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On Relativistic Fluid Space Time Admitting Heat Flux of a Generalized Recurrent and Ricci-recurrent Kenmotsu Manifold*

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Abstract: The aim of the present paper is to study generalized recurrent and ricci-recurrent Kenmotsu manifold for relativistic fluid space time admitting heat flux with the time like generator U as the velocity vector field of the fluid and generator V as the heat flux vector field.

Keywords: Generalized recurrent Kenmotsu manifold, relativistic fluid space time, heat flux vector field
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1. Introduction

A Riemannian manifold (M_n, g) is called generalized recurrent[1] if its curvature tensor R satisfies the condition

(1) $$(\nabla_X R)(Y,Z)W = \alpha(X)R(Y,Z)W + \beta(X)[g(Z,W)Y - g(Y,W)Z],$$

where α and β are two 1-forms, β is non- zero and these are defined by

(2) $$\alpha(X) = g(X,A),\ \beta(X) = g(X,B),$$

where A, B are vector fields associated with 1-forms α and β respectively. A Riemannian manifold (M_n, g) is called generalized Ricci-recurrent[1] if its Ricci-tensor S satisfies the condition

(3) $$(\nabla_X S)(Y,Z) = \alpha(X)S(Y,Z) + (n-1)\beta(X)\,g(Y,Z),$$

where α and β are defined as in (2).

2. Kenmotsu Manifold

Let M be an almost contact manifold[2] equipped with an almost contact metric structure (ϕ, ξ, η, g) consisting of a (1, 1) tensor field ϕ, a vector

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field ξ, a 1-form η and a compatible Riemannian metric g satisfying

(4) $\phi^2 = -I + \eta \otimes \xi,\ \eta(\xi) = 1,\ \phi(\xi) = 0\ \ \eta \circ \phi = 0,$

(5) $g(X,Y) = g(\phi X, \phi Y) + \eta(X)\eta(Y),$

(6) $g(X,\phi Y) = -g(\phi X, Y), g(X,\xi) = \eta(X)$, for all $X,Y \in \chi(M)$.

An almost contact metric manifold M is called a kenmotsu manifold if it satisfies[3]

(7) $(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X,\ X,Y \in \chi(M),$

where ∇ is Levi-Civita connection of the Riemannian metric g.
From the above equation it follows that

(8) $\nabla_X \xi = X - \eta(X)\xi,$

(9) $(\nabla_X \eta)Y = g(X,Y) - \eta(X)\eta(Y).$

Moreover, the curvature tensor R and Ricci tensor S satisfy[3]

(10) $R(\xi, X)Y = \eta(Y)X - g(X,Y)\xi$

and

(11) $S(X,\xi) = (1-n)\eta(X)$, where n = 2m +1.

In 1980, D. ray[4] defined the energy momentum tensor representing the matter distribution of a fluid space time admitting heat flux as

(12) $T(X,Y) = (\rho + \sigma)A(X)A(Y) + \sigma g(X,Y) + A(X)B(Y) + A(Y)B(X),$

where ρ and σ are the density and isotropic pressure of the fluid, U is the unit time vector field such that $g(X,U) = A(X)$ and V is the heat flux vector field such that $g(X,V) = B(X)$. In 1983, B.O'Neil[5] defined the form of Einstein's equation without cosmological constant as

(13) $$S(X,Y) - \frac{1}{2} r g(X,Y) = kT(X,Y),$$

where k is the gravitational constant and r being the scalar curvature of the manifold.

3. Theorems

In this paper, we establish the following:

Theorem: *If a fluid space time of general relativity admitting heat flux is a generalized Ricci- recurrent Kenmotsu manifold with the time like generator U as the velocity vector field of the fluid and the generator V as the heat flux vector field, then*

$$r = 2(1-n) \quad , \quad provided\ k = 0 .$$

Proof: In view of (12), (13) can be written as

$$S(X,Y) = \frac{1}{2} r g(X,Y) + k[(\rho+\sigma)A(X)A(Y) + \sigma g(X,Y) + \{A(X)B(Y) + A(Y)B(X)\}],$$

or

$$S(X,Y) = (\frac{1}{2} r + k\sigma) g(X,Y) + k(\sigma+\rho)A(X)B(Y) + k[A(X)B(Y) + A(Y)B(X)]. \tag{14}$$

Assume that (M_n, g) be a generalized recurrent Kenmotsu manifold, then the Ricci-tensor S of (M_n, g) satisfies the condition (3) for all vector fields X, Y, Z.Taking $Z = \xi$ in (3) we have

$$(\nabla_X S)(Y,\xi) = (1-n)[\alpha(X) - \beta(X)]\eta(Y) . \tag{15}$$

By the definition of covariant derivative of S we have

$$(\nabla_X S)(Y,\xi) = \nabla_X S(Y,\xi) - S(\nabla_X Y,\xi) - S(Y,\nabla_X \xi) , \tag{16}$$

which in view of (8) and (11), can be written as

$$(\nabla_X S)(Y,\xi) = (1-n)\nabla_X \eta(Y) - (1-n)\eta(\nabla_X Y) - S(X,Y) + (1-n)\eta(X)\eta(Y). \tag{17}$$

So using (9) we get

$$(\nabla_X S)(Y,\xi) = (1-n)g(X,Y) - S(X,Y). \tag{18}$$

From the equality of left hand side of the equation (15) and (18) we can write

$$(1-n)[\alpha(X) - \beta(X)]\eta(Y) = (1-n)g(X,Y) - S(X,Y) . \tag{19}$$

Hence taking $Y = \xi$ in (19) and using (11), we obtain

(20) $\alpha(X)=\beta(X)$, for any vector field X.

Using (20) in (19), we have

(21) $S(X,Y)=(1-n)g(X,Y)$.

From (21) and (14), we have $\frac{r}{2}=(1-n)$, provided k = 0. This implies $r=2(1-n)$, provided k = 0. Hence the theorem.

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