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Magnetohydrodynamic Flow of a Dusty Fluid between Two Inclined Parallel Plates*

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Abstract: In this paper, we have studied the MHD flow of a dusty viscous non Newtonian fluid flowing between two parallel inclined plates influenced by gravitational force. Within the frame work of some physically realistic approximation and suitable boundary conditions, derivations for velocity profile have been obtained by varying the magnetic field and inclination of the plates. The result shows that on increasing the magnetic field, the velocities of both phases increase in the beginning and taking a maximum value, it starts decreasing for every inclination of the plates. Similar results have also been obtained for different inclination of the plates and fixed magnetic field. The results may be applicable in such flows which occur in the industries concerning oil, gases, and molten metals flowing through inclined tubes.

Keywords: MHD flow, dusty viscous non Newtonian fluid, incompressible fluid

2000 Mathematics Subject Classification No.: 76W05

1. Introduction

The multiphase flow of non Newtonian fluids through different channels in the presence of transverse magnetic field has got remarkable applications in various fields' involving the flow of oil, gases, and molten metals. Due to growing use of these non Newtonian materials in many manufacturing and processing industries having these flows in inclined channels, we have studied the MHD flow of a dusty viscous non Newtonian fluid flowing between two parallel inclined plates influenced by gravity.

Following the equations given by Saffman[1] and others, Gourla and Katock[2] discussed the unsteady free convection of fluids through the vertical plates in the presence of uniform magnetic field. Ganguly and Lahri[3]

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examined the motion of an isothermal dusty viscous incompressible fluid between two infinite parallel plates where both plates were oscillating harmonically. Sharma and Kumar[4] investigated the unsteady flow of a non Newtonian fluid down an open inclined channel with respect to pressure gradient. Patidar and Purohit[5] studied free convection flow of a viscous incompressible fluid in porous medium between two long wavy walls. Conforoto[6] found an axi symmetric model of a dusty gas using the wave features and group analysis. Kannan and Ramurthy[7] discussed a two dimensional flow of a dusty assuming the particles as point sources. The motion induced by a normal oscillation of a wavy sinusoidal wall was calculated by the matching by the asymptotic expulsion technique. Volkov and Tsirkunov[8] derived basics dynamical equations for the kinetics of a poly dispersed admixture of solid particle in a dilute dusty gas flow by the method of statistical physics. He also discussed particle rotation and interaction in inelastic collision with the carrier gas on the basis of kinetic model. Ahmed and Sharma[9] investigated the flow of a non Newtonian fluid in helical pipe with an elliptical cross section. Attia[10] discussed the Hall Effect on the transient flow of a dusty Bingham fluid in circular pipe. Rao and Radhakrishnamacharya[11] studied the flow of magnetic fluid of through a non uniform wavy tube. Siddabasappa et al[12] discussed the flow of a viscous dusty fluid where velocity of the dust particle was everywhere parallel to that of the fluid and analyzed the variation of pressure on it. Gireesha, Bagawedi et al[13] discussed the unsteady laminar flow of an electrically conducting viscous fluid under the influence of a uniform magnetic field and derived important results by differential geometry techniques. Madhura et al[13] studied the motion of a dusty gas through porous media in an open rectangular channel and obtained the expression of skin friction at the boundaries. Khare and Singh[14] studied the MHD flow of a dusty viscous incompressible fluid confined between two vertical walls with volume fraction of dust.

2. Formulation of the Problem and Basic Equation

Consider a non-Newtonian, incompressible, and viscous fluid flowing between two inclined parallel plates at a distance d apart and making an angle β with the horizontal direction. We assume that the plates are very wide and very long, so that the flow is occurring along the axial direction which is the axis of the lower plate along which the x-axis is considered. Therefore the y-axis becomes perpendicular to the plates in the region of the flow of fluid. For an isothermal flow, the equations of motion of an unsteady dusty viscous incompressible fluid under gravity are

(2.1) $$\nabla . U = 0,$$

(2.2) $$\frac{\partial U}{\partial t} + (U.\nabla)U = -\frac{1}{\rho}\nabla p - \frac{\sigma B_0^2 U}{\rho(1+m^2)} + \vartheta \nabla^2 U + \frac{KN}{\rho}(V-U) + g\sin\beta,$$

(2.3) $$\frac{\partial V}{\partial t} + (V.\nabla)V = g\cos\beta + \frac{K}{m}(U-V),$$

(2.4) $$\frac{\partial N}{\partial t} + \nabla.(NV) = 0,$$

where U and V denotes the local velocity vectors of fluid and dust particles.
ρ is the density of the dust particles, p is the static fluid pressure, ϑ is the kinetic viscosity,
N is the number of dust particles per unit volume, K is a Strokes resistance coefficient,
B_0 is a constant external magnetic field parameter, g is the acceleration due to gravity,
m is the mass of the particles, σ is the electrical conductivity of the dust particles.
From the equation of continuity

$$\nabla . U = 0, \quad i.e. \quad \frac{\partial u}{\partial x} = 0 \Rightarrow u = u(y)$$

and

$$\nabla . V = 0, \quad i.e. \quad \frac{\partial v}{\partial x} = 0 \Rightarrow v = v(y).$$

Thus equations (2.1) – (2.4) reduce to

(2.5) $$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{x}} - \frac{\sigma B_0^2 u}{\rho(1+m^2)} + \vartheta\frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho}(v-u) + g\sin\beta,$$

(2.6) $$\frac{\partial v}{\partial t} = g\cos\beta + \frac{K}{m}(u-v),$$

where $u \; and \; v$ are the velocities of the fluid and dust particles respectively along the field of flow.

Oscillating parallel plates

We suppose that the lower plate oscillates at amplitude a_1 and frequency λ_1 while the upper plate oscillates at amplitude a_2 and frequency λ_2. We shall further assume that flow has a negligible convection; the pressure gradient is constant and N is fixed. The boundary conditions may be taken as

$$(2.7) \qquad u = a_1 e^{\lambda_1 t}, y = 0 \quad and \quad u = a_2 e^{\lambda_2 t}, y = \mathrm{d}.$$

We eliminate v between (2.5) and (2.6) by substituting for $v_t \; and \; v$ in (2.5) to give

$$(2.8) \qquad \frac{\partial^2 u}{\partial t^2} = \vartheta \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{\vartheta K}{m}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial t}\left(\frac{K}{m} + \frac{\sigma B_0^2}{\rho\left(1+m^2\right)} + \frac{KN}{\rho}\right)$$
$$- \frac{K}{m}\frac{\sigma B_0^2 u}{\rho\left(1+m^2\right)} + \left(\frac{KN}{\rho} g \cos\beta - \frac{K}{m\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{x}} + \frac{K}{m} g \sin\beta\right).$$

We may assume that

$$(2.9) \qquad \frac{dp}{dx} = e^{i\lambda t}.$$

Thus (2.8) reduces to

$$(2.10) \qquad \frac{\partial^2 u}{\partial t^2} = \vartheta \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{\vartheta K}{m}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial t}\left(\frac{K}{m} + \frac{\sigma B_0^2}{\rho\left(1+m^2\right)} + \frac{KN}{\rho}\right) - \frac{K}{m}\frac{\sigma B_0^2 u}{\rho\left(1+m^2\right)}$$
$$+ \left(\frac{KN}{\rho} g \cos\beta - \frac{K}{m\rho} e^{i\lambda t} + \frac{K}{m} g \sin\beta\right).$$

From the boundary condition (2.7) we seek a solution of the form

$$(2.11) \qquad a_1 f\left(y\right) e^{-i\lambda_1 t} + a_2 g\left(y\right) e^{-i\lambda_2 t},$$

where

$$A = \frac{\vartheta K}{m}, B = \left(\frac{K}{m} + \frac{\sigma B_0^2}{\rho} + \frac{KN}{\rho} \right), C = \frac{K}{m} \frac{\sigma B_0^2}{\rho\left(1+m^2\right)}.$$

$f\left(y\right) and\ g(y)$ are to be determined. The substitution of equation (2.11) into (2.10) gives

$$-\lambda_1^2 a_1 f e^{-i\lambda_1 t} - \lambda_2^2 a_2 g e^{-i\lambda_2 t} = \vartheta\left(-i\lambda_1 a_1 f'' e^{-i\lambda_1 t} - i\lambda_2 a_2 g'' e^{-i\lambda_2 t}\right) + A\left(a_1 f'' e^{-i\lambda_1 t} + a_2 g'' e^{-i\lambda_2 t}\right)$$
$$+ B\left(i\lambda_1 a_1 f e^{-i\lambda_1 t} + i\lambda_2 a_2 g e^{-i\lambda_2 t}\right) - C\left(a_1 f(y) e^{-i\lambda_1 t} + a_2 g(y) e^{-i\lambda_2 t}\right)$$
$$+ \left(\frac{KN}{\rho} g \cos\beta - \frac{K}{m\rho} e^{i\lambda t} + \frac{K}{m} g \sin\beta\right).$$

Collecting terms

$$(2.12) \qquad -\lambda_1^2 f = -\vartheta\lambda_1 f'' + Af'' + Bi\lambda_1 f - Cf + \left(\frac{KN}{\rho} g \cos\beta - \frac{K}{m\rho} e^{i\lambda t} + \frac{K}{m} g \sin\beta\right)\frac{e^{i\lambda_1 t}}{a_1}$$

and

$$(2.13) \qquad -\lambda_2^2 g = -\vartheta\lambda_2 g'' + Ag'' + Bi\lambda_2 g - Cg + \left(\frac{KN}{\rho} g \cos\beta - \frac{K}{m\rho} e^{i\lambda t} + \frac{K}{m} g \sin\beta\right)\frac{e^{i\lambda_2 t}}{a_2}.$$

We re-write (2.12) and (2.13) as

$$(2.14) \qquad f'' + Pf + R = 0,$$

Similarly

$$(2.15) \qquad g'' + Qg + S = 0,$$

where

$$P = \frac{\left(\lambda_1^2 + Bi\lambda_1 - C\right)}{\left(A - \vartheta\lambda_1\right)},\ R = \left(\frac{KN}{\rho} g \cos\beta - \frac{K}{m\rho} e^{i\lambda t} + \frac{K}{m} g \sin\beta\right)\frac{e^{i\lambda_1 t}}{a_1\left(A - \vartheta\lambda_1\right)},$$

$$Q=\frac{\left(\lambda_2^2+Bi\lambda_2-C\right)}{\left(A-\vartheta\lambda_2\right)},\ S=\left(\frac{KN}{\rho}g\cos\beta-\frac{K}{m\rho}e^{i\lambda t}+\frac{K}{m}g\sin\beta\right)\frac{e^{i\lambda_2 t}}{a_2\left(A-\vartheta\lambda_2\right)}.$$

The solution of (2.14) is

$f(y)=C_0\cos Py+C_1\sin Py-\dfrac{R}{P}$ with boundary conditions

$f(0)=1$ and $f(d)=0.$

Hence

$$f(y)=\frac{\sin P(d-y)}{\sin Pd}+\frac{R}{P}\left(\frac{\sin P(d-y)+\sin Py}{\sin Pd}\right)-\frac{R}{P}.$$

Similarly the solution to (2.15) is

$$g(y)=C_2\cos Qy+C_3\sin Qy-\frac{S}{Q},$$

with boundary conditions

$g(0)=0$ and $g(d)=1$

$$g(y)=\frac{\sin Qy}{\sin Qd}+\frac{S}{Q}\left(\frac{\sin Q(d-y)+\sin Qy}{\sin Qd}\right)-\frac{S}{Q},$$

where C_0, C_1, C_2 and C_3 are constant of integration.

So

$$(2.16)\qquad u=a_1\left[\frac{\sin P(d-y)}{\sin Pd}+\frac{R}{P}\left(\frac{\sin P(d-y)+\sin Py}{\sin Pd}\right)-\frac{R}{P}\right]e^{-i\lambda_1 t}$$
$$+a_2\left[\frac{\sin Qy}{\sin Qd}+\frac{S}{Q}\left(\frac{\sin Q(d-y)+\sin Qy}{\sin Qd}\right)-\frac{S}{Q}\right]e^{-i\lambda_2 t}.$$

Substituting (2.16) into (2.5), we get

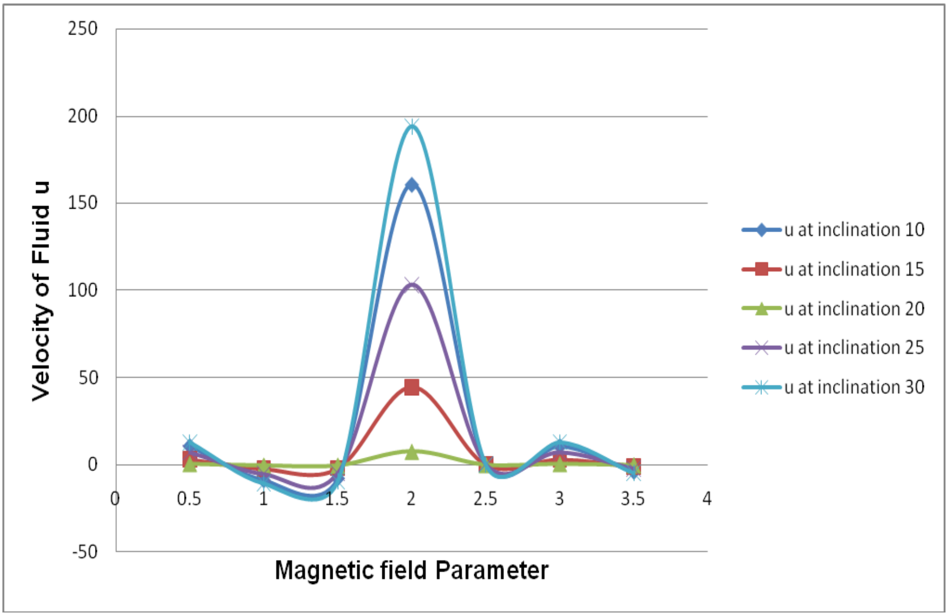
$$(2.17)\qquad v=\frac{1}{KN}e^{i\lambda t}-\frac{\rho}{KN}g\sin\beta+a_1\left[-\frac{R}{P}\left(1-i\lambda_1\frac{\rho}{KN}+\frac{\sigma B_0^2}{KN\left(1+m^2\right)}\right)\right.$$

$$+\left(1-i\lambda_1\frac{\rho}{KN}+\frac{\sigma B_0^2}{KN\left(1+m^2\right)}+\frac{\rho}{KN}\vartheta P^2\right)\left(\frac{\left(1+\frac{R}{P}\right)\sin P\left(d-y\right)+\frac{R}{P}\sin Py}{\sin Pd}\right)$$

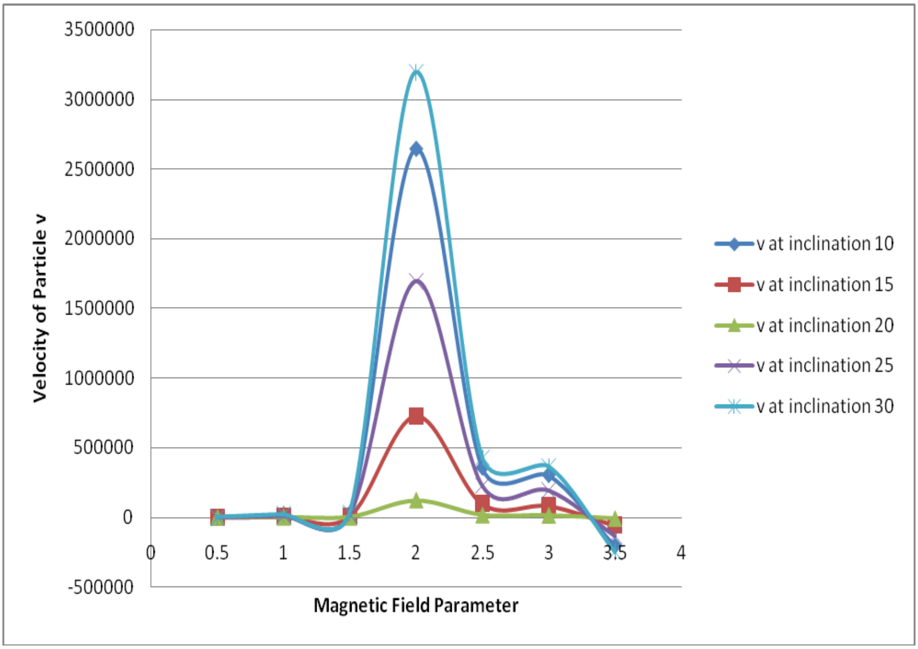
$$\left.+\frac{\rho}{KN}\frac{1}{P}\left(\frac{\sin P\left(d-y\right)+\sin Py}{\sin Pd}-1\right)\frac{\partial R}{\partial t}\right]e^{-i\lambda_1 t}+a_2\left[-\frac{S}{Q}\left(1-i\lambda_2\frac{\rho}{KN}+\frac{\sigma B_0^2}{KN\left(1+m^2\right)}\right)\right.$$

$$+\left(1-i\lambda_2\frac{\rho}{KN}+\frac{\sigma B_0^2}{KN\left(1+m^2\right)}+\frac{\rho}{KN}\vartheta Q^2\right)\left(\frac{\frac{S}{Q}\sin Q\left(d-y\right)+\left(1+\frac{S}{Q}\right)\sin Qy}{\sin Qd}\right)$$

$$\left.+\frac{\rho}{KN}\frac{1}{Q}\left(\frac{\sin Q\left(d-y\right)+\sin Qy}{\sin Qd}-1\right)\frac{\partial S}{\partial t}\right]e^{-i\lambda_2 t}.$$



Graph between Magnetic field parameter & Velocity of Fluid



Graph between Magnetic field parameter & Velocity of Particles

Result and Discussion

(1) The study indicates that for every inclination both velocities show a resonances character which occurs nearly magnetic field parameter and that has been verified mathematically also.

(2) The value of the velocity in case of particle is very very high in comparison to that of fluid. The reason is clear that magnetic field is more effective on magnetic sensitive particle.

(3) Both graphs are appearing similar in nature but the magnitudes of the changes in parameter are different.

(4) Also the maximum/minimum velocities for both phases are occurring at the same inclinations.

Thus, the results obtained may be applied in the aforesaid mentioned field having multiphase flow through inclined channels placed in magnetic field.

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