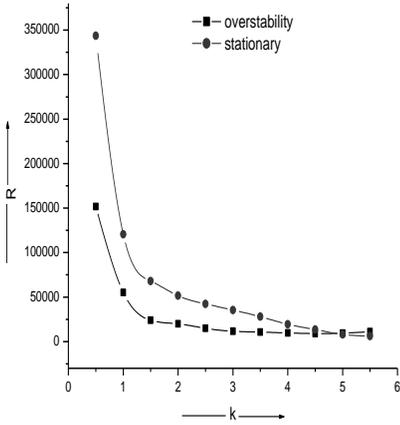


Fig.1

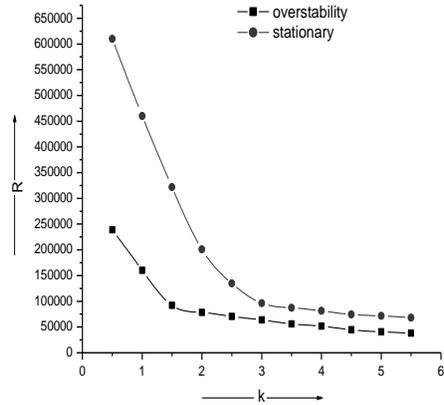


Fig.2

Figures 1 and 2 correspond to two values of the angular velocity of rotation  $\Omega = 4$  and  $6 \text{ rev. min}^{-1}$ , respectively. It is evident from the graphs that the Rayleigh number increases with the increases in rotation parameter, depicting the stabilizing effect of rotation

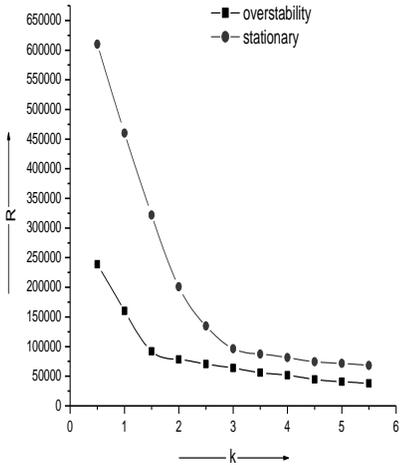


Fig.3

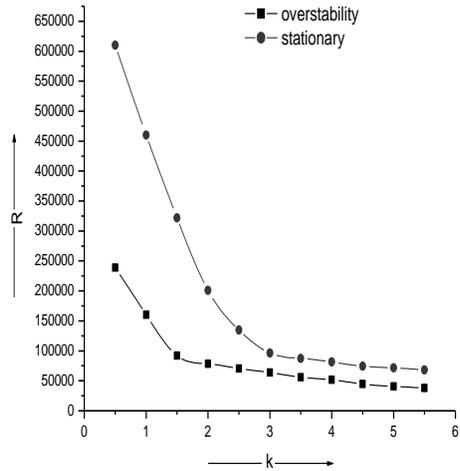


Fig.4

Figures 3 and 4 correspond to two values of the magnetic field  $H = 10$  Gauss and  $15$  Gauss, respectively. The graphs show that the Rayleigh number increases with the increases in magnetic field depicting the stabilizing effect of magnetic field.

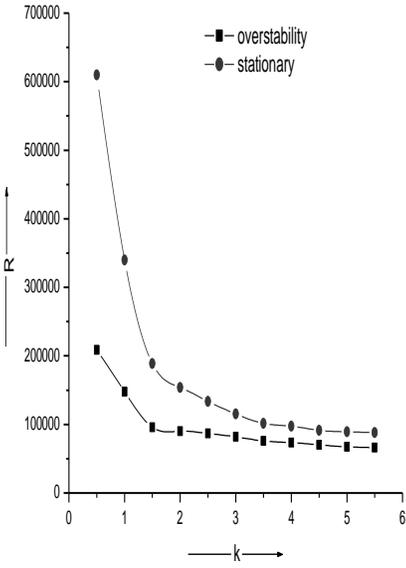


Fig.5

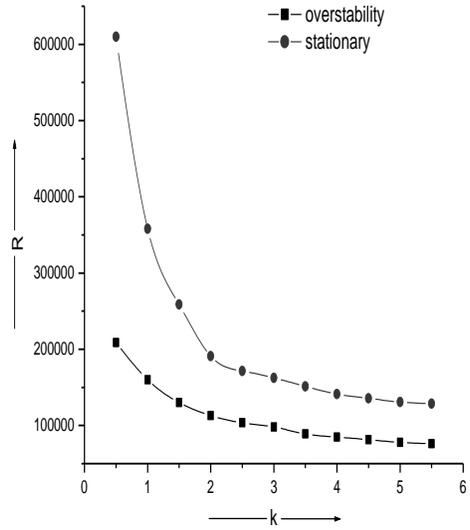


Fig.6

Figures 5 and 6 correspond to two values of medium permeability  $\bar{k}_1 = 3$  and 1, respectively. It is evident from the graphs that the Rayleigh number decreases with the increases in permeability depicting destabilizing effect of permeability

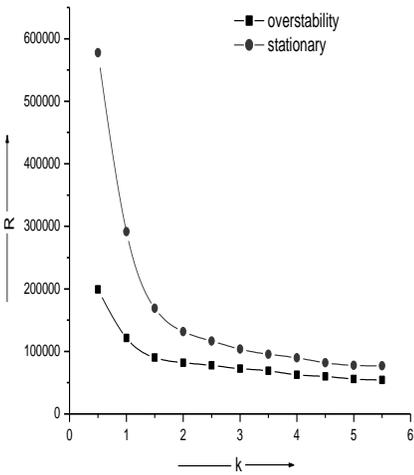


Fig.7

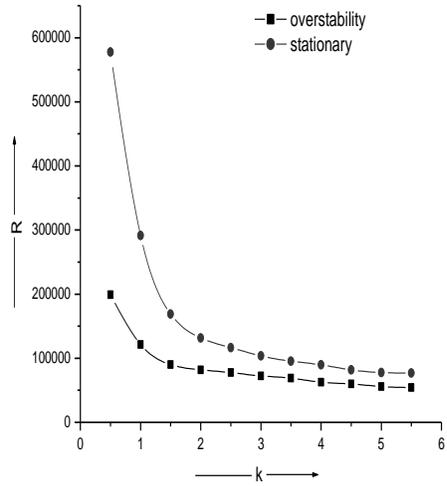


Fig.8

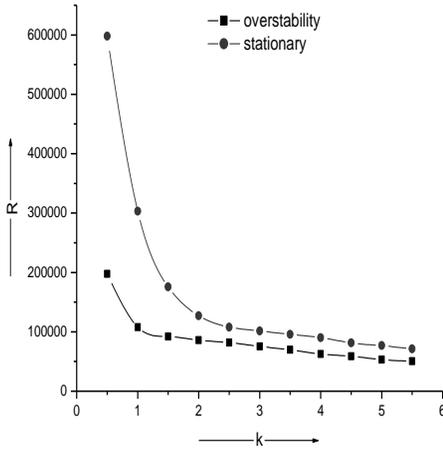


Fig.9

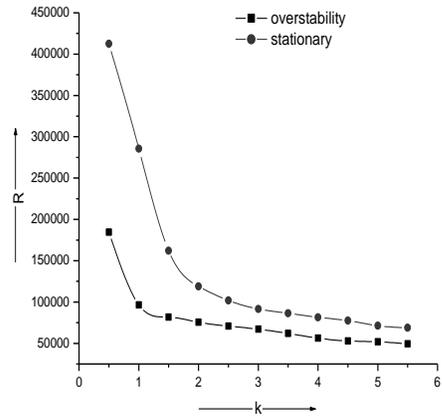


Fig10

Figures 7–10 correspond to two values of micropolar coefficients  $\kappa = 1.5, 1.0$  and  $\gamma' = 2.5, 1.2$  accounting for dynamic microrotation viscosity and coefficient of angular viscosity, respectively. The graphs show that the Rayleigh number for the stationary convection and for the case of overstability increases with the increase in micropolar coefficients implying thereby the stabilizing effect of dynamic microrotation viscosity and coefficient of angular viscosity on the system

It is clear from graphs that the Rayleigh number for overstability is always less than the Rayleigh number for stationary convection, for a fixed wave number. Moreover, rotation and magnetic field introduce oscillatory modes in the system. The presence of coupling between thermal and micropolar effects may bring over stability in the system.

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