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Minimizing Rental Cost under Specified Rental Policy in Two Stage Flow Shop, the Processing Time Associated with Probabilities Including Break-down Interval and Job – Block Criteria*

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Abstract: In real world scheduling applications, machines might not be available during certain time periods due to deterministic or stochastic causes. This paper is an attempt to study the two machine general flow shop problem in which the processing time of the jobs are associated with probabilities, following some restrictive renting policy including break-down interval and equivalent job-block criteria. The objective of the paper is to find an algorithm to minimize the rental cost of the machines under specified rental policy with break-down interval and job block criteria. The proposed method is very simple and easy to understand and also, provide an important tool for decision makers. The method is illustrated with the help of numerical example.

Keyword: Equivalent-job, Rental Policy, Make span, Elapsed time, Idle time, Break-down interval, Johnson's technique, Optimal sequence.
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1. Introduction

The classical scheduling literature commonly assumes that the machines are never unavailable during the process. This assumption might be justified in some cases but it dose not apply if certain maintenance requirements, break-downs or other constraints that causes the machine not to be available for processing have to be considered. The temporal lack of machine availability is known as '***break-down'*** (due to failure of electric current, the

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non-supply of raw material, shift pattern or other technical interruption). Before 1954, the concept of break-down of machines had not considered by any author. In 1954 Johnson had considered the effect of break-down of machines on the completion times of jobs in an optimal sequence. Later on many researchers such as *Adiri*[1], *Akturk and Gorgulu*[2], *Schmidt*[3], *Chandramouli*[4], *Singh T. P.*[5], *Belwal and Mittal*[6] etc. have discussed the various concepts of break-down of machines. The functioning of machines for processing the jobs on them is assumed to be smooth with having no disturbance on the completion times of jobs. But there are feasible sequencing situations in flow shops where machines while processing the jobs get sudden break-down due to failure of a component of machines for a certain interval of time or the machines are supposed to stop their working for a certain interval of time due to some external imposed policy such as stop of flow of electric current to the machines may be a government policy due to shortage of electricity production. In each case this may be well observed that working of machines is not continuous and is subject to break for a certain interval of time.

In flow-shop scheduling, the object is to obtain a sequence of jobs which when processed in a fixed order of machines, will optimize some well defined criteria. Various Researchers have done a lot of work in this direction. Johnson[3] *Ignall* and *Scharge*[7] *Szwarch*[8] *Chandra Shekhran*[9] *Maggu & Das*[10] *Bagga P. C.*[11], *Singh T. P., Gupta Deepak* [12] etc. derived the optimal algorithm for two, three or multi stage flow shop problems taking into account the various constraints and criteria. *Maggu & Das* [10] introduced the concept of equivalent-job blocking in the theory of scheduling. The concept is useful and significant in the sense to create a balance between the cost of providing priority in service to the customer and cost of giving services with non priority customers. The decision maker may decide how much to charge extra from the priority customer. Further, *Maggu* [9] *Singh T.P.and Gupta Deepak* [12] associated probabilities with processing time and set up time in their studies. Recently, *Singh T. P., Gupta Deepak* [13] studied n x 2 general flow shop problem to minimize rental cost under a pre-defined rental policy in which the probabilities have been associated with processing time on each machine including job block criteria. We have extended the study made by *Singh T.P., Gupta Deepak* [13] by introducing the concept of break-down interval. We have developed an algorithm minimizing the utilization time of second machine combined with Johnson's algorithm in order to minimize the rental cost machines.

2. Practical Situation

Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology.

Sometimes the priority of one job over the other is preferred. It may be because of urgency or demand of its relative importance, the job block criteria becomes important.

Another event which is mostly considered in the models is the break-down of machines. There may also be delays due to material, changes in release and tail dates, tools unavailability, failure of electric current, the shift pattern of the facility and fluctuations in processing times. All of these events complicate the scheduling problem in most cases. Hence the criterion of break-down interval becomes significant.

3. Notations

S : Sequence of jobs 1,2,3,….,n
M_j : Machine j, j= 1,2,…….
A_i : Processing time of i^{th} job on machine A.
B_i : Processing time of i^{th} job on machine B.
$A_i^{'}$: Expected processing time of i^{th} job on machine A.
$B_i^{'}$: Expected processing time of i^{th} job on machine B.
p_i : Probability associated to the processing time A_i of i^{th} job on machine A.
q_i : Probability associated to the processing time B_i of i^{th} job on machine B.
β : Equivalent job for job – block.
L : Length of the break-down interval.

$A_i^{''}$: Expected processing time of i^{th} job after break-down effect on machine A .

$B_i^{''}$: Expected processing time of i^{th} job after break-down effect on machine B.

S_i : Sequence obtained from Johnson's procedure to minimize rental cost.

C_j : Rental cost per unit time of machine j.

U_i : Utilization time of B (2^{nd} machine) for each sequence S_i

$t_1(S_i)$: Completion time of last job of sequence S_i on machine A.

$t_2(S_i)$: Completion time of last job of sequence S_i on machine B.

$R(S_i)$: Total rental cost for sequence S_i of all machines.

$CT(S_i)$:Completion time of 1^{st} job of each sequence S_i on machine A.

4. Assumptions

1. We assume rental policy that all the machines are taken on rent as and when they are required and are returned as when they are no longer required for processing. Under this policy second machine is taken on rent at time when first job completes its processing on first machine. Therefore idle time of second machine for first job is zero.
2. Jobs are independent to each other.
3. Machine break-down interval is deterministic, .i.e. the break-down intervals are well known in advance. This simplifies the problem by ignoring the stochastic cases where the break-down interval is random.
4. Pre- emption is not allowed, .i.e. once a job started on a machine, the process on that machine can't be stopped unless the job is completed.

5. Definitions

Definition 1: An operation is defined as a specific job on a particular machine.

Definition 2: Sum of idle time of M_2 (for all jobs)

$$\sum_{i=1}^{n} I_{i2} = \max\left[\left(\sum_{i=1}^{n} A_i^{'} - \sum_{i=1}^{n-1} B_i^{'}\right), \left(\sum_{i=1}^{n-1} A_i^{'} - \sum_{i=1}^{n-2} B_i^{'}\right), \left(\sum_{i=1}^{n-2} A_i^{'} - \sum_{i=1}^{n-3} B_i^{'}\right), \ldots\ldots, \left(\sum_{i=1}^{2} A_i^{'} - \sum_{i=1}^{2-1} B_i^{'}\right), A_i^{'}\right]$$

$$= \max\left[P_1, P_2, P_3, P_4, ----------- P_2, P_1\right]$$

$= \max\limits_{1\le k\le n}[P_k]$, where $P_K = \sum_{i=1}^{k} A^{'}_i - \sum_{i=1}^{k-1} B^{'}_i$ and $A^{'}_i = A_i\, p_i$, $B^{'}_i = B_i\, q_i$.

Definition 3: Total elapsed time for a given sequence.

= Sum of expected processing time on 2 nd machine (M_2) + Total idle time on M_2

$$= \sum_{i=1}^{n} B'_i + \sum_{i=1}^{n} I_{i2} = \sum_{i=1}^{n} B'_i + \max_{1\le k\le n}\left[P_k\right], \text{ where } P_K = \sum_{i=1}^{k} A'_i - \sum_{i=1}^{k-1} B'_i .$$

Note 1: Break-down time interval (a, b) for which the machines remain unavailable is already known to us, that is deterministic in nature. The break-down interval length L = b - a is known**.**

Note 2: Idle time of 1 st machine is always zero i.e. $\sum_{i=1}^{n} I_{i1} = 0$.

Note 3: Idle time of 1 st job on 2 nd machine $\left(I_{i2}\right)$

= Expected processing time of 1st job on machine = A'_i.

Note 4: Rental cost of machines will be minimum if idle time of 2nd machine is minimum.

6. Algorithm

Based on the equivalent job block theorem by *Maggu & Das* [10] and by considering the effect of break-down interval (a ,b) on different jobs, the algorithm which minimize the total rental cost of machines under specified rental policy with the minimum makespan can be depicted as below**:**

Step 1: Define expected processing time A'_i & B'_i on machine A & B respectively as follows:

$A'_i = A_i \times p_i$

$B'_i = B_i \times q_i$

Step 2: Define expected processing time of job block β = (k ,m) on machine A & B using equivalent job block given by *Maggu & Das* [10] i.e. find A'_β and B'_β as follows:

$$A'_\beta = A'_k + A'_m - \min(B'_k, A'_m)$$

$$B'_\beta = B'_k + B'_m - \min(B'_k, A'_m)$$

Step 3: Using *Johnson's* two machine algorithm [3] obtain the sequence S, while minimize the total elapsed time.

Step 4: Prepare a flow time table for the sequence obtained in step 3 and read the effect of break-down interval (a ,b) on different jobs on the lines of *Singh T.P.* [5]

Step 5: Form a reduced problem with processing times $A_i^{''}$ and $B_i^{''}$.
If the break-down interval (a, b) has effect on job i then
$A_i^{''} = A_i^{'} + L, \quad B_i^{''} = B_i^{'} + L,$ where L = b – a, the length of break-down interval.
If the break-down interval (a, b) has no effect on job i then

$A_i^{''} = A_i^{'}, \; B_i^{''} = B_i^{'}$

Step 6: Find the processing times $A_\beta^{''}$ and $B_\beta^{''}$ of job-block $\beta(k,m)$ on machine A and B using equivalent job-block β as in step 2.

Step 7: Now repeat the procedure to get the sequence S_i, using Johnson's two machine algorithm as in step 3.

Step 8: Observe the processing time of 1 st job of S_1 on the first machine A. Let it be α.

Step 9: Obtain all the jobs having processing time on A greater than α. Put these job one by one in the 1 st position of the sequence S_1 in the same order. Let these sequences be S_2, S_3, S_4 ,……,S_r.

Step 10: Prepare in-out flow table only for those sequence S_i (i=1,2,…r) which have job block β(k, m) and evaluate total completion time of last job of each sequence, .i.e. $t_1(S_i)$ & $t_2(S_i)$ on machine A & B respectively.

Step 11: Evaluate completion time CT (S_i) of 1 st job of each of above selected sequence S_i on machine A.

Step 12: Calculate utilization time U_i of 2 nd machine for each of above selected sequence S_i as:

$U_i = t_2(S_i) - CT(S_i)$ for i=1, 2 , 3,…r.

Step 13: Find Min {U_i}, i=1, 2, …r. let it be corresponding to i = m, then S_m is the optimal sequence for minimum rental cost.
Min rental cost = $t_1(S_m) \times C_1 + U_m \times C_2$,
where C_1 & C_2 are the rental cost per unit time of 1 st & 2 nd machines respectively.

7. Numerical Iilustration

Consider 5 jobs and 2 machines problem to minimize the rental cost. The processing times with their respective associated probabilities are given as follows:

Jobs	Machine M_1		Machine M_2	
i	A_i	p_i	B_i	q_i
1	11	0.1	8	0.2
2	15	0.3	11	0.2
3	14	0.1	15	0.1
4	17	0.2	16	0.2
5	12	0.3	18	0.3

Rental costs per unit time for machines M_1 & M_2 are 16 and 14 units respectively, and jobs (2, 5) are to be processed as an equivalent group job. Also given that the break-down interval is (5,10).

Solution

Step 1: The expected processing times A_i' and B_i' on machine A and B are as under:

(Tableau – 1)

Jobs	A_i'	B_i'
1	1.1	1.6
2	4.5	2.2
3	1.4	1.5
4	3.4	3.2
5	3.6	5.4

Step 2: The processing times of equivalent job block β = (2,5) by using *Maggu* and *Das* [10] criteria are given by

$A_\beta' = 4.5 + 3.6 - 2.2 = 5.9$ and $B_\beta' = 2.2 + 5.4 - 2.2 = 5.4$

(Tableau – 2)

Jobs	A_i'	B_i'
1	1.1	1.6
β	5.9	5.4
3	1.4	1.5
4	3.4	3.2

Step 3: Using *Johnson's* two machines algorithm, the optimal sequence is

S = 1, 3, β, 4, i.e. S = 1, 3, 2, 5, 4.

Step 4: The in-out flow table for the sequence S = 1- 3- 2- 5- 4 is as follows

(Tableau – 3)

Jobs	A	B
	In-Out	In-Out
1	0.0- 1.1	1.1 – 2.7
3	1.1 – 2.5	2.7 – 4.2
2	2.5 – 6.9	6.9 – 9.1
5	6.9 – 10.5	10.5 – 15.9
4	10.5 – 13.9	15.9 – 19.1

Step 5: On considering the effect of break-down interval (5, 10), the revised processing times A_i'' and B_i'' of machines A and B are as follows:

(Tableau – 4)

Jobs	A_i''	B_i''
1	1.1	1.6
2	9.5	7.2
3	1.4	1.5
4	3.4	3.2
5	8.6	5.4

Step 6: The new processing times of equivalent job block β = (2,5) by using *Maggu* and *Das* [10] criteria are given by

$A_\beta'' = 9.5 + 8.6 - 7.2 = 10.9$ and $B_\beta'' = 7.2 + 5.4 - 7.2 = 5.4$

(Tableau – 5)

Jobs	A_i''	B_i''
1	1.1	1.6
β	10.9	5.4
3	1.4	1.5
4	3.4	3.2

Step 7: Using *Johnson's* two machines algorithm, the optimal sequence is S_1= 1, 3, β, 4 .i.e. S_1= 1 – 3 – 2 – 5 – 4

Step 8: The processing time of 1st job on S_1 = 1.1, .i.e. α = 1.1

Step 9: The other optimal sequences for minimizing rental cost are

S_2= 2 – 1 – 3 – 5 – 4

$S_3 = 3 – 1 – 2 – 5 – 4$
$S_4 = 4 – 1 – 3 – 2 – 5$
$S_5 = 5 – 1 – 3 – 2 – 4$

Step 10: The in-out flow tables for sequences S_1, S_3 and S_4 having job block (2, 5) are as follows:

For $S_1 = 1 – 3 – 2 – 5 – 4$

(Tableau – 6)

Jobs	A	B
	In-Out	In-Out
1	0.0- 1.1	1.1 – 2.7
3	1.1 – 2.5	2.7 – 4.2
2	2.5 – 12.0	12.0 – 19.2
5	12.0 – 20.6	20.6 – 26.0
4	20.6 – 24.0	26.0 – 29.2

Total time elapsed on machine A = $t_1(S_1) = 24.0$
Total time elapsed on machine B = $t_2(S_1) = 29.2$
Utilization time of 2nd machine (B)= $U_1 = 29.2 – 1.1 = 28.1$.
For $S_3 = 3 – 1 – 2 – 5 – 4$

(Tableau – 7)

Jobs	A	B
	In-Out	In-Out
3	0.0- 1.4	1.4 – 2.9
1	1.4 – 2.5	2.9 – 4.5
2	2.5 – 12.0	12.0 – 19.2
5	12.0 – 20.6	20.6 – 26.0
4	20.6 – 24.0	26.0 – 29.2

Total time elapsed on machine A = $t_1(S_3) = 24.0$
Total time elapsed on machine B = $t_2(S_3) = 29.2$
Utilization time of 2nd machine (B)= $U_2 = 29.2 – 1.4 = 27.8$.
For $S_4 = 4 – 1 – 3 – 2 – 5$

(Tableau – 8)

Jobs	A	B
	In-Out	In-Out
4	0.0- 3.4	3.4 – 6.6
1	3.4 – 4.5	6.6 – 8.2
3	4.5 – 5.9	8.2 – 9.7
2	5.9 – 15.4	15.4 – 22.6
5	15.4 – 24.0	24.0 – 29.4

Total time elapsed on machine A = $t_1(S_4)$ = 24.0

Total time elapsed on machine B = $t_2(S_4)$ = 29.4

Utilization time of 2nd machine (B)= U_3 = 29.4 – 3.4 = 26.0

The total utilization of machine A is fixed 24.0 units and minimum utilization of B is 26.0 units for the sequence S_4. Therefore the optimal sequence is **S_4 = 4 – 1 – 3 – 2 – 5**.

Therefore minimum rental cost is = 24.0 x 16 + 26.0 x 14 = 748 units.

Remarks: In case the break-down interval criteria is not taken into consideration then result tally with T. P. Singh and Gupta Deepak [13].

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