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Effect of Initial Stress and Magnetic Field on Shear Wave Propagation*

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Abstract: The study of shear-wave propagation is very important because of its extensive application in various branches of Science and Technology, like Geophysics, Plasma physics, Earthquake science and Optics. Therefore in this paper, the shear wave propagation in a non-homogeneous anisotropic incompressible stressed medium with magnetic field and gravity field has been studied. The dependence of velocity of wave propagation on the anisotropy, magnetic field, gravity field, non-homogeneity of the medium, and the initial stress have been analyzed. The frequency equation that determines the velocity of the shear wave has been obtained. The dispersion equations have been solved numerically and investigated for different cases. The results have been presented graphically.

Keyword: Incompressible, initial-stress, anisotropic, shear-wave.
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1. Introduction

The study of shear-wave propagation is very important because of its extensive application in various branches of Science and Technology, like Geophysics, Plasma physics, Earthquake science and Optics.

Shear waves propagating over the surface of homogeneous and inhomogeneous elastic half-spaces are a well known and prominent feature of wave theory. The materials may be considered as incompressible media and the velocities of longitudinal waves in them are very high. The varieties of hard rocks present in the earth are also almost incompressible. Due to the factors like external pressure, slow process of creep, difference in temperature, manufacturing processes, nitriding,

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pointing etc., the medium stay under high stresses. These stresses are regarded as initial stresses.

Earlier, many researchers have worked on shear-wave propagation in anisotropic media. Pal[1] has studied Generation of SH-type waves in layered anisotropic elastic media. Abd-Alla[2] studied the effect of initial stress and orthotropy on the propagation waves in a hollow cylinder. Garg[3] considered effect of initial stress on harmonic plane homogeneous waves in viscoelastic anisotropic media, Vecsey, et al.[4] investigated shear wave splitting as a diagnostic of variable anisotropic structure of the upper mantle beneath central Fennoscandia. Singh[5] studied wave propagation in a generalized thermoelastic material with voids. Willis and Movchan[6] discussed propagation of elastic energy in a general anisotropic medium. Zhang and Batra[7] wave propagation in functionally graded materials by modified smoothed particle hydrodynamic method. Zhu and Shi[8] discussed wave propagation in non-homogeneous magneto- electro-elastic hollow cylinders. Jiangong, et al.[9] studied wave propagation in non-homogeneous magneto-electro-elastic plates. Stuart and Sheila[10] investigated shear horizontal waves in transversely inhomogeneous plates. Rayleigh waves in a magnetoelastic initially stressed conducting medium with the gravity field are investigated by El-Naggare et al.[11]

In this paper, the shear wave propagation in a non-homogeneous anisotropic incompressible stressed medium with magnetic field and gravity field has been studied. The dependence of velocity of wave propagation on the anisotropy, magnetic field, gravity field, non-homogeneity of the medium, and the initial stress have been analyzed. The frequency equation that determines the velocity of the shear wave has been obtained.

2. Formulation of the Problem

We consider an unbounded incompressible anisotropic medium under initial stresses s_{11} and s_{22} along the x, y directions, respectively. When the medium is slightly disturbed, the incremental stresses s_{11}, s_{12} and s_{22} are developed, and the equations of motion in the incremental state become

$$\frac{\partial s_{11}}{\partial x}+\frac{\partial s_{12}}{\partial x}-P\frac{\partial \Omega}{\partial y}-\rho g\frac{\partial v}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}, \tag{2.1}$$

$$\frac{\partial s_{12}}{\partial x}+\frac{\partial s_{22}}{\partial y}-P\frac{\partial \Omega}{\partial x}+\rho g\frac{\partial u}{\partial x}+\mu_e H_0^2\left(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial x\partial y}\right)=\rho\frac{\partial^2 v}{\partial t^2}, \tag{2.2}$$

where μ_e is the magnetic permeability and H_0, the intensity of the uniform magnetic field, parallel to x-axes, also, s_{ij} is incremental stresses, Ω is the rotational component about z-axis, g is the acceleration due to gravity. The incremental stress-strain relation for an incompressible medium may be taken as

(2.3) $$s_{11} = 2Ne_{xx} + s,\ s_{22} = 2Ne_{yy} + s \ \text{ and } s_{12} = 2Qe_{xy}.$$

$s_{33} = s_{13} = s_{23}$, since the problem is treated in x-y plane where $s = (s_{11} + s_{22})/2$, e_{ij} is an incremental strain component, and N and Q are the rigidities of the medium. The incompressibility condition $e_{xx} + e_{yy} = 0$ is satisfied by

(2.4) $$u = -\frac{\partial\phi}{\partial y},\ v = \frac{\partial\phi}{\partial x}.$$

Substituting from equations (2.3) and (2.4) in equations (2.1) and (2.2), we get

(2.5) $$\frac{\partial s}{\partial x} - 2N\frac{\partial^3\phi}{\partial x^2\partial y} + \frac{\partial}{\partial y}\left[Q\left(\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\phi}{\partial y^2}\right)\right] - \frac{P}{2}\left(\frac{\partial^3\phi}{\partial x^2\partial y} + \frac{\partial^3\phi}{\partial y^3}\right) - \rho g\frac{\partial^2\phi}{\partial x^2} = -\rho\left(\frac{\partial^3\phi}{\partial t^2\partial y}\right),$$

(2.6) $$\frac{\partial s}{\partial y} + Q\left(\frac{\partial^3\phi}{\partial x^3} - \frac{\partial^3\phi}{\partial x\partial y^2}\right) + \frac{\partial}{\partial y}\left(2N\frac{\partial^2\phi}{\partial x\partial y}\right) - \frac{P}{2}\left(\frac{\partial^3\phi}{\partial x^3} + \frac{\partial^3\phi}{\partial x\partial y^2}\right) - \rho g\frac{\partial^2\phi}{\partial x\partial y}$$
$$+\mu_e H_0^2\left(\frac{\partial^3\phi}{\partial x^3} + \frac{\partial^3\phi}{\partial x\partial y^2}\right) = \rho\left(\frac{\partial^3\phi}{\partial t^2\partial x}\right).$$

Assuming non-homogeneities as

(2.7) $$\left.\begin{aligned} Q &= Q_0(1+ay) \\ N &= N_0(1+by) \\ \rho &= \rho_0(1+cy) \end{aligned}\right\}.$$

Substituting from equation (2.7) in equations (2.5) and (2.6), we get

(2.8) $$\frac{\partial^2 s}{\partial x\partial y} - \left[2N_0 b - 2aQ_0 + \rho_0 g(1+cy)\right]\frac{\partial^3\phi}{\partial x^2\partial y} - \left[2N_0(1+by) - Q_0(1+ay) + \frac{P}{2}\right]\frac{\partial^4\phi}{\partial x^2\partial y^2}$$
$$-2aQ_0\frac{\partial^3\phi}{\partial y^3} - \left[Q_0(1+ay) + \frac{P}{2}\right]\frac{\partial^4\phi}{\partial y^4} - \rho_0 cg\frac{\partial^2\phi}{\partial x^2} = -\rho_0(1+cy)\frac{\partial^4\phi}{\partial t^2\partial y^2} - \rho_0 c\frac{\partial^3\phi}{\partial t^2\partial y}$$

(2.9) $$\frac{\partial^2 s}{\partial x\partial y} + \left[Q_0(1+ay) - \frac{P}{2} + \mu_e H_0^2\right]\frac{\partial^4\phi}{\partial x^4} + \left[2N_0 b - \rho_0 g(1+cy)\right]\frac{\partial^3\phi}{\partial x^2\partial y}$$

$$+\begin{bmatrix} 2N_0\left(1+by\right)-Q_0\left(1+ay\right) \\ -\dfrac{P}{2}+\mu_e H_0^2 \end{bmatrix}\frac{\partial^4\phi}{\partial x^2\partial y^2}=\rho_0\left(1+cy\right)\frac{\partial^4\phi}{\partial t^2\partial x^2}$$

Eliminating s from equations (2.8) and (2.9), we get

$$\begin{aligned}(2.10)\quad &\left[Q_0\left(1+ay\right)-\frac{P}{2}+\mu_e H_0^2\right]\frac{\partial^4\phi}{\partial x^4}+\left[4N_0\left(1+by\right)-2Q_0\left(1+ay\right)-\frac{P}{2}+\mu_e H_0^2\right]\frac{\partial^4\phi}{\partial x^2\partial y^2}\\ &+\left[Q_0\left(1+ay\right)+\frac{P}{2}\right]\frac{\partial^4\phi}{\partial y^4}+2aQ_0\frac{\partial^3\phi}{\partial y^3}-\left[2aQ_0-4N_0 b\right]\frac{\partial^3\phi}{\partial x^2\partial y}+\rho_0 cg\frac{\partial^2\phi}{\partial x^2}\\ &=\rho_0\left(1+cy\right)\left[\frac{\partial^4\phi}{\partial t^2\partial y^2}+\frac{\partial^4\phi}{\partial t^2\partial x^2}\right]+\rho_0 c\frac{\partial^3\phi}{\partial t^2\partial y}.\end{aligned}$$

3. Solution of The Problem

For propagation of sinusoidal waves in any arbitrary direction, we take the solution of equation (2.10) as

$$(3.1)\qquad \phi\left(x,y,t\right)=Ae^{ik\left(x\cos\theta+y\sin\theta-c_1 t\right)},$$

where θ is the angle made by the direction of propagation with the x-axes, and c_1 and k are the velocity of propagation and wave number, respectively.

Using equation (3.1) in equation (2.10) and equating real and imaginary parts separately, we get

$$(3.2)\qquad \left(\frac{c_1}{\beta}\right)^2=\frac{1}{1+cy}\left\{\begin{aligned}&\left[\left(1+ay\right)-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0}\right]\cos^4\theta+\left[4\frac{N_0}{Q_0}\left(1+by\right)-2\left(1+ay\right)+\frac{\mu_e H_0^2}{Q_0}\right]\cos^2\theta\sin^2\theta\\ &+\left[\left(1+ay\right)+\frac{P}{2Q_0}\right]\sin^4\theta-\frac{cg}{k^2\beta^2}\cos^2\theta\end{aligned}\right\}$$

$$(3.3)\qquad \left(\frac{c_1}{\beta}\right)^2=\left[4\frac{N_0}{Q_0}\frac{b}{c}-\frac{2a}{c}\right]\cos^2\theta+2\frac{a}{c}\sin^2\theta$$

4. Analysis of Problem in Homogeneous Medium

(i) Analysis of equation (3.2) obtained by equating the real part of equations of motion:

Case I: In this case Q is homogeneous $(a \to 0)$ i.e., rigidity along vertical direction is constant

$$(4.1) \qquad \left(\frac{c_1}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\begin{array}{l}\left[1-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0}\right]\cos^4\theta + \left[\begin{array}{l} 4\frac{N_0}{Q_0}(1+by) \\ -2+\frac{\mu_e H_0^2}{Q_0}\end{array}\right]\cos^2\theta\sin^2\theta \\ +\left[1+\frac{P}{2Q_0}\right]\sin^4\theta - \frac{cg}{k^2\beta^2}\cos^2\theta \end{array}\right\}$$

The velocity along x-direction $(\cos\theta = 1, \sin\theta = 0, c_1 = c_{11})$ as

$$(4.2) \qquad \left(\frac{c_{11}}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\left[1-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0}\right]-\frac{cg}{k^2\beta^2}\right\}$$

$$(4.3) \qquad c_{11}^2 = \frac{1}{1+cy}\left\{\beta^2\left[1-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0}\right]-\frac{cg}{k^2}\right\},$$

which depends on the initial stress, gravity field and magnetic field. Similarly the velocity of propagation along y-direction $(\cos\theta = 0, \sin\theta = 1, c_1 = c_{22})$, is obtained as

$$(2.1) \qquad c_{22}^2 = \frac{\beta^2}{1+cy}\left[1+\frac{P}{2Q_0}\right].$$

Case II: In this case N is homogeneous $(b \to 0)$ i.e., rigidity along horizontal direction is constant

$$(2.2) \qquad \left(\frac{c_1}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\left[(1+ay)-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0}\right]\cos^4\theta + \left[4\frac{N_0}{Q_0}-2(1+ay)+\frac{\mu_e H_0^2}{Q_0}\right]\right.$$
$$\left.\cos^2\theta\sin^2\theta + \left[(1+ay)+\frac{P}{2Q_0}\right]\sin^4\theta - \frac{cg}{k^2\beta^2}\cos^2\theta\right\}.$$

The velocity along x-direction $(\cos\theta = 1, \sin\theta = 0, c_1 = c_{11})$ is given by

$$(2.3) \qquad c_{11}^2 = \frac{1}{1+cy}\left\{\beta^2\left[(1+ay)-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0}\right]-\frac{cg}{k^2}\right\},$$

which depends on depth y, gravity field and magnetic field. The velocity of propagation along y-direction $\left(\cos\theta = 0, \sin\theta = 1, c_1 = c_{22}\right)$, is given by

(2.4) $$c_{22}^2 = \frac{\beta^2}{1+cy}\left[\left(1+ay\right)+\frac{P}{2Q_0}\right].$$

For $P > 0$, the velocity along y-direction may increase considerably at a distance from free surface and the wave becomes dispersive.

Case III: In this case N, Q and ρ are homogeneous $\left(a \to 0, b \to 0, c \to 0\right)$

(2.5) $$\left(\frac{c_1}{\beta}\right)^2 = \left\{\begin{array}{l}\left[1-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0}\right]\cos^4\theta + \left[4\frac{N_0}{Q_0}-2+\frac{\mu_e H_0^2}{Q_0}\right]\cos^2\theta\sin^2\theta \\ +\left[1+\frac{P}{2Q_0}\right]\sin^4\theta\end{array}\right\}.$$

In the absence of initial stress the velocity equation becomes

(2.6) $$\left(\frac{c_1}{\beta}\right)^2 = \left\{\frac{\mu_e H_0^2}{Q_0}\cos^4\theta + 1 + \left(4\left(\frac{N_0}{Q_0}-1\right)+\frac{\mu_e H_0^2}{Q_0}\right)\right. \left.\left[4\frac{N_0}{Q_0}-2+\frac{\mu_e H_0^2}{Q_0}\right]\cos^2\theta\sin^2\theta\right\}.$$

In x-direction $\left(\cos\theta = 1, \sin\theta = 0, c_1 = c_{11}\right)$, the velocity is given by

(2.7) $$c_{11}^2 = \beta^2\left[1+\frac{\mu_e H_0^2}{Q_0}\right],$$

and in y-direction $\left(\cos\theta = 0, \sin\theta = 1, c_1 = c_{22}\right)$, the velocity is given by

(2.8) $$c_{22}^2 = \beta^2.$$

(ii) Analysis of equation (3.3) obtained by equating imaginary parts of equation of motion.

In absence the initial stress P in equation (3.3), following three cases have been analyzed.

Case I: In this case Q is homogeneous $(a \to 0)$ i.e., rigidity along vertical direction is constant

(2.9) $$\left(\frac{c_1}{\beta}\right)^2 = \left[4\frac{N_0}{Q_0}\frac{b}{c}\right]\cos^2\theta.$$

This shows that velocity of shear wave is always damped. The velocity of wave along x-direction $(\cos\theta = 1, \sin\theta = 0, c_1 = c_{11})$ is obtained as

(2.10) $$\left(\frac{c_{11}}{\beta}\right)^2 = \left[4\frac{N_0}{Q_0}\frac{b}{c}\right].$$

This shows that actual velocity in x-direction is damped by $(4N_0 b/Q_0 c)$, and no damping takes place along y-direction.

Case II: In this case N is homogeneous $(b \to 0)$, i.e., rigidity along horizontal direction is constant.

(2.11) $$\left(\frac{c_1}{\beta}\right)^2 = \left[-\frac{2a}{c}\right]\cos^2\theta + 2\frac{a}{c}\sin^2\theta.$$

The velocity of wave along x-direction $(\cos\theta = 1, \sin\theta = 0, c_1 = c_{11})$ is given by

(2.12) $$\left(\frac{c_{11}}{\beta}\right)^2 = \left[-\frac{2a}{c}\right].$$

The existence of negative sign shows that damping does not take place along x-direction for $(b \to 0)$. The velocity along y-direction is given by

(2.13) $$\left(\frac{c_{22}}{\beta}\right)^2 = 2\frac{a}{c}.$$

Indicating that a damping of magnitude $(2a/c)$ takes place along y -direction.

Case III: In this case N and Q are homogeneous $(a \to 0, b \to 0)$ but density is linearly varying with depth:

(2.14) $$\left(\frac{c_1}{\beta}\right)^2 = 0,$$

i.e. no damping takes place. Rather than perform the tedious analysis required in obtaining higher-order approximations in the manner outlined above.

5. Numerical Analysis and Discussion

We wish to investigate the variation of shear-waves velocity in a perfectly conducting medium under effect of magnetic field, initial stress and gravity field. To get numerical information on the velocity of shear wave in the nonhomogenous initially stressed medium we introduce the following non-dimensional parameters:

$$\bar{a} = \frac{a}{b}, \bar{b} = by, \ \bar{c} = \frac{c}{b}, \ \bar{c}_1 = \frac{c_1}{b}; \ \bar{N} = \frac{N}{Q_0}, \ \bar{P} = \frac{P}{2Q_0}, \ \bar{g} = \frac{g}{\rho c^2}, \ \bar{H} = \frac{H}{2Q_0}.$$

Using these parameters in the equation (3.2), we obtain

(3.1) $$\bar{c}_1^{\,2} = \frac{1}{1+\bar{c}\,\bar{b}}\left\{ \begin{array}{l} \left[\left(1+\bar{a}\,\bar{b}\right) - \bar{P} + \bar{H}\right]\cos^4\theta + \left[\begin{array}{l} 4\bar{N}\left(1+\bar{b}\right) \\ -2\left(1+\bar{a}\,\bar{b}\right) + \bar{H} \end{array}\right]\cos^2\theta\sin^2\theta \\ +\left[\left(1+\bar{a}\,\bar{b}\right) + \bar{P}\right]\sin^4\theta - \bar{c}\,\bar{g}\cos^2\theta \end{array} \right\}$$

Figure (1) shows the variation in velocities of shear wave in the direction of $\theta = 30^\circ$ with x-axis at different depth and different values of density parameter: $\bar{c} = 0.7, 0.8, 0.9, 1.0;$ taking $\bar{a} = 4.0; \bar{P} = 0.5; \bar{g} = 0.1 cm/\sec^2; \bar{N} = 2.5$ and $\bar{H} = 0.3$. The velocity of the shear-wave increases as depth increases and proportional to gravity field. Figure (2) shows the variation in velocities of shear-wave in the direction of $\theta = 30^\circ$ with x-axis at different depth and different values of rigidities parameter: $\bar{a} = 3.0, 3.5, 4.0, 4.5$ taking $\bar{c} = 0.8; \bar{P} = 0.5;$ $\bar{g} = 0.1 cm/\sec^2; \bar{N} = 2.5$ and $\bar{H} = 0.3$. The shear-wave velocity increases as depth increases. Figure (3) shows the variation in velocities of shear wave in the direction of $\theta = 30^\circ$ with x-axis at different depth and different values of $\bar{N}$ (anisotropy): $\bar{N} = 2, 2.5, 3.0, 3.5$ taking $\bar{c} = 0.8; \bar{P} = 0.5; \bar{g} = 0.1 cm/\sec^2; \bar{a} = 4$

and $\overline{H} = 0.3$. The shear-wave velocity increases as depth increases and Proportional to anisotropy. Figure (4) shows the variation in velocities of shear wave in the direction of $\theta = 30^\circ$ with x-axis at different depth and different values initial of stress parameter $\overline{P}$: $\overline{P} = -0.8, 0.0, 0.8, 1.6$ taking $\bar{c} = 0.8$; $\bar{g} = 0.1 cm/\sec^2$; $\bar{a} = 4$; $\overline{N} = 2.5$ and $\overline{H} = 0.3$. The velocity of the wave increases as depth increases and it is inversely proportional to initial stress. Figure (5) shows the variation in velocities of shear wave in the direction of $\theta = 30^\circ$ with x-axis at different depth and different values of magnetic parameter: $\overline{H} = 0.1, 0.5, 0.9, 1.3$ taking $\bar{c} = 0.8$; $\overline{P} = 0.5$; $\bar{g} = 0.1 cm/\sec^2$; $\overline{N} = 2.5$ and $\bar{a} = 4$. The velocity of the wave increases as depth increases and it is proportional to magnetic field.

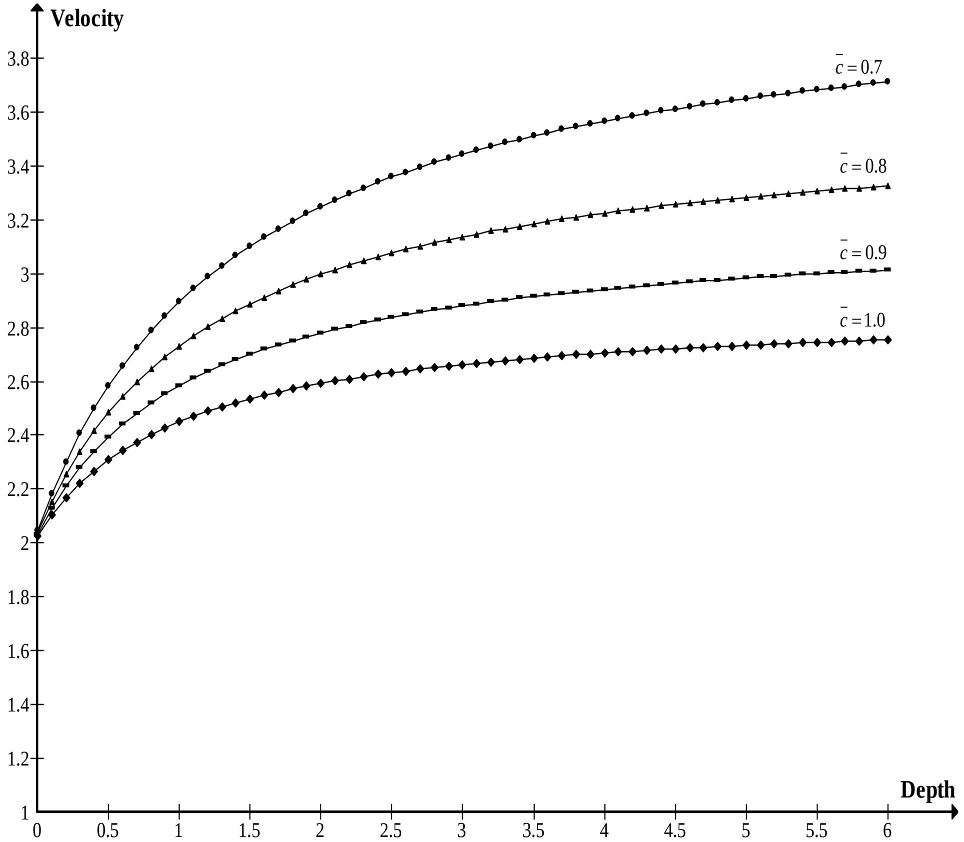

Figure1: Variation of velocities of shear-waves in the direction of $\theta = 30^\circ$ with x-axis at different depth and different values of density parameter : $\bar{c} = 0.7, 0.8, 0.9, 1.0$; taking $\bar{a} = 4.0$; $\overline{P} = 0.5$; $\bar{g} = 0.1 cm/\sec^2$; $\overline{N} = 2.5$ and $\overline{H} = 0.3$.

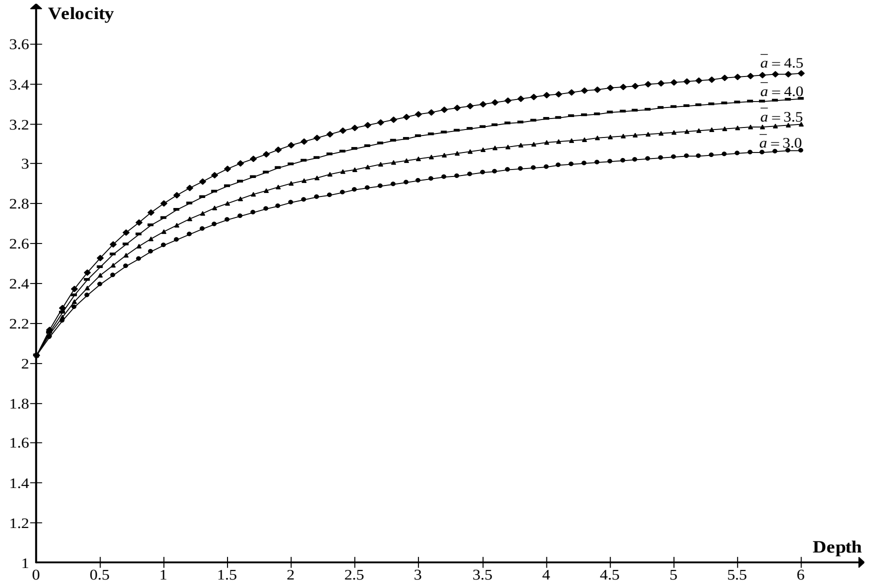

Figure2 : Variation of velocities of shear-waves in the direction of $\theta = 30^\circ$ with x -axis at different depth and different values of rigidities parameter : $\bar{a} = 3.0, 3.5, 4.0, 4.5$ taking $\bar{c} = 0.8; \bar{P} = 0.5; \bar{g} = 0.1 cm/\sec^2; \bar{N} = 2.5$ and $\bar{H} = 0.3$.

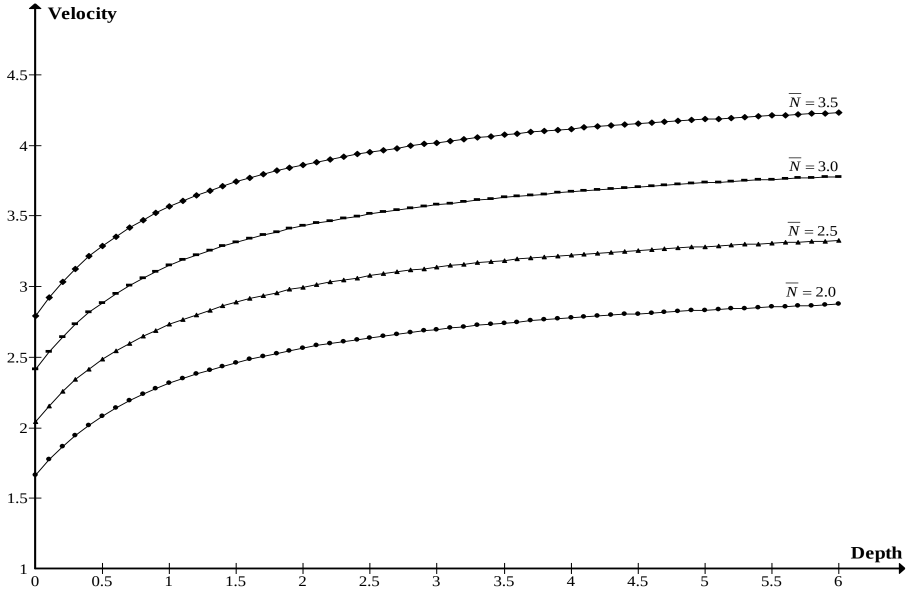

Figure3: Variation of velocities of shear-waves in the direction of $\theta = 30^\circ$ with x -axis at different depth and different values of $\bar{N}$ (anisotropy): $\bar{N} = 2, 2.5, 3.0, 3.5$ taking $\bar{c} = 0.8; \bar{P} = 0.5; \bar{g} = 0.1 cm/\sec^2; \bar{a} = 4$ and $\bar{H} = 0.3$.

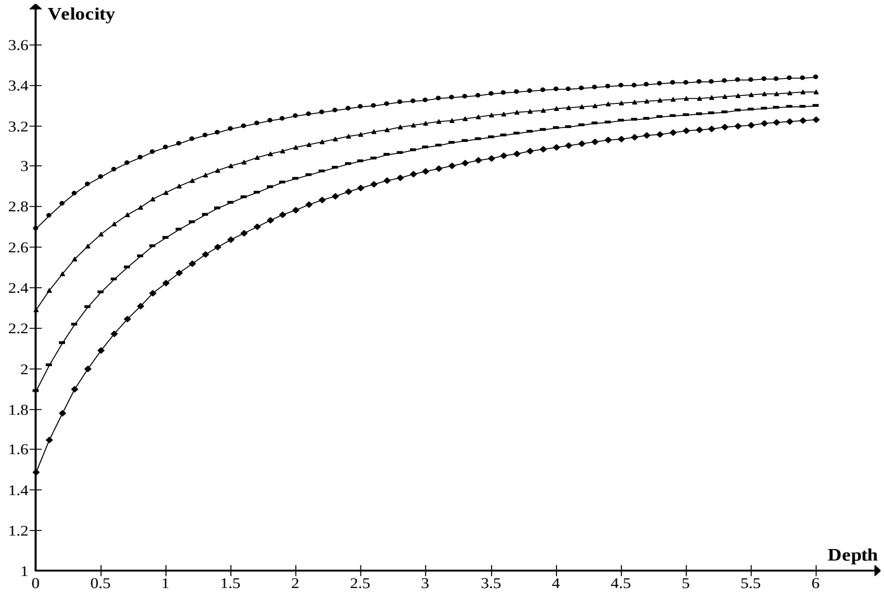

Figure4: Variation of velocities of shear-waves in the direction of $\theta = 30^\circ$ with x -axis at different depth and different values of stress parameter $\overline{P}$: $\overline{P} = -0.8, 0.0, 0.8, 1.6$ taking $\bar{c} = 0.8; \bar{g} = 0.1 cm/\sec^2; \bar{a} = 4; \overline{N} = 2.5$ and $\overline{H} = 0.3$

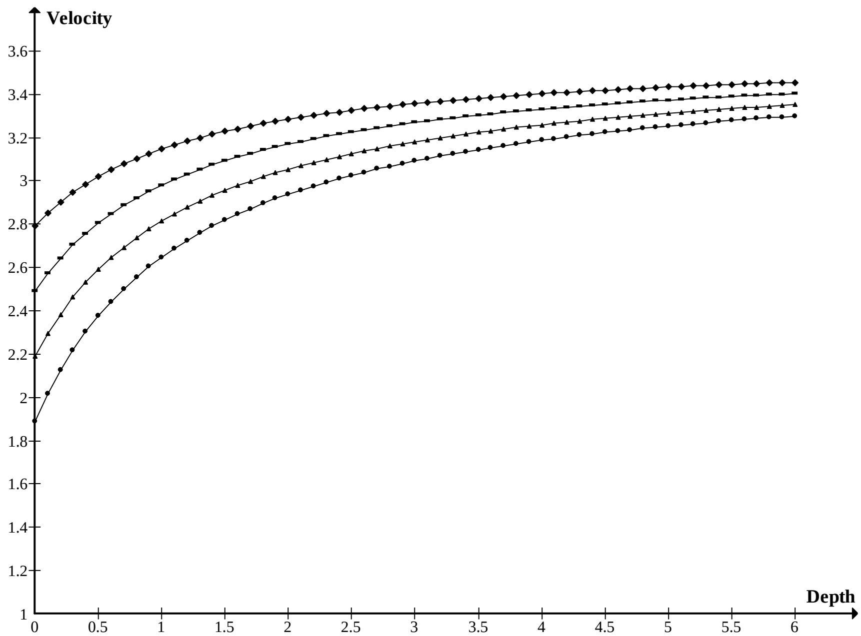

Figure5: Variation of velocities of shear-waves in the direction of $\theta = 30^\circ$ with x -axis at different depth and different values of initial stress medium: $\overline{H} = 0.1, 0.5, 0.9, 1.3$ taking $\bar{c} = 0.8; \overline{P} = 0.5; \bar{g} = 0.1 cm/\sec^2; \overline{N} = 2.5$ and $\bar{a} = 4$.

6. Conclusion

Thus we conclude that the anisotropy, magnetic field, gravity field, nonhomogeneity of the medium, the initial stress and the depth have considerable effect on the velocity of propagation of shear wave. The shear wave velocity is inversely proportional to the initial stress and it is proportional to anisotropy, magnetic field and gravity field.

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