ISSN 0974 - 9373

Vol.15(2011) Special Issue 1 **Journal of International Academy of Physical Sciences** pp. 47-57

Impact of Ignored Faults on Reliability and Availability of Centrifuge System That Undergoes Periodic Rest*

Rajeev Kumar and Pooja Bhatia
Department of Mathematics
M.D. University, Rohtak (Haryana)-124001 INDIA
Email: drrajeevkr@yahoo.co.in

(Received June 14, 2011)

Abstract The paper deals with a single unit centrifuge system that undergoes periodic rest wherein faults are classified as minor, ignored and major faults. The occurrence of a minor fault leads to degradation of the system whereas occurrence of a major fault leads to failure of the system. Ignored faults are the faults such as vibration, abnormal sound, etc that are ignored /delayed for repair during operation of the system until the system goes to rest or complete failure. However these faults may lead to failure of the system. The system undergoes periodic rest during rest period and upon complete failure of the system, the repairman first inspects whether the fault is repairable or non repairable and accordingly carries out repair or replacement of the system. Two repairman - an ordinary and other expert are considered. Various measures of system effectiveness such as mean sojourn times, mean time to system failure and steady-state availability of the system, are obtained using Markov processes and regenerative point technique. The conclusions regarding the reliability and availability of the system are drawn on the basis of the graphical studies.

Keyword: centrifuge system, ignored fault, mean time to system failure, steady-state availability, Markov process, regenerative point technique.
2010 Subject Classification No.: 68M15

1. Introduction

A large number of researchers[1-7] in the field of reliability modeling analyzed several systems considering various aspects such as different types of failure, repairs/ replacements policies, inspections, etc. In many practical situations, for instance, in thermal power plant for oil purification, in milk plants, laboratories, blood fractionation, wine clarification, etc. centrifuge systems are used and act as the main systems or sub-systems. The reliability and availability of centrifuge systems plays a very important and crucial role

*Paper presented in CONIAPS XIII at UPES, Dehradun during June 14-16, 2011.

these situations and hence need to be analyzed. In the literature of reliability not much work has been reported in this direction.

Recently R. Kumar and P. Bhatia[8] carried out reliability and availability analyses of a centrifuge system considering minor, ignored and major faults. In the paper it is assumed that a minor fault leads to down state while a major fault leads to complete failure of the system and the ignored faults are these minor faults that are not repair in time that may lead to complete failure of the system.

However practically it was observed a centrifuge system working in Thermal Power plant, Panipat (Haryana), while collecting data on faults/ failures and repairs, that a minor fault leads to degradation of the system whereas a major fault leads to complete failure of the system. Some faults such as vibration, abnormal sound, etc are ignored/ delayed for repair during the operation of the system until system goes to rest or to complete failure. These faults sometimes may lead to complete failure of the system and the system undergoes to periodic rest in regular intervals of time.

Keeping this in view, the present paper deals with a single unit centrifuge system considering above mentioned faults wherein a minor fault degrades the system whereas a major fault leads to complete failure of the system. The ignored fault is taken as the fault that may be ignored/delayed for repair during the operation of the system until system goes to rest or to complete failure. The system undergoes periodic rest. During the rest period and complete failure the repairman first inspect whether the fault is repairable or non repairable and accordingly carry out repair or replacement of the faulty components. Various measures of system effectiveness, such as mean sojourn time, mean time to system failure and steady- state availability are obtained using Markov processes and regenerative point technique. Various conclusions regarding the reliability and availability of the system on the basis of graphical analysis.

Other Assumptions

1. Faults are self- announcing.
2. There are two repairman one is ordinary and other is expert. The ordinary repairman handles minor faults and ignored faults whereas expert repairman will repair major faults only.
3. The repairman reaches the system in negligible time.
4. The system is as good as new after each repair/replacement.
5. Switching is perfect and instantaneous.
6. The failure time distributions are exponential while other time distributions are general.

Notations

λ_1	Rate of occurrence of a major fault
λ_2	Rate of occurrence of a minor fault
λ_3	Rate of occurrence of an ignored/delayed fault
a	Probability that a fault is non repairable
b	Probability that a fault is repairable
p	Probability that an ignored fault lead to complete failure
q	Probability that an ignored fault don't lead to complete failure
α	Rate at which the unit goes to rest
β	Rate at which the unit restarts after rest
i(t)/I(t)	p.d.f./c.d.f. of time to inspection of the unit
$\begin{cases} g_1(t)/G_1(t); \\ g_2(t)/G_2(t); \\ g_3(t)/G_3(t) \end{cases}$	p.d.f./ c.d.f. of times to repair the unit
$\begin{cases} h_1(t)/H_1(t); \\ h_2(t)/H_2(t) \end{cases}$	p.d.f./c.d.f. of times to replacement of the unit
k(t)/K(t)	p.d.f./c.d.f. of time to delay the ignored fault
O	Operative state
Or	Operative unit under repair
Od	Operative unit under ignored fault
R	Rest state
Rrp	Rest unit under replacement
Rr	Rest unit under repair
F_i	Failed unit under inspection
F_r	Failed unit under repair
F_{rp}	Failed unit under replacement
©	Symbol for Laplace Convolution

A diagram showing the various states of transition of the system is shown in Fig. 1. The epochs of entry in to state 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are regenerative point and thus all the states are regenerative states.

Transition Probabilities and Mean Sojourn Times

The transition probabilities are

$$dQ_{01}(t)=\lambda_1 e^{-(\lambda_1+\lambda_2+\lambda_3+\alpha)t}dt \qquad dQ_{02}(t)=\lambda_2 e^{-(\lambda_1+\lambda_2+\lambda_3+\alpha)t}dt$$

$$dQ_{03}(t)=\lambda_3 e^{-(\lambda_1+\lambda_2+\lambda_3+\alpha)t}dt \qquad dQ_{04}(t)=\alpha e^{-(\lambda_1+\lambda_2+\lambda_3+\alpha)t}dt$$

$$dQ_{15}(t)=ai_1(t)dt \qquad dQ_{16}(t)=bi_1(t)dt$$

$$dQ_{20}(t)=g_1(t)e^{-\lambda_3(t)}dt \qquad dQ_{27}(t)=\lambda_3 e^{-\lambda_3(t)}\overline{G_1}(t)dt$$

$$dQ_{31}(t)=pk(t)dt=dQ_{71}(t) \qquad dQ_{34}(t)=qk(t)dt=dQ_{74}(t)$$

$$dQ_{40}(t)=\beta e^{-\beta(t)}\overline{I_2}(t)dt \qquad dQ_{48}(t)=ai_2(t)e^{-\beta(t)}dt$$

$$dQ_{49}(t)=bi_2(t)e^{-\beta(t)}dt \qquad dQ_{50}(t)=h_1(t)dt$$

$$dQ_{84}(t)=h_2(t)dt \qquad dQ_{60}(t)=g_2(t)dt$$

$$dQ_{94}(t)=g_3(t)dt\ .$$

The non-zero elements p_{ij} obtained using $p_{ij}=\lim_{s\to 0} Q_{IJ}^*(s)$, are as under:

$$p_{01}=\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3+\alpha} \qquad p_{02}=\frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3+\alpha}$$

$$p_{03}=\frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3+\alpha} \qquad p_{04}=\frac{\alpha}{\lambda_1+\lambda_2+\lambda_3+\alpha}$$

$$p_{15}=ai_1^*(0) \qquad p_{16}=bi_1^*(0)$$

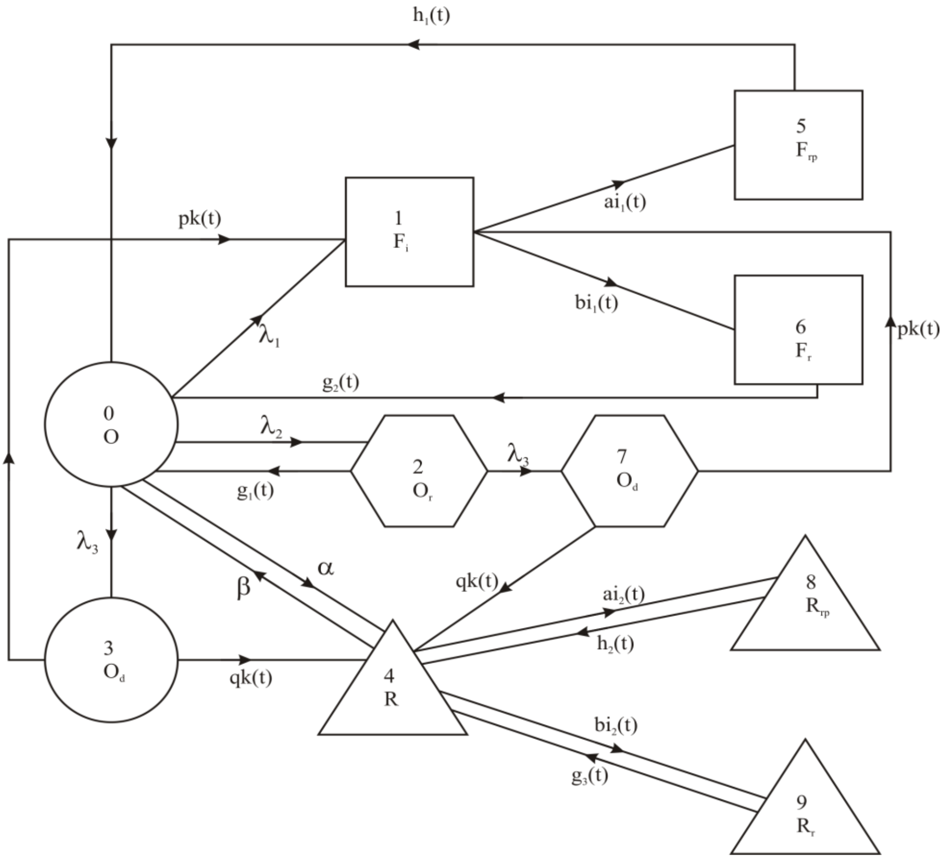
$$p_{27}=1-g_1^*(\lambda_3) \qquad p_{20}=g_1^*(\lambda_3)$$

$$p_{50}=h_1^*(0) \quad p_{60}=g_2^*(0) \qquad p_{84}=h_2^*(0)$$

$$p_{94}=g_3^*(0) \qquad p_{40}=1-i_2^*(\beta)$$

$$p_{48}=ai_2^*(\beta) \qquad p_{49}=bi_2^*(\beta)$$

$$p_{31}=pk^*(0)=p_{71} \qquad p_{34}=qk^*(0)=p_{74}$$



Operative State, Failed State, Degraded State, Rest State

Figure1 State Transition Diagram

By these transition probabilities, it can be verified that

$$p_{01}+p_{02}+p_{03}+p_{04} = p_{15}+p_{16} = p_{27}+p_{20} = p_{34}+p_{31}$$

$$= p_{40}+p_{48}+p_{49} = p_{74}+p_{71} = p_{50} = p_{60} = p_{84} = p_{94} = 1.$$

The mean sojourn time in the regenerative state i (μ_i) is defined as the time of stay in that state before transition to any other state then we have

$$\mu_0 = \frac{1}{\lambda_1+\lambda_2+\lambda_3+\alpha},\ \mu_1 = -i_1^{*\prime}(0),\ \mu_2 = \frac{1-g_1^*(\lambda_3)}{\lambda_3},$$

$$\mu_3 = -k^{*\prime}(0) = \mu_7,\ \mu_4 = \frac{1-i_2^*(\beta)}{\beta},\ \mu_5 = -h_1^{*\prime}(0),$$

$\mu_6 = -g_2^{*/}(0)$, $\mu_8 = -h_2^{*/}(0)$, $\mu_9 = -g_3^{*/}(0)$.

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as

$$m_{ij} = \int td\, Q_{ij}(t)$$

Thus,

$m_{01} + m_{02} + m_{03} + m_{04} = \mu_0$, $m_{15} + m_{16} = \mu_1$, $m_{27} + m_{20} = \mu_2$, $m_{34} + m_{31} = \mu_3$,

$m_{40} + m_{49} = \mu_4$, $m_{50} = \mu_5$, $m_{60} = \mu_6$, $m_{74} + m_{71} = \mu_7$, $m_{84} = \mu_8$, $m_{94} = \mu_9$.

Mean Time to System Failure

The expression for the mean time to system failure (MTSF) is obtained on taking the failed states of the system as absorbing states. By probabilistic arguments, we obtain the following recursive relations for $\phi_i(t)$, c. d. f of the first passage time from regenerative state *i* to failed state:

$$\phi_0(t) = Q_{01}(t) + Q_{02}(t) \quad \phi_2(t) + Q_{03}(t) \quad \phi_3(t) + Q_{04}(t)$$

$$\phi_2(t) = Q_{27}(t) \quad \phi_7(t) + Q_{20}(t) \quad \phi_0(t)$$

$$\phi_3(t) = Q_{34}(t) + Q_{31}(t),\ \phi_7(t) = Q_{71}(t) + Q_{74}(t).$$

Taking Laplace Stejling Transformatin of these equations and solving for $\phi_0^{**}(s)$, we obtain

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)},$$

The mean time to system failure when the system starts from the state 0, is

$$T_0 = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D},$$

where

$$N = \mu_0 + p_{02}\,\mu_2 + p_{02}\,p_{27}\,\mu_7 + \mu_3 p_{03}, \quad D = 1 - p_{02}p_{20}$$

Availability Analysis

Using the probabilistic arguments and the theory of regenerative processes, the availability $A_i(t)$, the probability that the system is up at instant t given that it entered regenerative state i at t=0, satisfies the following recursive relations

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}©A_2(t) + q_{03}(t)©A_3(t) + q_{04}©A_4(t)$$

$$A_1(t) = q_{15}©A_5(t) + q_{16}©A_6(t)$$

$$A_2(t) = M_2(t) + q_{27}©A_7(t) + q_{20}©A_0(t)$$

$$A_3(t) = M_3(t) + q_{34}©A_4(t) + q_{31}©A_1(t)$$

$$A_4(t) = q_{40}©A_0(t) + q_{48}©A_8(t) + q_{49}©A_9(t)$$

$$A_5(t) = q_{50}©A_0(t)$$

$$A_6(t) = q_{60}©A_0(t)$$

$$A_7(t) = M_7(t) + q_{71}©A_1(t) + q_{74}©A_4(t)$$

$$A_8(t) = q_{84}©A_4(t)$$

$$A_9(t) = q_{94}©A_4(t)$$

Here $M_0(t) = e^{-(\lambda_1+\lambda_2+\lambda_3+\alpha)t}$; $M_2(t) = \overline{G_1}(t)e^{-\lambda_3(t)}$;

$M_3(t) = \overline{K}(t) = M_7(t)$;

Taking Laplace transform of the above equations and solving for A_0**(s), we have

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)}.$$

The steady state availability of the system is given by

$$A_0 = \lim_{s\to 0} (s\, A_0^*(s)) = \frac{N_1}{D_1},$$

where $N_1 = p_{40}(\mu_0 + p_{02}\mu_2 + p_{03}\mu_3 + p_{02}p_{27}\mu_7)$.

$$\begin{aligned} D_1 = {} & p_{40}(\mu_0 + p_{20}\mu_2 + p_{03}\mu_3 + p_{02}p_{27}\mu_7) + (p_{01} + p_{03}p_{31} + p_{02}p_{27}p_{71})\times \\ & (\mu_1 + p_{15}\mu_5 + p_{16}\mu_6) + (p_{48}\mu_8 + p_{49}\mu_9 + \mu_4) \\ & (1 - p_{01} - p_{02}p_{20} - p_{03}p_{31} - p_{02}p_{27}p_{71}). \end{aligned}$$

Graphical Interpretation and Conclusions

For graphical analysis the following particular cases are considered:

$$g_1(t) = \beta_1 e^{-\beta_1(t)} \quad g_2(t) = \beta_2 e^{-\beta_2(t)} \quad g_3(t) = \beta_3 e^{-\beta_3(t)} \quad k(t) = \delta e^{-\delta(t)}$$

$$h_1(t) = \gamma_1 e^{-\gamma_1(t)} \quad h_2(t) = \gamma_2 e^{-\gamma_2(t)} \qquad i_1(t) = \alpha_1 e^{-\alpha_1(t)} \qquad i_2(t) = \alpha_2 e^{-\alpha_2(t)}$$

Various graphs are drawn for the MTSF and the steady-state availability (A_0) for the different values of the rates of occurrence of faults (λ_1, λ_2, λ_3), repairs (β_1, β_2, β_3), replacement (γ_1, γ_2), inspection (α_1,α_2) and delay (δ) on the basis of these plotted graphs.Following conditions are drawn.

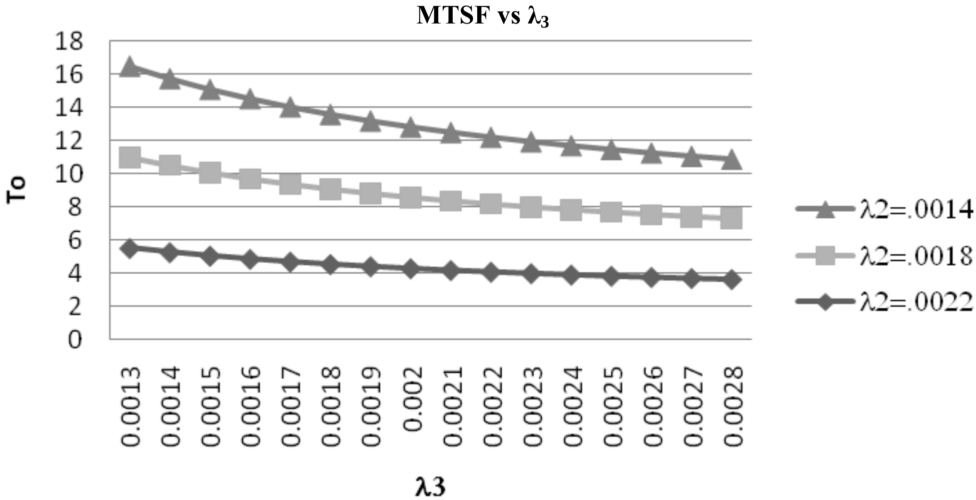


Figure 2

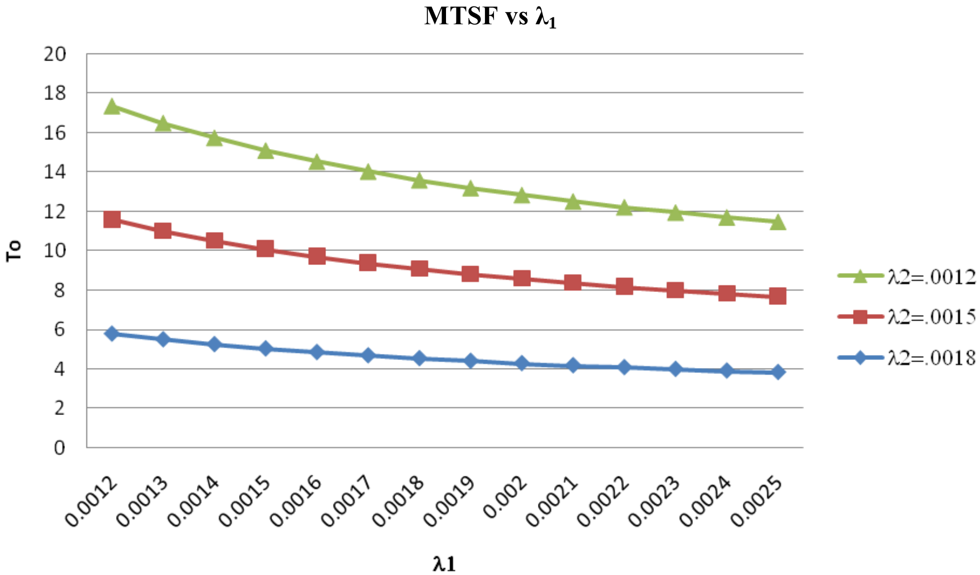


Figure 3

Mean Time to System Failure verses Rate of occurrence of Ignored Fault (λ_3) for different values of λ_2

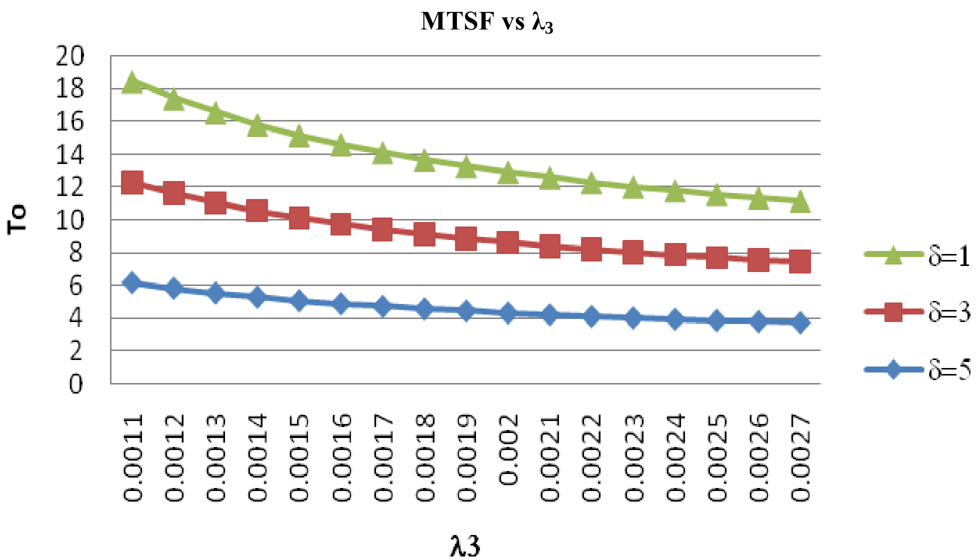


Figure 4

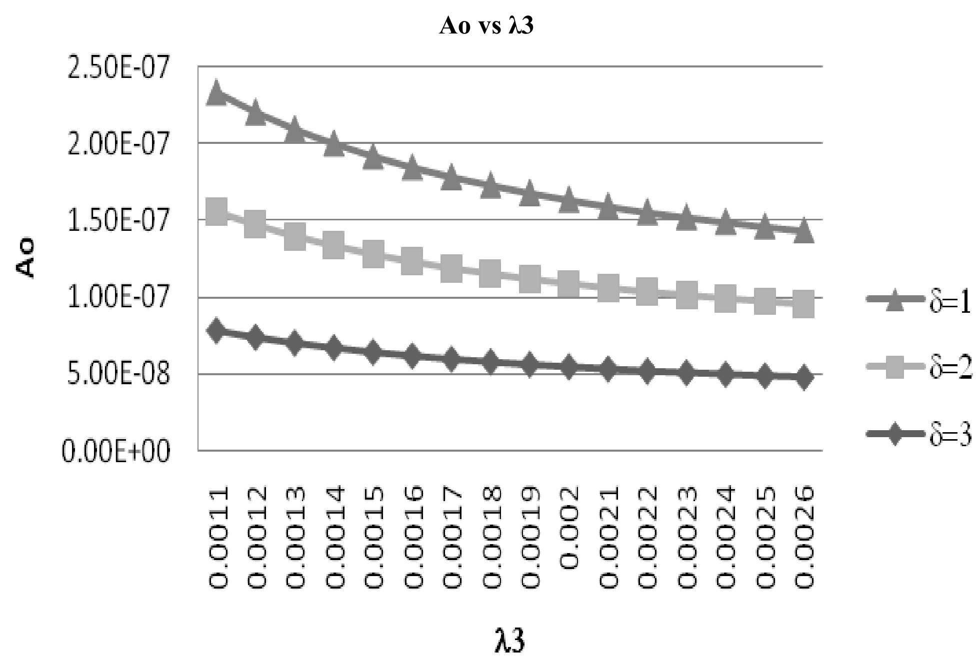


Figure 5

Availability verses Rate of occurrence of major fault (λ_1) for different values of λ_2

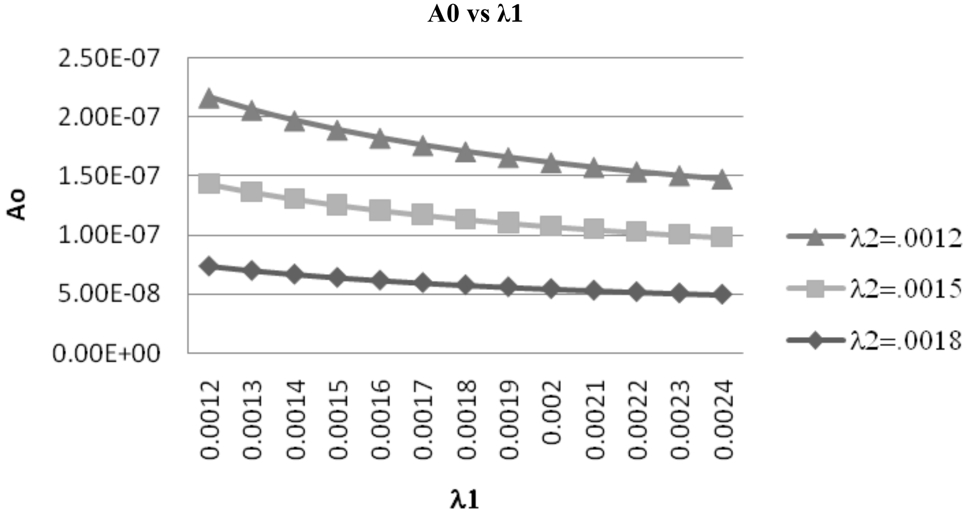


Figure 6

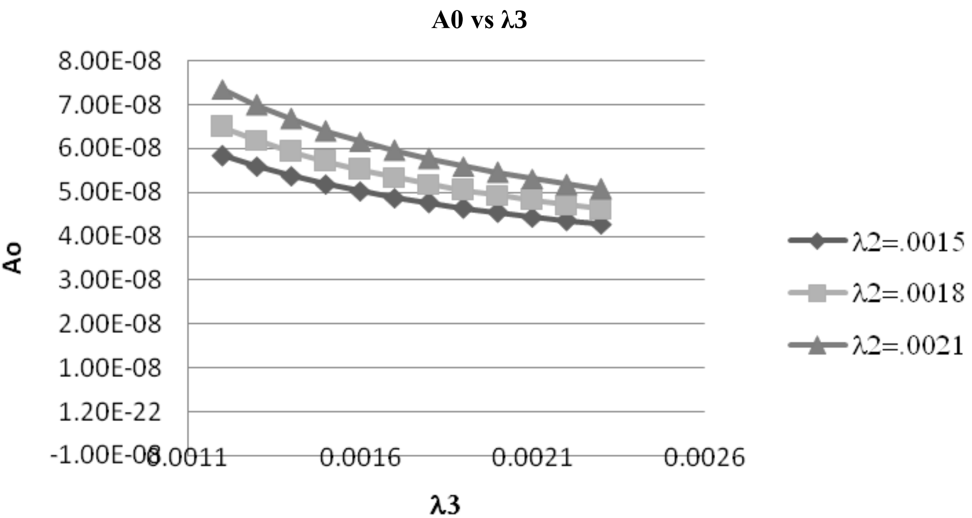


Figure 7

The graphs in Fig.2 to Fig.4 reveal that the mean time to system failure (MTSF) deceases with the increase in the values of the rate of occurrence of major fault (λ_1), minor fault (λ_2) and ignored fault (λ_3), respectively. Also MTSF decreases with the increase in delay in repair (δ) of the ignored fault

Thus we conclude that higher the rate of occurrence of major, minor and ignored faults in the centrifuge systems lesser is the reliability of the system. Also it has been observed through the graphs Fig.5 and Fig.7 that the availability of the system decreases with the increase in the values of the above mentioned faults.

Also it is evident that the availability decreases with the increase in delay time.

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