ISSN 0974 - 9373

Vol. 15(2011) Special Issue 1 **Journal of International Academy of Physical Sciences** pp. 109-118

# Bioconvection in Porous Media: The Effect of Small Particles*

**Virendra Kumar and Y. D. Sharma**
Department of Mathematics
National Institute of Technology, Hamirpur- 177005, India
Email: veeru.nith@gmail.com, ydsnith@gmail.com

(Received June 14, 2011)

**Abstract:** A linear stability analysis is performed to investigate the effect of small particles on the stability of a suspension of motile gyrotactic microorganisms in a horizontal porous fluid layer. Small solid particles which are heavier than water are added into the fluid layer when bioconvection has already attained its steady state. If bioconvection develops, it enhances mixing and slows down the settling of particles. This problem may be relevant to a number of applications in bio-engineering such as microbial enhanced oil Recovery (MEOR), a biological based technology used to improve the recovery of oil entrapped in porous media.

**Key Words:** Bioconvection, Gyrotactic microorganisms, Small particles, Linear stability.

**2010 MS Classification No.**: 76E15

## 1. Introduction

The phenomena of bioconvection has been refer to a pattern forming convective motion of fluid, caused by upwardly swimming microorganisms whose average density is slightly larger than that of water[1,2]. Gyrotaxis is behavior typical for algal suspensions. when such organisms are in a flow field with a horizontal component of vorticity; their swimming is direction determined by the balance between the torques due to viscous drag arising from shear flow and gravitational toque acting on the microorganisms [3-5]. Experimental studies about the development of bioconvection plumes in algal suspensions have been investigated by Kessler[6]. Ghorai and Hill[7-10] examined the stability and structure of a single plume in a chamber with either stress-free side walls or periodic side walls. Geng & Kuznetsov[11] investigated the effect of small particles on the developments of bioconvection plumes and found that particles affect the system and is the origin of transition of bioconvection plume to a different steady state. In the

*Paper presented in CONIAPS XIII at UPES, Dehradun during June 14-16, 2011.

present article we analyze the effect of porous medium on the problem studied by Kuznetsov and Avramenko[12], in which the effect of small solid particles on the stability of gyrotactic microorganisms in a horizontal fluid layer was investigated. The particles we considered in the present problem are assumed to be heavier then ambient water having diameter about 1 micrometer or less so that compete between gravitational settling and Brownian diffusion may take place. It is reported that due to porous medium the system becomes less stable and smaller particles have the strong destabilizing effect on the system as compare to the larger particles.

## 2. Mathematical formulation and Solution of the problem

An infinite horizontal shallow fluid layer of thickness $H$ is considered. Cartesian axes coordinate system with $z$-axis in vertical direction is utilized. For developing a model of bioconvection in a porous medium, it suggested that the medium is sufficiently porous to allow the swimming and falling microorganisms to penetrate throw it. Also, it is assumed that porous matrix does not absorb microorganisms; so that, their gyrotactic behavior is not affected by the presence of the porous matrix[1]. Governing equations for this system can be obtained by the volume-averaged equations for a horizontal fluid layer in a suspension of gyrotactic microorganisms in the presence of small particles developed in Kuznetsov and Avramenko[12]. Whitaker[13] has given a detailed description about the volume- averaging procedure. This scheme results in the replacement of the Laplacian viscous term with the Darcian terms that describe the viscous resistance in porous medium. The linearized governing equations are:
Equation of continuity

(2.1) $$\nabla.\mathbf{q} = 0.$$

Equation of momentum balance

(2.2) $$\rho_{\mathrm{w}}\mathrm{c}_{\mathrm{a}}\left(\partial\mathbf{q}/\partial t\right) = -\nabla p - \left(\mu/K\right)\mathbf{q} + \left[n'\theta\Delta\rho + n'_p\theta_p\Delta\rho_p\right]\mathbf{g}.$$

Equation of Cell conservation for microorganisms

(2.3) $$\partial n/\partial t = -div\left(\mathbf{j}\right), \quad \mathbf{j} = n\mathbf{q} + n\mathrm{q}_{\mathrm{c}}\hat{\mathbf{p}} - D\nabla n.$$

Equation of Cell conservation for small particles

(2.4) $$\partial n_p/\partial t = -div\left(\mathbf{j}_{\mathbf{p}}\right), \quad \mathbf{j}_{\mathbf{p}} = n_p\mathbf{q} + n_p\mathrm{q}_{\mathrm{p}}\left(\mathbf{g}/|\mathbf{g}|\right) - D_p\nabla n_p,$$

where $D$ is the diffusivity of microorganisms (it is assumed that all random aspects of swimming of the microorganisms can be utilized by a diffusive process); $\mathrm{c_a}$ is the acceleration coefficient introduced by Nield and Bejan[14]; its value depends upon the porous medium, $K$ is the permeability of porous medium; $D_p$ is the diffusivity of the small particles due to the Brownian motion; $n$ is the number density of the microorganisms; $n_p$ is the number density of small solid particles; $p$ is the excess pressure over hydrostatic; $\hat{\mathbf{p}}$ is the unit vector indicating the direction of swimming of gyrotactic microorganisms; $\mathbf{q}=(u_1,u_2,u_3)$ is the fluid filtration velocity vector; $\rho_{\mathrm{w}}$ is the density of water; $\mathrm{q_c}\hat{\mathbf{p}}$ is the vector of microorganisms' average swimming velocity relative to the fluid ($\mathrm{q_c}$ is assumed to be constant); $\mathrm{q_p}(\mathbf{g}/|\mathbf{g}|)$ is the vector of particles' settling velocity relative to the fluid ($\mathrm{q}_p$ is assumed to be constant, the particles move straight downward); $\mathbf{g}$ is the gravitational acceleration; $t$ is the time; $\theta$ is the volume of microorganisms; $\theta_p$ is the volume of small particles; $\mu$ is the dynamic viscosity, assumed to be approximately the same as that of water; $\Delta\rho=\rho_{\mathrm{cell}}-\rho_{\mathrm{w}}$, is the density difference between cell and water; $\Delta\rho_p=\rho_p-\rho_{\mathrm{w}}$, is the density difference between particles and water. It suggested that particles do not interact with each other or with microorganisms. According to Stokes law[15] the settling velocity for spherical particles can be found as given by following expression

$$\mathrm{q_p}=\left\{\theta_p\Delta\rho_p g\right\}\Big/\left\{6\pi\mu\left(\sqrt[3]{\left(3\theta_p/4\pi\right)}\right)\right\}. \tag{2.5}$$

Equations (2.1)-(2.4) have to be solved subject to following boundary conditions. At the bottom as well as top surface of the layer, the following no-slip boundary conditions are satisfied:

$$\mathbf{q}=0,\quad \mathbf{j}.\hat{\mathbf{k}}=0,\quad \mathbf{j_p}.\hat{\mathbf{k}}=0,\qquad \text{at}\quad z=0,\ H. \tag{2.6}$$

The basic state is considered to be quiescent state and is given as follows:

$$\begin{cases} \mathbf{q}_{\mathrm{b}}=(0,0,0),\quad n_{p_0}(z)=\nu_p\exp\left(-\mathrm{q_p}z/D_p\right) \\ \qquad\qquad\Rightarrow \nu_p=\bar{n}_p Q_p\big/\exp\left(Q_p\right)-1,\ Q_p=\dfrac{-\mathrm{q_p H}}{D_p}, \\ n_0(z)=\nu\exp\left(\mathrm{q_c}z/D\right)\Rightarrow \nu=\bar{n}Q\big/\exp(Q)-1, Q=\mathrm{q_c H}/D. \end{cases} \tag{2.7}$$

The subscript'b' denotes the basic state. The integration constants, $\nu$ and $\nu_p$, represents the value of basic number densities of the microorganisms and particles, respectively, at the bottom of the layer. The parameters $Q$ and $Q_p$, are the bioconvection Peclet numbers for microorganisms and particles respectively and defined by. We introduce the following perturbations to the basic state:

$$\left[\overline{\mathbf{q}}, p, n, n_p, \hat{\mathbf{p}}\right] = \left[0, p_b(z), n_0(z), n_{p_0}(z), \hat{\mathbf{k}}\right] + \varepsilon\left[\overline{\mathbf{q}}, \overline{p}, \overline{n}, \overline{n}_p, \overline{\hat{\mathbf{p}}}\right] \tag{2.8}$$

where the bar denote the perturbation quantities and $\varepsilon$ is small perturbation amplitude. Substituting Eq. (2.8) into the system of Eqs. (2.1)-(2.4), linearizing the results, we obtain the following equations for perturbation quantities:

$$\nabla.\overline{\mathbf{q}} = 0. \tag{2.9}$$

$$\rho_{\mathrm{w}}\mathrm{c}_{\mathrm{a}}\left(\partial\overline{\mathbf{q}}/\partial t\right) = -\nabla\overline{p} - \left(\mu/K\right)\overline{\mathbf{q}} + \left[\overline{n}\theta\Delta\rho + \overline{n}_p\theta_p\Delta\rho_p\right]\mathbf{g}. \tag{2.10}$$

$$\partial\overline{n}/\partial t = -div\left[n_0\left(\overline{\mathbf{q}} + \mathrm{q_c}\overline{\hat{\mathbf{p}}}\right) + \overline{n}\mathrm{q_c}\hat{\mathbf{k}} - D\nabla\overline{n}\right]. \tag{2.11}$$

$$\partial\overline{n}_p/\partial t = -div\left[n_{p0}\overline{\mathbf{q}} + \overline{n}_p\mathrm{q_p}\left(\mathbf{g}/|\mathbf{g}|\right) - D_p\nabla\overline{n}_p\right]. \tag{2.12}$$

It is suggested that gyrotaxis nature of the microorganisms remains unaffected to the imposed small particles across the porous fluid layer[3]. Therefore, the perturbation to the unit vector indicating the cell swimming direction:

$$\overline{\hat{\mathbf{p}}} = \mathrm{B}\left(\eta, -\xi, 0\right), \tag{2.13}$$

where

$$\begin{cases} \xi = \left(1-\alpha_0\right)\partial\overline{u}_3/\partial y - \left(1+\alpha_0\right)\partial\overline{u}_2/\partial z, & \alpha_0 = a^2 - b^2/a^2 + b^2, \\ \eta = -\left(1-\alpha_0\right)\partial\overline{u}_3/\partial x + \left(1+\alpha_0\right)\partial\overline{u}_1/\partial z, & \mathrm{B} = \alpha_1\mu/2h\rho_{\mathrm{w}}g \end{cases} \tag{2.14}$$

where $a$ and $b$ are the semi-major and semi-minor axes of the spheroidal cell, therefore $\alpha_0$ is a measure of cell eccentricity: B is the "gyrotactic orientation parameter" introduced by Pedley and Kessler[2] and has dimensions of time; $\alpha_1$ is dimensionless constant relating viscous torques to the relative angular velocity of the cell; and $h$ is the displacement of centre of mass of the cell from the centre of buoyancy. Taking operator $\operatorname{curl}\operatorname{curl}$ on Eq. (2.10) and considering 3rd component only

$$\rho_{\mathrm{w}}\mathrm{c}_{\mathrm{a}}\left\{\partial\left(\nabla^2\overline{u}_3\right)/\partial t\right\}=-\left(\mu/K\right)\nabla^2\overline{u}_3-g\theta\Delta\rho\left(\partial^2\overline{n}/\partial x^2+\partial^2\overline{n}/\partial y^2\right)$$
$$-g\theta_p\Delta\rho_p\left(\partial^2\overline{n}_p/\partial x^2+\partial^2\overline{n}_p/\partial y^2\right). \tag{2.15}$$

Using (2.13) and (2.14) into (2.11) and (2.12), we have

$$\partial\overline{n}/\partial t=\mathrm{q_c}\mathrm{B}n_0\left\{\left(1-\alpha_0\right)\left(\partial^2\overline{u}_3/\partial x^2+\partial^2\overline{u}_3/\partial y^2\right)+\left(1+\alpha_0\right)\partial^2\overline{u}_3/\partial z^2\right\}$$
$$+D\nabla^2\overline{n}-\overline{u}_3\,\partial n_0/\partial z-\mathrm{q_c}\,\partial\overline{n}/\partial z, \tag{2.16}$$

$$\partial\overline{n}_p/\partial t=-u_3^{'}\,\partial n_{p_0}/\partial z-\mathrm{q_p}\,\partial\overline{n}_p/\partial z+D_p\nabla^2\overline{n}_p. \tag{2.17}$$

A normal mode expansion is introduced as follows:

$$\left[\overline{u}_3,\overline{n},\overline{n}_p\right]=\left[\overline{U}_3(z),\overline{N}(z),\overline{N}_p(z)\right]g(x,y)\exp(\sigma t), \tag{2.18}$$

where $\left(\partial^2/\partial x^2+\partial^2/\partial y^2\right)g+m^2g=0$, $'m'$ is the horizontal wave number (used as a separation constant). In the numerical monograph[4] of the suspension of gyrotactic microorganisms (with no small particles), the regions of overstability has been investigated and it is established that small gyrotactic number always corresponds to the monotonic instability. Therefore, it is concluded that the principal of exchange of stabilities[16] is valid for the present problem. Therefore $\sigma$ is set to zero and using Eq. (2.18), the linearized dimensionless perturbation equations for the amplitudes $W(z), N(z)$ and $N_p(z)$ are

$$\left[\left\{d^2\tilde{\mathrm{U}}_3(\tilde{z})/d\tilde{z}^2\right\}-\tilde{a}^2\tilde{\mathrm{U}}_3(\tilde{z})\right]-\tilde{a}^2\mathrm{Rb}\left\{\tilde{\mathrm{N}}(\tilde{z})+\Delta\tilde{\rho}\,\tilde{\mathrm{N}}_{\mathrm{p}}(\tilde{z})\right\}=0, \tag{2.19}$$

$$\left[\tilde{a}^2\tilde{\mathrm{N}}(\tilde{z})+Q\left\{d\tilde{\mathrm{N}}^{'}(\tilde{z})/d\tilde{z}\right\}-\left\{d^2\tilde{\mathrm{N}}^{''}(\tilde{z})/d\tilde{z}^2\right\}\right]+\tilde{\mathrm{U}}_3(\tilde{z})\left\{d\tilde{n}_0(\tilde{z})/d\tilde{z}\right\}$$
$$-\tilde{n}_0(\tilde{z})GQ\left[\left(1+\alpha_0\right)\left\{d^2\tilde{\mathrm{U}}_3(\tilde{z})/d\tilde{z}^2\right\}-\tilde{a}^2\left(1-\alpha_0\right)\tilde{\mathrm{U}}_3(\tilde{z})\right]=0 \tag{2.20}$$

$$\left[\tilde{a}^2\tilde{\mathrm{N}}_p(\tilde{z})+Q_p\left\{d\tilde{\mathrm{N}}_p^{'}(\tilde{z})/d\tilde{z}\right\}-\left\{d^2\tilde{\mathrm{N}}_p^{''}(\tilde{z})/d\tilde{z}^2\right\}\right]+\tilde{\mathrm{U}}_3(\tilde{z})\tilde{D}\left\{d\tilde{n}_{p_0}(\tilde{z})/d\tilde{z}\right\}=0, \tag{2.21}$$

where the following dimensionless variables and parameter are introduced:

$$\text{(2.22)}\quad \begin{cases} \tilde{z}=z/\mathrm{H}, \quad \tilde{a}=m\mathrm{H}, \quad \tilde{\mathrm{U}}_3=\nu\theta\mathrm{H}\bar{U}_3/D, \quad \tilde{\mathrm{N}}=\bar{N}\theta, \quad \tilde{\mathrm{N}}_p=\bar{N}_p\theta_p, \\ G=BD/\mathrm{H}^2, \tilde{D}=D/D_p, \ \mathrm{Rb}=\nu\theta g\Delta\rho\mathrm{H}^3/\mu D, \quad \tilde{n}_0=n_0/\nu, \\ \tilde{n}_{p_0}=n_{p_0}\theta_p/\nu\theta, \quad \Delta\tilde{\rho}=\Delta\rho_p/\Delta\rho, \quad \mathrm{Da}=K/\mathrm{H}^2. \end{cases}$$

Here, $\mathrm{Rb}$ is the bioconvection Rayleigh number, $\mathrm{Da}$ is the Darcy number and $G$ is the gyrotaxis number. Also we introduce the modified bioconvection Rayleigh-Darcy number as $\tilde{\mathrm{R}}\mathrm{b}=\mathrm{RbDa}$. Equations (2.19)-(2.21) must be solved subject to the following non-dimensionalized boundary conditions:

$$\text{(2.23) at } \tilde{z}=0,1 \ \ \mathrm{U}_3(\tilde{z})=0, \ Q\tilde{\mathrm{N}}(\tilde{z})=d\left(\tilde{\mathrm{N}}(\tilde{z})\right)/d\tilde{z}, Q_p\tilde{\mathrm{N}}_p(\tilde{z})=d\left(\tilde{\mathrm{N}}_p(\tilde{z})\right)/d\tilde{z}$$

To solve the above dimensionless system of equations (2.19)-(2.21), we employ single term Galerkin scheme[17]. The trail solutions satisfying the boundary conditions (2.23) are assumed as follows:

$$\text{(2.24)}\quad \begin{cases} \tilde{\mathrm{U}}_3(\tilde{z})=\tilde{z}-\tilde{z}^2, \ \ \tilde{\mathrm{N}}(\tilde{z})=2-Q(1-2\tilde{z})-Q^2\left(\tilde{z}-\tilde{z}^2\right) \\ \tilde{\mathrm{N}}_p(\tilde{z})=2-Q_p(1-2\tilde{z})-Q_p{}^2\left(\tilde{z}-\tilde{z}^2\right). \end{cases}$$

Using (2.24) into (2.19)-(2.21) and utilizing the standard Galerkin procedure[17], we get the system of equations involving the coefficients of $\tilde{\mathrm{U}}_3(\tilde{z}), \tilde{\mathrm{N}}(\tilde{z})$ and $\tilde{\mathrm{N}}_p(\tilde{z})$. The determinant of the coefficients of $\tilde{\mathrm{U}}_3(\tilde{z}), \tilde{\mathrm{N}}(\tilde{z})$ and $\tilde{\mathrm{N}}_p(\tilde{z})$ must vanish for the existence of non-trivial solutions and this determinant can be simplified in the following equation for the stability boundary

(2.25)

$$\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}=\min_{\bar{a}\geq 0}\left\{\frac{\left[\left\{\bar{a}^2\left(120-10Q_p{}^2+Q_p{}^4\right)+10Q_p{}^4\right\}\right]\left[\bar{a}^2\left(120-10Q^2+Q^4\right)+10Q^4\right]\left(10+\bar{a}^2\right)}{\left\{\begin{array}{l}\Delta\tilde{\rho}\Phi_1\left(Q_p\right)\left\{\bar{a}^2\left(120-10Q^2+Q^4\right)+10Q^4\right\}\left(10-Q_p{}^2\right) \\ \qquad +\Phi_2(Q)\left\{\bar{a}^2\left(120-10Q_p{}^2+Q_p{}^4\right)+10Q_p{}^4\right\}\left(10-Q^2\right)\end{array}\right\}}\right\}$$

where $\Phi_1\left(Q_p\right)=30\left[\left\{8\left(e^{Q_p}+1\right)/\left(e^{Q_p}-1\right)\right\}-\left(Q_p+16/Q_p\right)\right]\tilde{D}\bar{n}_p,$

$$\Phi_2(Q)=30\bar{n}\left[4QG\left(1+\alpha_0\right)+\left[\left\{8\left(e^Q+1\right)/\left(e^Q-1\right)\right\}-\left(Q+16/Q\right)\right]\left(1+\bar{a}^2\left(1-\alpha_0\right)G\right)\right].$$

For the case $\bar{n}_p = 0$, i.e., in the absence of small particles, equation (2.25) gives the following expression for the critical bioconvection Rayleigh number

$$(2.26)\quad \tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}} = \min_{\bar{a}\geq 0}\left[\left(\bar{a}^2+10\right)\left\{\bar{a}^2\left(120-10Q^2+Q^4\right)+10Q^4\right\}\Big/\left\{\bar{a}^2\left(10-Q^2\right)\Phi_2(Q)\right\}\right]$$

## 3. Results and Discussions

In this section we are discussing the numerical interpretation, different parameters have following values $\bar{n}=10, \Delta\tilde{\rho}=5, \alpha_0=0.3$. Along with the different pairs of the values of $\tilde{D}, Q, Q_p, G$ and $\hat{n}$, has been shown in Figs. (1-3). The dependence of critical modified bioconvection Rayleigh-Darcy number $\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}$ on the relative average number density ($\tilde{n}=\bar{n}_p/\bar{n}$) of small particles has been analyzed numerically and the results have been plotted graphically. In the absence of small particles ($\bar{n}_p=0$), from Eq. (2.26), it is predicted that suspension is most stable and the critical bioconvection Rayleigh number $\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}$ takes on its greatest value (92.02 approximately). Figure 1 depicts the dependence of $\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}$ on the relative average number density $\tilde{n}$, for different fixed values of gyrotaxis number. It is found that increasing $G$ the Raleigh number $\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}$, decreases rapidly which shows that, $G$ destabilizes the suspension. Figure 2 illustrates the effect of $\tilde{n}$ on the critical values of $\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}$ for various fixed values of bioconvection Peclet number $Q$ and it is observed that increasing $Q$ the modified bio-convection Rayleigh-Darcy number $\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}$ increases rapidly which shows that, $Q$ stabilizes the suspension. On adding the small particles to the suspension the decrease of $\bar{\mathrm{R}}\mathrm{b}_{cr}$ with increase of $\tilde{n}$ means that increasing the relative average number density across the porous layer containing small particles destabilizes the suspension and helps the development of bioconvection. An important conclusion is the effect of the diffusivity of small particles $D_p$. From equation (2.22), it is clear that if we increase $D_p$ by a factor of two then it results in decrease in $\tilde{D}$ and $Q_p$ by a factor of two. According to Einstein's relation that determines the diffusivity of small particles because of the Brownian motion, the diffusivity is inversely proportional to particle's radius, which means that the larger particles have smaller diffusivity. Therefore the smaller particles will destabilize the suspension better than larger particles. On adding the small particles having the larger diffusivity results in a faster decrease of the critical Rayleigh number. This analysis is valid when suspension is taken as dilute and particles are small so that gravitational settling and diffusion competes, otherwise the interaction between particles as well as particles and microorganisms must be considered. An interesting phenomenon occurs when values of $\tilde{n}$ changes from 0.1 to 0.2. It is observed that the convection develops in a suspension of buoyancy-neutral micro-organisms containing small particles due to unstable density stratification. Since physically negative values of $\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}$ is not possible, therefore for $\tilde{n} \in [0.1, 0.2]$, the values of $\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}$ considered in Figs. 1, 2 and 3 shows the mathematical features of solutions only. When $\tilde{n}$ exceeds 0.2 the suspension is destabilized asymptotically.

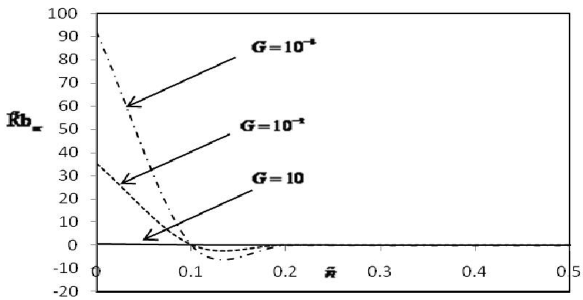


Figure 1: Effect of relative average number density of small particles $\left(\tilde{n} = \overline{n}_p / \overline{n}\right)$ on the critical bioconvection Rayleigh number, $\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}$ for different values of gyrotaxis number $G$

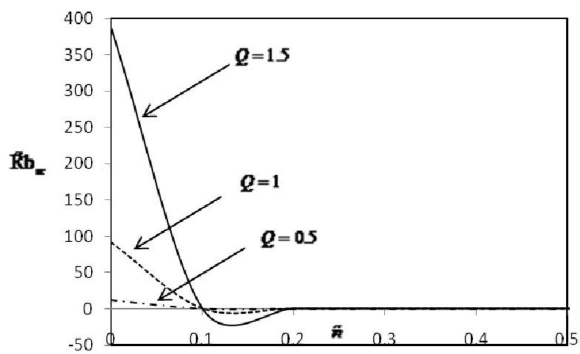


Figure 2: Effect of relative average number density of small particles $\left(\tilde{n} = \overline{n}_p / \overline{n}\right)$ on the critical bioconvection Rayleigh number, $\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}$ for different values of gyrotaxis number $Q$ .

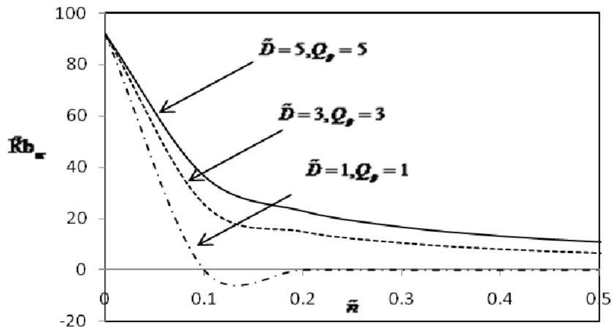


Figure 3: Effect of relative average number density of small particles $\left(\tilde{n}=\bar{n}_p/\bar{n}\right)$ on the critical bioconvection Rayleigh number, $\tilde{\mathrm{R}}\mathrm{b}_{\mathrm{cr}}$ for the different modes of $\tilde{D}$ and $Q_p$.

## 4. Conclusions

A linear stability analysis is applied to study the onset of bioconvection in a dilute suspension of gyrotactic microorganisms in a horizontal fluid layer saturating a porous medium in the presence of small particles. The system is solved analytically using Galerkin technique and the stability criterion depends upon the values of cell eccentricity, Gyrotactic number, measure of average diffusivity of small particles, bioconvection Peclet numbers for microorganisms and small particles respectively. The major conclusions of the present problem are listed as follows:

(i) The gyrotaxis number stands for the deviation of cell swimming direction strictly form vertical therefore the utilization of more gyrotactic species of microorganisms makes the suspension more unstable.

(ii) Since $\mathrm{Q}$ represents the ratio of the swimming speed of micro-organisms to the speed of bulk fluid flow therefore larger of $\mathrm{Q}$ corresponds to the rapid species of cells and it is reported that the suspension containing faster swimmers is more stable than a suspension of slower swimmers.

(iii) It found that the smaller particles will destabilize the suspension better than larger particles because the larger particles with small diffusivity concentrate near the bottom of the layer creating more stable density stratification and small particles with large diffusivity have almost uniform distribution across the layer.

## Acknowledgments

The authors are grateful to Prof. D. A. Nield, University of Auckland and Prof. A.V. Kuznetsov, North Carolina State University, Rayleigh, USA for providing us the necessary literature.

## References

1. J. O. Kessler, The external dynamics of swimming microorganisms, *Prog. Phycol. Res.*, **4** (1986) 257-307.
2. T. J. Pedley and J. O. Kessler, The orientation of spheroidal micro-organisms swimming in a flow field, *Proc. R. Soc. Lond.*, **B 231** (1987) 47–70.
3. T. J. Pedley, N. A. Hill and J. O. Kessler, The growth of bioconvection patterns in a uniform suspension of gyrotactic micro-organisms, *J. Fluid Mech.*, **195** (1988) 223–338.
4. N. A. Hill, T. J. Pedley and J. O. Kessler, Growth of bioconvection patterns in a suspension of gyrotactic microorganisms in a layer of finite depth, *J. Fluid Mech.*, **208** (1989) 509-543.
5. T. J. Pedley and J. O. Kessler, A new continuum model for suspensions of gyrotactic micro-organisms, *J. Fluid Mech.*, **212** (1990) 155–182.
6. J. O. Kessler, Hydrodynamic focusing of motile algal cells, *Nature*. **313** (1985) 218–220.
7. S. Ghorai, *Bioconvection and Plumes*, Ph.D. Thesis, University of Leeds, 1997.
8. S. Ghorai and N. A. Hill, Development and stability of gyrotactic plumes in bioconvection, *J. Fluid Mech.*, **400** (1999) 1–31.
9. S. Ghorai and N.A. Hill, Wavelengths of gyrotactic plumes in bioconvection, *Bull. Math. Biol.*, **62** (2000) 429–450.
10. S. Ghorai and N. A. Hill, Periodic arrays of gyrotactic plumes in bioconvection, *Phys. Fluids.*, **12** (2000) 5–22.
11. P. Geng and A.V. Kuznetsov, Settling of bidispersed small solid particles in a dilute suspension containing gyrotactic micro-organisms, *Int. J. Eng. Sc.*, **43** (2005) 992–1010.
12. A. V. Kuznetsov and A. A. Avramenko, Effect of small particles on the stability of bioconvection in a suspension of gyrotactic microorganisms in a layer of finite depth, *Int. Comm. Heat Mass Transfer*, **31** (2004) 1-10.
13. S. Whitaker, *The Method of Volume Averaging*, Kluwer Academic Publishers, Dordrecht, 1999.
14. D. A. Nield and A. Bejan, *Convection in Porous Media*, third ed. Springer, New York, 2006.
15. G. K. Batchelor, Sedimentation in a dilute polydisperse system of interacting spheres. Part 1. General theory, *J. Fluid Mech.*, **119** (1982) 379–408.
16. S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*, Oxford University Press, Oxford, 1961.
17. B. A. Finlayson, *The Method of Weighted Residuals and Variational Principles*, Academic Press, New York, 1972.