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Derivation of Eigen value Equation by Using Equivalent Transmission Line method for the Case of Symmetric/ Asymmetric Planar Slab Waveguide Structure

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Abstract: The symmetric/asymmetric planar slab waveguide is simplest waveguide structure to be analyzed. These waveguides are used in optical communication systems. In this paper we have derived the Eigen value equations by using the transmission line (TL) method for the case of symmetric/asymmetric planar slab waveguide structure. Earlier also the equations have been derived but no where the intermediate steps of solution found by author knowledge. The derived results have been exactly matched with the existing results found into the literatures.

1. Introduction

The symmetric/asymmetric planar waveguide structures have impact on WDM optical communication systems. The transmission line (TL) method has great application to analyze waveguide structure having arbitrary refractive index profile [1-2]. There is large number of current research papers on application of TL method of waveguide analysis [3-11]. The asymmetric waveguide is somewhat tough to analyze due to their asymmetric mode field profile. The asymmetric waveguide have found certain advantage over the symmetric waveguide structure due to their easiness [4]. In section-2, we have shown the calculation of Eigen value equation for symmetric planar slab waveguide followed with asymmetric waveguide structure in section-3.

Maxwell Equations

Let consider the planar slab optical waveguide as shown in Fig.1 having refractive index variation in x-direction and direction of wave propagation in

$+z$ direction. Consider the following Maxwell equations ($time\,harmonic\;e^{j\omega t}$),

(1.1) $$\nabla\times E=-\mu_0\frac{\partial H}{\partial t},$$

(1.2) $$\nabla\times H=\varepsilon\frac{\partial E}{\partial t},$$

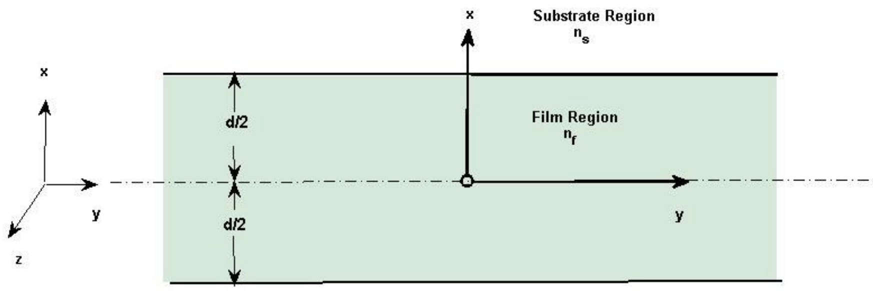

Fig 1: A thin wave guiding element with coordinate

$For\,the\,TE\,mode\,case\;propagating\;in+z\;direction\;as\;shown\;in\;Fig.1,$
$have\;\;E_x=0,H_y=0\;\;and\;\;\;E_z=0\;\;\;hence\;from\,equations\,(1.1)\;and\,(1.2),$

(1.3) $$\frac{\partial E_y}{\partial x},$$

(1.4) $$\frac{\partial H_z}{\partial x}+j\beta Hx=-j\omega\varepsilon_0\{n(x)^2\,Ey,$$

(1.5) $$\beta E_y=-\omega\mu_0 Hx,$$

2. Derivation of Eigen Value equation for Symmetric Planar Slab Dielectric Waveguide

In this section the derivation of Eigen value equations have been done for odd/even TE modes by Equivalent T L method. Let us define new variables as follows

(2.1) $$v=\beta H_z,$$

(2.2) $$I=\beta E_y=-\omega\mu_0 Hx,$$

Equation (1.4) can be written as

(2.3) $$\frac{\partial V}{\partial x} = -\frac{\gamma^2}{j\omega\mu_0} I,$$

and equation (1.3) can be written as

(2.4) $$\frac{\partial I}{\partial x} = -j\omega\mu_0 V,$$

with

(2.5) $$\gamma^2 = \beta^2 - \omega^2 \mu_0 \varepsilon_0 \{n(x)\}^2,$$

The characteristics impedance in equations (2.3) and (2.4) is given by

(2.6) $$Z = \frac{\gamma}{j\omega\mu_0},$$

The planar layer of thickness d can be represented by an equivalent T-circuit as shown in Fig. 2, with series and parallel elements given by

(2.7) $$Z_s = Z \tanh\left(\gamma \frac{d}{2}\right),$$

(2.8) $$Z_p = Z \frac{1}{\sinh(\gamma d)},$$

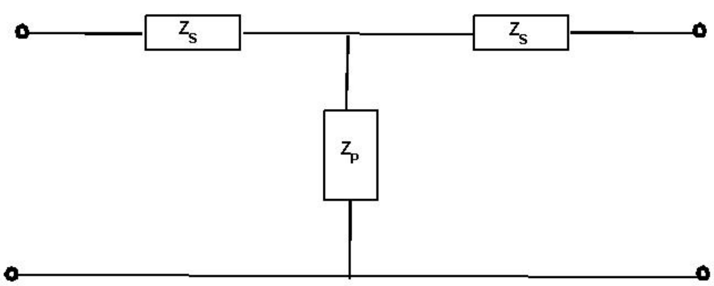

Fig 2: Equivalent T-circuit of TE modes of planar waveguide layer of thickness d.

If the layer is homogeneous and infinite of thickness and by using the following relations

(2.9) $$\lim_{d\to\infty} sinh(\gamma d) = \frac{e^{\gamma d} - e^{\gamma d}}{2} = \infty,$$

(2.10) $$\lim_{d\to\infty} tanh(\gamma d) = \frac{e^{\gamma d/2} - e^{-\gamma d/2}}{e^{\gamma d/2} + e^{-\gamma d/2}} = 1,$$

Then layer may be represented by its characteristics impedance Z eq. (2.6). At the center of the film region where thickness $d = 0$,

$$\lim_{d \to 0} \sinh(\gamma d) = \frac{e^{\gamma d} - e^{-\gamma d}}{2} = 0, \tag{2.11}$$

$$\lim_{d \to 0} \tanh(\gamma d/2) = \frac{e^{\gamma d/2} - e^{-\gamma d/2}}{e^{\gamma d/2} + e^{-\gamma d/2}}, \tag{2.12}$$

This condition will lead

$$Z_s = 0,$$

$$Z_p = \infty.$$

It means that at the center of waveguide the layer may be represented by an open circuit. Thin waveguide shown in Fig. 1 can be represented in equivalent form for a region $x \geq 0$ as shown in Fig. 3.

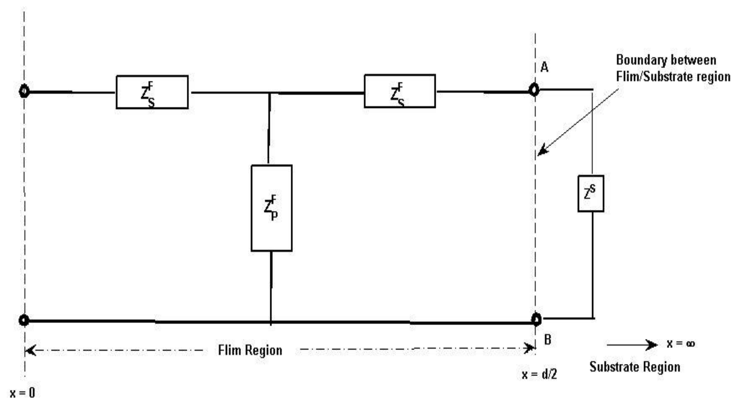

Fig 3: Equivalent T network representation of symmetric slab waveguide $(x \geq 0)$ as shown in Fig. 1

In the Fig. 3 the impedance of the various branches are shown as follows

$$Z_S^F = Z^F \tanh\left(\gamma \frac{d}{4}\right), \tag{2.13}$$

$$Z_P^F = Z^F \frac{1}{\sin(\gamma d/2)}, \tag{2.14}$$

(2.15) $$Z^s = Z^s,$$

Here subscripts S, P and superscripts F, S, s represent the series, parallel, film, substrate respectively. Where,

(2.16) $$Z^F = \frac{\sqrt{\beta^2 - k_0^2 n_f^2}}{j\omega\mu_0},$$

and

(2.17) $$Z^s = \frac{\sqrt{\beta^2 - k_0^2 n_s^2}}{j\omega\mu_0},$$

2.1 Derivation of Eigen value equation for Odd Modes

The Eigen value equation for odd modes can be obtained when the impedance seen from either side of terminal $A-B$ $A-B$ in Fig.3 is same as

(2.1.1) $$Z_S^F + Z\frac{F}{P} = Z^s,$$

or

(2.1.2) $$Z^F \, tanh\left(\gamma\frac{d}{4}\right) + Z^F \frac{1}{sinh(\gamma d/2)} = Z^s,$$

In this equation γ is given by

(2.1.3) $$\gamma = \sqrt{\beta^2 - k_0^2 n_f^2} = jk,$$

where $\kappa = \sqrt{k_0^2 n_f^2 - \beta^2}$, Simple algebraic manipulation gives

(2.1.4) $$tan\left(\frac{\kappa d}{2}\right) = -\frac{\kappa}{\gamma_s}(odd\, mode),$$

Here we have used the following relations

(2.1.5)
$$\begin{cases} sinh\left(\frac{\gamma d}{2}\right) = \dfrac{\frac{\gamma d}{e^2} - \frac{\gamma d}{e^2}}{2} \\ cos(\gamma d/4) = \dfrac{e^{\gamma d/4} + e^{-\gamma d/4}}{2} \end{cases},$$

and also $\gamma_s = \sqrt{\beta^2 - \kappa_0^2 n_s^2}$. This equation is exactly same for the Eigen value equation of odd TE mode of planar slab symmetric waveguide having thickness d[12].

2.2 Derivation of Eigen value equation for Even Modes

To derive the Eigen value equation for even mode, we define the new parameters

(2.2.1.) $$Y_S^F = Y^5 \, tang\left(\gamma \frac{d}{4}\right),$$

(2.2.2) $$Y_p^F = Y^F \frac{1}{sinh(\gamma d/2)},$$

(2.2.3.) $$Y^s = Y^s,$$

Here

$$Y^F = \frac{1}{Z^F},$$

$$Y^s = \frac{1}{Z^s}.$$

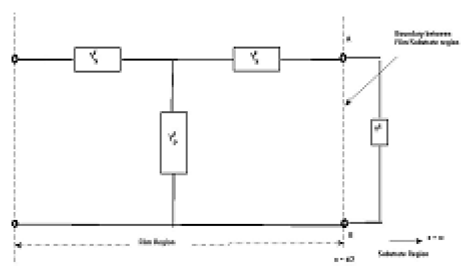

Fig 4: Equivalent T- network representation $(x \geq 0)$ of even mode symmetric slab waveguide.

The Eigen value equation can be obtained when the impedance seen from either side of terminal A-B in Fig. 4 is same.

(2.2.4.) $$Y_S^F + Y\frac{F}{P} = Y^s,$$

Substituting the equations (2.2.1)-(2.2.3) into eq. (2.2.4) and after some trivial algebra it can shown [12],

(2.2.5.) $$tan\left(\frac{\kappa d}{2}\right) = \frac{\gamma_s}{\kappa} \qquad (Even\, mode),$$

3. Derivation of Eigen Value equation of Asymmetric Planar Slab Dielectric Waveguide

In this section we derive the rigorous and exact Eigen value equation for the case of asymmetric planar slab waveguide structure having the refractive index variation[12],

(3.1) $$n(x) = \begin{cases} n_c, & x \geq 0, \\ n_f, & -d \leq x \leq 0, \\ n_s, & x \leq -d \end{cases}$$

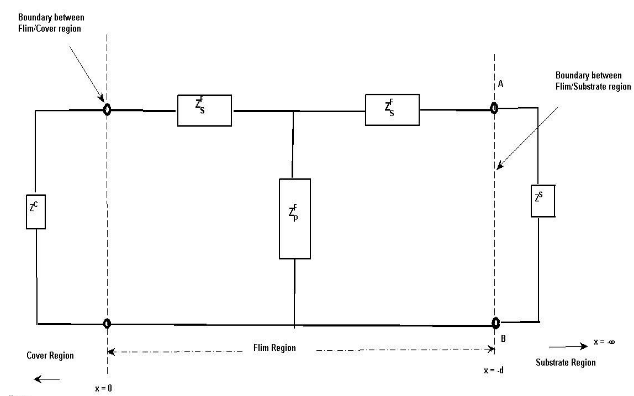

Fig 5: Equivalent T network representation of asymmetric slab waveguide.

where

(3.2) $$Z_S^F = Z^F \frac{1}{sin(\gamma d)},$$

(3.3) $$Z_P^F = Z^F \frac{1}{sinh(\gamma d)},$$

(3.4) $$Z^s = -Z^s,$$

(3.5) $$Z^c = Z^c,$$

and

(3.6) $$\begin{cases} Z^F = \dfrac{\sqrt{\beta^2 - k_0^2 n_f^2}}{j\omega\mu_0}, \\ Z^s = \dfrac{\sqrt{\beta^2 - k_0^2 n_s^2}}{j\omega\mu_0} = \dfrac{\gamma_s}{j\omega\mu_0}, \\ Z^c = \dfrac{\sqrt{\beta^2 - k_0^2 n_s^2}}{j\omega\mu_0} = \dfrac{\gamma_c}{j\omega\mu_0}. \end{cases}$$

After some trivial algebraic manipulation and by using eq. (2.1.3) it can also be shown

(3.7) $$Z_S^F = \frac{j\kappa}{\omega\mu_0} tan\left(\kappa \frac{d}{2}\right),$$

(3.8) $$Z_p^F = \frac{\kappa}{j\omega\mu_0} \frac{1}{sin(\kappa d)}.$$

Minus in eq. (3.4) is due to limit $x = -\infty$. Finally the Eigen value equation can be derived from the following expression when the impedance seen from either side of terminal A-B in Fig. 5 is same

(3.9) $$\frac{(Z^c + Z_S^F) Z_p^F}{(Z^c + Z_S^F) Z_p^F} + Z_S^F = Z^s.$$

After some trivial mathematical manipulation leads the following results

(3.10) $$\frac{\left[\kappa\gamma_c - \kappa^2 tan\left(\kappa\frac{d}{2}\right)\right]}{[\gamma_c\, sin(\kappa d) + \kappa cos(\kappa d)]} = \left[\kappa tan(\kappa\frac{d}{2}) - \gamma_s\right],$$

This implies

(3.11) $$tan(\kappa d) = \frac{\gamma_c + \gamma_s}{\kappa\left(1 - \frac{\gamma_s\,\gamma_c}{\kappa^2}\right)}.$$

This is exactly same Eigen value equation for the asymmetric planar slab waveguide structure [12].

4. Conclusion

First time we have derived the exact Eigen value equation for the case of symmetric/asymmetric planar slab waveguide. We have shown the intermediate step of calculation with substantial assumption. In most of the papers being published on TL method, have not clarifies the trivial calculation to achieve some specific expression. The derivation presented into this paper is useful to the beginners who want to gain inside into TL method. One can easily extended these results to simulate the mode field profile, mode cutoff condition, dispersion relation of multilayer dielectric waveguide structure.

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