ISSN 0974 - 9373

Vol. 20 No. 4 (2016) **Journal of International Academy of Physical Sciences** pp. 257-271

Bianchi-III Cosmological Model in Saez-Ballester Theory of Gravity with BVDP

Avtar Chand and R. K. Mishra

Department of Mathematics
SLIET Deemed University, Under MHRD
Govt .of India, Longowal-148106, Punjab, India

Email: avi24maths@gmail.com, ravkmishra@yahoo.co.in

(Received October 06, 2016)

Abstract: In present communication an exact solution of modified EFE (Einstein's field equations) for spatially homogeneous and anisotropic Bianchi type-III space-time have been investigated in scalar-tensor Saez-Ballester theory of gravity (Phys. Lett. A113, 467 (1986)) along with time dependent deceleration parameter. We have presented some accelerating cosmological models by assuming bilinearly varying deceleration parameter (BVDP, as suggested by Mishra et al. (2016). Present study indicates that our universe shows deceleration expansion for early time and accelerating expansion at present time i.e. transitional phase of universe expansion. We have also presented the physical and geometrical properties of the models with the help of pictorial representation.

Keywords: Bianchi type-III space-time, Bilinear varying deceleration parameter, EoS parameter.

2010 Mathematics Subject Classification: 83F05.

1. Introduction

Now in these days many cosmologists published their research work related to cosmological constants problems (Weinberg 1989; Padmanabhan 2003)[1-2]. Recent observations of supernovae SNe Ia and the power spectrum of fluctuation in the cosmic microwave background (CMB) have provided the direct evidences for a universe dominated by unknown quantity so called ‘dark energy’ (Riess 1998 2001; Perlmutter 1998, 1999; Speergel 2003)[3-7]. Both the above mentioned observations suggested to study the accelerating universe model with $\Omega_m \approx 0.3$ and $\Omega_\Lambda \approx 0.7$, Here Ω_m and Ω_Λ are denoted the dark matter and dark energy density parameters respectively. These values of density parameters corresponds to cosmological constant Λ is positive, non-zero and smaller quantity at present time. Therefore it is required to calculate the dependence of Λ upon scale factor or cosmic time with desired level of accuracy. During investigation of literature it has also been observed that the recent cosmic observations (Riess 1998, 2001; Perlmutter 1998, 1999; Speergel 2003)[3-7] indicate that the present universe is expanding in nature at an acceleration rate. Therefore to study the rate of expansion of the universe is important issue in Cosmology. As we know that the scale factor $a(t)$, the Hubble parameter $H(t)$ and the deceleration parameter $\mathrm{q}(t)$ are important parameters to know the dynamics history of the universe. Also the sign of $H(t)$ and $\mathrm{q}(t)$ categorized the expansion rate as follow:

- For $H > 0, \ \ q > 0,$ universe is expanding and decelerating,
- For $H > 0, \ \ q < 0,$ universe is expanding at accelerating rate,
- For $H > 0,\ q = 0,$ universe shows constant expansion (Bolotin et al 2015)[8].

It has also noticed from literature, a group of researchers believed that DP as a constant (Berman 1988)[9] while some others believed that it is a variable quantity (Pradhan 2007; Akarsu et al. 2012, 2014; Chawla et al. 2012)[10-13] and accordingly they had published their research work. In this regard Berman had proposed constant DP along with relation $q = m - 1$, here $m \geq 0$, known as Berman's law of constant deceleration parameter[9]. In 2012 Akarsu and Dereli had published worked by assuming DP as a linearly varying function of cosmic time with suitable relation $q(t) = -kt + m - 1$, here k and m are non negative constants (Akarsu and Dereli 2012)[11]. In continuation of study of cosmological models with time dependent DP, our peer research group (Chawla et al. 2012; Mishra et al. 2013, 2014, 2016a; Chand et al. 2016b)[13-17] had also derived some cosmological models with time dependent DP along with proper anastz $q(t) = n.\sec^2(\alpha\, \mathrm{t}) - 1$ and $q(t) = -1 + \dfrac{n\alpha}{(\mathrm{t}+\alpha)^2}$, Here $n, \alpha > 0$ and published fruitful outcomes. Very recently we have published research paper entitled ‘Cosmological models in alternative theory of gravity with bilinear deceleration parameter' (Mishra & Chand 2016c) [18].

Under the motivation of above mention study here in present communication we have investigated the Bianchi type–III cosmological models in scalar-tensor Saez and Ballester theory of gravity (SB theory)[19] with bilinear varying deceleration parameter as suggested by Mishra et al. 2016c[18]. We may believe that the scalar-tensor theories have the potential to address a large section of problems related with expanding nature of universe. Under this motivation our peer group have already published few results (Mishra et al. 2014)[15]. As suggested by Saez-Ballester theory the metric is coupled with a dimensionless scalar field in a simple way[19]. This coupling provide a reasonable description of the weak field in which an accelerated expansion regime appears despite of the dimensionless behavior of a scalar field. Numerous modification of general relativity accepted the theory of variable gravitation constant G, based on different arguments proposed by several authors (Dirac 1937; Wesson 1980; Canuto 1980; Arbab 1997)[20-23] and agreed that G varying cosmology is consistent with cosmological observations known as on date.

The paper is organized as follows: In section 2, we have presented basic equations governing the cosmological model. Section 3, deals with the solutions of field equations, section 4, we discussed the physical and geometric properties of the models. Section 5 results and discussion have been made. Finally, concluding remarks are given in the last of the paper i.e. section 6.

2. Equations Governing the Cosmological Model

We consider spatially homogeneous and anisotropic Bianchi type-III space-time as

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2\beta x}B^2dy^2 + C^2dz^2, \tag{2.1}$$

where A, B and C are functions of cosmic time t only and β is a positive constant. The modified Einstein's field equations (EFE) in scalar-tensor theory of gravity as proposed by Saez and Ballester[9] expressed as

(2.2) $$G_{ik} - w\phi^{r}\left(\phi_{,i}\phi_{,k} - \frac{1}{2}g_{ik}\phi_{,j}\phi^{,j}\right) = -8\pi T_{ik}\,,$$

where $G_{ik} = R_{ik} - \frac{1}{2}Rg_{ik,}$ Einstein tensor, T_{ik} is the energy momentum tensor, w is a dimensionless coupling constant, ϕ is the scalar field and r is an arbitrary constant. The scalar field ϕ satisfying the following equations:

(2.3) $$2\phi^{r}\phi^{,j}_{;j} + r\phi^{r-1}\phi_{,k}\phi^{,k} = 0\,,$$

where commas and semi-colon denote partial and co-variant derivatives respectively. The energy-momentum tensor for an anisotropic dark energy is expressed as

(2.4) $$\begin{aligned} T_{ik} &= diag(\rho, -\mathrm{p}_x, -\mathrm{p}_y, -\mathrm{p}_z) \\ &= diag(1, -\omega_x, -\omega_y, -\omega_z)\rho\,, \end{aligned}$$

where ρ and p are the energy density and pressure for cosmic fluid, $u^i = (0,0,0,1)$ is the four velocity vector with $g_{ik}u^i u^k = -1$.

(2.5) $$T_{ik} = diag(1, -\omega, -(\omega+\gamma), -(\omega+\delta))\rho\,,$$

where ω_x, ω_y, ω_z are directional EoS parameters along X, Y and Z axes respectively and $\omega(t) = \frac{p}{\rho}$, is the deviation free EoS parameter of the fluid. For sake of simplicity, we choose $\omega_x = \omega$ and the skewness parameter γ and δ are the deviation from ω on Y and Z axes respectively.
The Saez-Ballester field equation (2.2) and (2.3) for the metric (2.1) with the help of (2.5) yields the following set of field equations:

(2.6) $$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\beta^2}{A^2} - w\phi^{r}\dot{\phi}^2 = -(\omega+\delta)\rho\,,$$

(2.7) $$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{2}w\phi^{r}\dot{\phi}^2 = -\omega\rho\,,$$

(2.8) $$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{2}w\phi^r\dot{\phi}^2=-(\omega+\gamma)\rho,$$

(2.9) $$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{\beta^2}{A^2}+\frac{1}{2}w\phi^r\dot{\phi}^2=\rho,$$

(2.10) $$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0,$$

(2.11) $$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{r\dot{\phi}^2}{2\phi}=0,$$

where an over dot denotes the derivative with respect to cosmic time ‘t’. It is necessary to discuss some physical parameters before solving the field equations. The average scale factor $a(t)$ and the volume scale factor V are related by

(2.12) $$V=a^3=ABC.$$

The generalized means Hubble parameter H defined as

(2.13) $$H=\frac{1}{3}\left(H_x+H_y+H_z\right)=\frac{\dot{a}}{a},$$

where $H_x=\frac{\dot{A}}{A}, H_y=\frac{\dot{B}}{B}, H_z=\frac{\dot{C}}{C}$ are the directional Hubble parameters in the direction X, Y and Z respectively. The mean anisotropy parameter A_m is defined as

(2.14) $$A_m=\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2.$$

The shear scalar σ^2 is defined as

(2.15) $$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-3H^2\right).$$

The deceleration parameter (q) measure the expansion rate of the universe and it is defined as

(2.16) $$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(1 + \frac{\dot{H}}{H^2}\right).$$

After discussing the necessary equations and defining geometrical parameters, now we are going to solve the field equations (2.6)-(2.11) with help of physical plausible and mathematical suitable assumption in next section.

3. Solution of the Field Equations

The above mention set of field equations is the set of six non-linear differential equations with eight unknown $A, B, C, \rho, \omega, \gamma, \delta \,\&\, \phi$. Therefore for explicit solution we need two more constraints related to these unknown parameters. For exact solution of modified EFE we will apply following physically plausible relations as,

(i) The time dependent deceleration parameter: The scale factor measures the distance between pair of objects in 4-dimensional space, it also represents the relative expansion of the universe. The dynamics of the universe is described by time-dependent cosmic scale factor $a(\mathrm{t})$. The time dependence of $a(\mathrm{t})$ reflects important events in evolution history of the universe. Therefore to study the dynamic history of the universe with time we need more dynamical & kinematical parameters related to scale factor and its derivative.

Motivated from above mentioned study we wish to investigate the dynamic nature of the universe by assuming DP (q) as a bilinear function of cosmic time `t' with specific assumption as proposed by Mishra et al. 2016c[18]:

(3.1) $$q(t) = \frac{\alpha(1-\mathrm{t})}{1+t},$$

where α is a non-negative constant. The expression for Hubble parameter and scale factor are given by

(3.2) $$\mathrm{H}(t) = \frac{1}{(1-\alpha)t + 2\alpha\log(1+\mathrm{t})},$$

(3.3) $$a(t) = t^{\frac{1}{1+\alpha}}.e^{F_1(t)},$$

where

$$F_1(t) = \frac{\alpha}{(1+\alpha)^2}t + \frac{-2\alpha+\alpha^2}{6(1+\alpha)^3}t^2 + \frac{3\alpha - 2\alpha^2 + \alpha^3}{18(1+\alpha)^4}t^3$$
$$+\frac{-18\alpha + 11\alpha^2 - 14\alpha^3 + 2\alpha^4}{180(1+\alpha)^5}t^4 + O(t^5)$$

Now, we analyze following possibilities for derived model of universe from equation (3.1)-(3.3) as

- $0 < \alpha < 1, 0 < t < 1$ then $\mathrm{q} > 0, \mathrm{H} > 0, \mathrm{a} > 0$
- $0 < \alpha < 1, t > 1,$ then $\mathrm{q} < 0, \mathrm{H} > 0, \mathrm{a} > 0$

(ii) Secondly we assume that the shear scalar σ is propositional to expansion scalar θ, as suggested by Throne et al. 1967[24] , which may written as

(3.4) $$B = C^m,$$

(3.5) $$A = k_1 B,$$

(3.6) $$A = B,$$

(3.7) $$A = B = \left[t^{\frac{1}{1+\alpha}}e^{F_1(t)}\right]^{\frac{3m}{2m+1}},$$

(3.8) $$C=\left[t^{\frac{1}{1+\alpha}}e^{F_1(t)}\right]^{\frac{3}{2m+1}},$$

where m and k_1 are constants. For simplicity we take k_1=1.

The metric equation (2.1) can be written as after substituting the values of A, B and C,

(3.9) $$ds^2=-dt^2+\left[t^{\frac{1}{1+\alpha}}e^{F_1(t)}\right]^{\frac{6m}{2m+1}}dx^2+e^{-2\beta x}\left[t^{\frac{1}{1+\alpha}}e^{F_1(t)}\right]^{\frac{6m}{2m+1}}dy^2$$
$$+\left[t^{\frac{1}{1+\alpha}}e^{F_1(t)}\right]^{\frac{6}{2m+1}}dz^2\ .$$

In next section i.e. section 4, we will discuss the results and physical interpretation of obtained cosmological parameters.

4. Results and Discussions

The expression for physical and geometric parameter such as energy density parameter (ρ), the EoS parameter (ω) and the skewness parameter (δ) may expressed as

(4.1) $$\rho=\frac{9m(m+2)}{\left[(2m+1)\{(1-\alpha)t+2\alpha\log(1+t)\}\right]^2}-\frac{\beta^2}{[t^{\frac{1}{1+\alpha}}e^{F_1(t)}]^{\frac{6m}{2m+1}}}$$
$$+\frac{w}{2}k_1^2t^{\frac{-6}{1+\alpha}}e^{-6F_1(t)}+\frac{w}{2}k_1^2t^{\frac{-6}{1+\alpha}}e^{-6F_1(t)},$$

(4.2) $$\omega\rho=\frac{9(m^2+m+1)}{\left[(2m+1)\{(1-\alpha)t+2\alpha\log(1+t)\}\right]^2}$$
$$-\frac{3(m+1)\{1+3\alpha+(1+\alpha)t\}}{(2m+1)(1+t)[(1-\alpha)t+2\alpha\log(1+t)]^2}-\frac{w}{2}k_1^2t^{\frac{-6}{1+\alpha}}e^{-6F_1(t)}$$

,

(4.3) $$\delta\rho = \frac{9(1-2m^2-2m)}{\left[(2m+1)\{(1-\alpha)t+2\alpha\log(1+t)\}\right]^2}$$

$$-\frac{3(m-1)\{1+3\alpha+(1+\alpha)t\}}{(2m+1)(1+t)[(1-\alpha)t+2\alpha\log(1+t)]^2}+\frac{\beta^2}{[t^{\frac{1}{1+\alpha}}e^{F_1(t)}]^{\frac{6m}{2m+1}}},$$

(4.4) $$\gamma = 0,$$

(4.5) $$V = t^{\frac{3}{1+\alpha}}.e^{3F_1(t)},$$

(4.6) $$\sigma^2 = \frac{3(m-1)^2}{\left[(2m+1)\{(1-\alpha)t+2\alpha\log(1+t)\}\right]^2},$$

(4.7) $$A_m = 2\left[\frac{m-1}{2m+1}\right]^2,$$

(4.8) $$\phi(t) = \left[\frac{2+n}{2}\{\phi_0 F_2(t)+\phi_1\}\right]^{\frac{2}{2+n}},$$

where $F_2(t) = \int t^{\frac{-3}{1+\alpha}}e^{-3F_1(t)}dt$.

We have plotted the mean Hubble parameter H along with cosmic time t for different choice of α i.e. $\alpha = 0.5, 1.0, 1.5$. It is clear from Figure 1 that as $t \to 0$, then $H \to \infty$ i.e. at early universe expansion rate is very large, for evolution during time it decreases with time and converges to a constant value at present scenario.

The variation of DP q with cosmic time t is presented in Figure 2 for $\alpha = 0.5, 1.0, 1.5$. It is observed from Figure 2, q is positive for early time $0 < t < 1$, negative for $t > 1$ and $q = 0$ for $t = 1$.

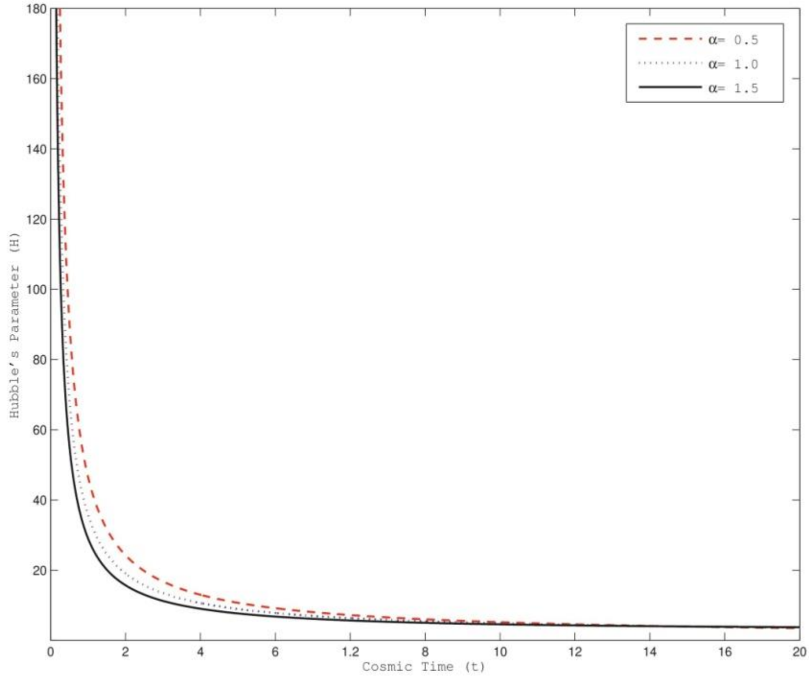

Figure 1: The Plot of Hubble Parameter $\boldsymbol{H}$ versus time $\boldsymbol{t}$.

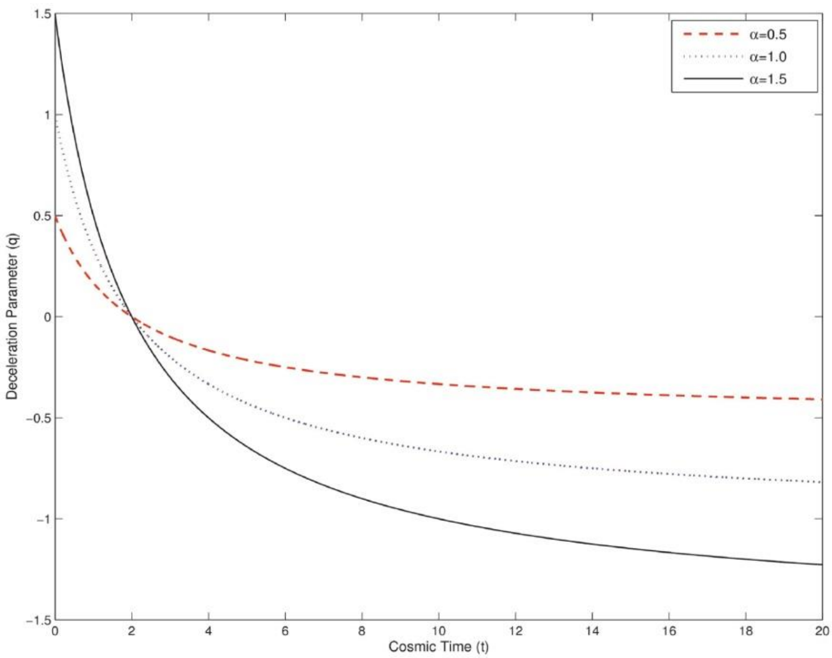

Figure 2: The plot of deceleration parameter q versus time t.

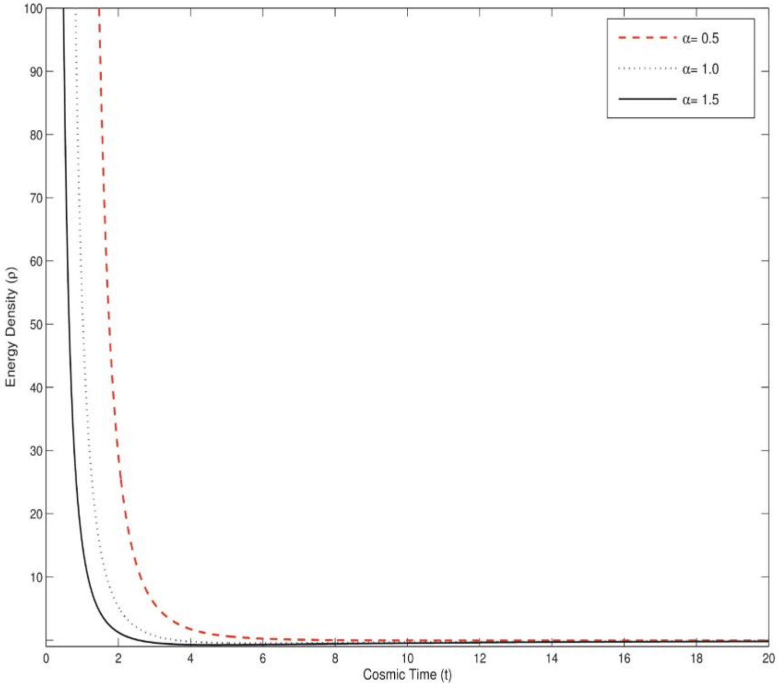

Figure 3: The plot of energy density $\boldsymbol{\rho}$ versus time $\boldsymbol{t}$ for $\boldsymbol{m = 1}$, $\boldsymbol{w = 0.1}, \boldsymbol{k_1 = 1}, \boldsymbol{\beta = 1}$.

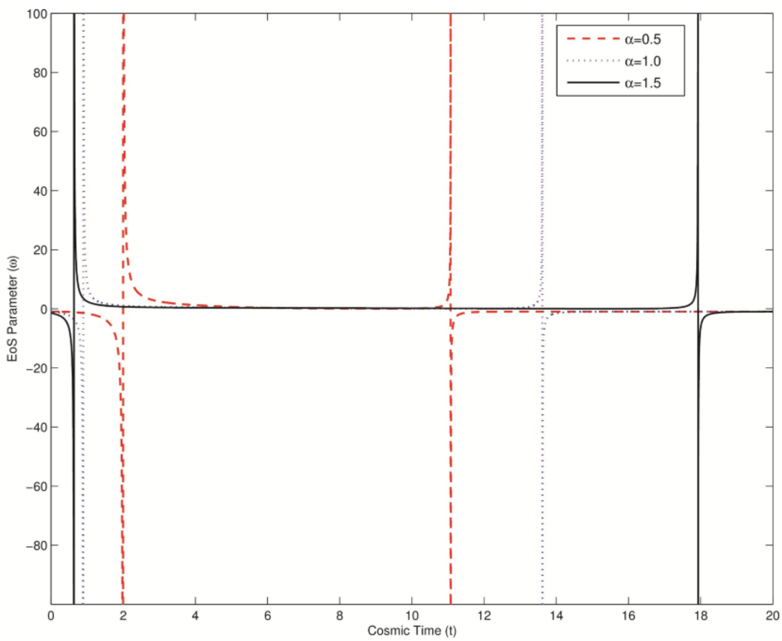

Figure 4: The plot of EoS parameter $\boldsymbol{\omega}$ versus time $\boldsymbol{t}$

for $\boldsymbol{m = 1,\ w = 0.1, k_1 = 1, \beta = 1.}$

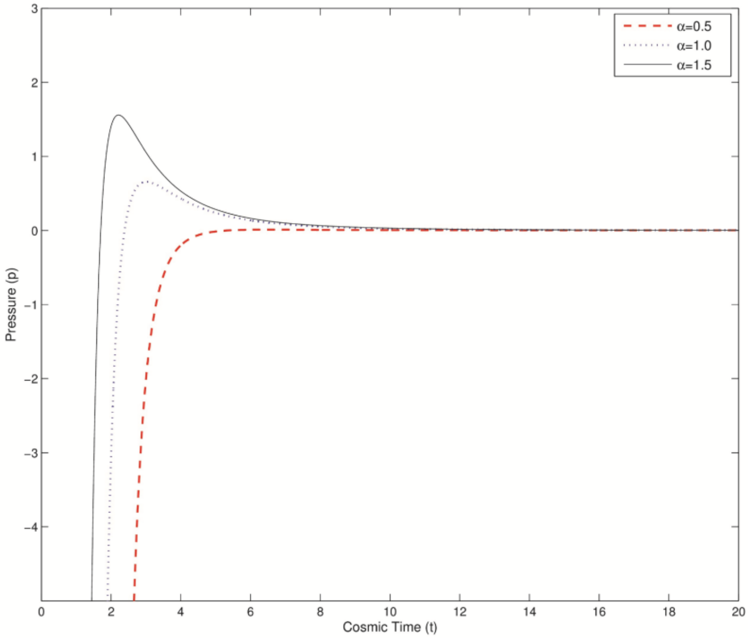

Figure 5: The plot of pressure $\boldsymbol{p}$ versus time $\boldsymbol{t}$ for $\boldsymbol{m = 1,\ w = 0.1, k_1 = 1, \beta = 1.}$

5. Concluding Remarks

In this paper we have investigated the Bianchi type-III cosmological models in scalar-tensor Saez-Ballester[19] theory of gravity (Phys. Lett. A 113, 46(1986)). The solution of modified Einstein field equations have been obtained with the help of bilinear varying deceleration parameter as suggested by Mishra et al (2016)[18]. The outcomes of the study have been summarized as below:

- It is clear from Eq. (4.5), the spatially volume (V) is zero at initially and then increasing exponentially with time. Hence our proposed model validate the initially singularity of the universe.
- The variation of the mean Hubble parameter H with cosmic time t has been presented in Figure 1 for different choices of α i.e. $\alpha = 0.5, 1.0, 1.5$ We analyze that as $t \to 0$, the mean Hubble parameter i.e. at early universe expansion rate is very very large (inflationary phase) after that it decreases with time and converges to a constant value at present time.

- In Figure 2, we presented the variation of the deceleration parameter (q) with cosmic time t. The detailed analysis of the of universe expansion with BVDP may summarized below:

  (i) For $0 < t < 1,\ \alpha > 0,\ q > 0$, we have decelerating nature of expansion,

  (ii) For $t = 1, \alpha > 0, q = 0,$ we have constant nature of expansion,

  (iii) For $t > 1, 0 < \alpha < 1, -1 < q < 0$, we have exponential expansion,

  (iv) For $t > 1, \alpha > 1, \mathrm{q} < -1$, we have super exponential expansion.

- It is clear from Eq. (4.2) the EoS parameter ω is function of time. Also variation of EoS parameter (dark energy parameter) ω with cosmic time for $\alpha = 0.5, 1.0, 1.5$ presented in Figure 4. The concern figure indicates that ω can take both positive and negative values for different choices of α .

- In Figure 5 shows the nature of isotropic pressure p with cosmic time. It observed that at initial inflationary phase it increased very rapidly from large negative value to positive value, then it approached to its maximum value and after that decreasing & converges to small value (nearly zero) at present time.

In this communication we have studied the Bianchi type-III cosmological models in scalar-tensor Saez-Ballester theory of gravity (Phys. Lett. A 113, 467 (1986)) with BVDP as suggested by Mishra et al. 2016. As published in the cited paper, authors had constructed several cosmological models by taking BVDP and concluded that the proposed law is generalized in nature and validated the Berman's and Akarsu's laws for DP, where DP is assumed as constant and linearly varying with time respectively. In other words we can say that BVDP has provided an envelope for all such laws. Under motivation to such study, here in this paper we have already presented the solution of modified EFE by assuming DP as a bilinear function of cosmic time with relation $q = \frac{\alpha(1-\mathrm{t})}{1+t}$. Although all the outcomes of the study have been already presented in section 5 of this paper in detail but we may also conclude that the variation of DP with cosmic time 't' is positive for early time i.e. $0 < t < 1$, zero for $t = 1$ and negative for $t >$

1, which indicate the deceleration expansion, the constant expansion and super exponential expansion of the universe respectively. Fig.3 indicates that the energy density parameter ρ is decreasing function of time and it approaches to small value at present time. Also the graph of EoS, (ω) parameter indicate that it may have both positive and negative values for different choices of α. It is observe that at initial inflationary phase the isotropic pressure increases very rapidly from negative value to small value at present time.

References

1. S. Weinberg, The cosmological constant problem, *Rev. Mod. Phys.* **61(1)** (1989).
2. T. Padmanabhan, Cosmological Constant-the Weight of the Vacuum, *Phys. Rep.*, **380** (2003) 235-320.
3. A. G. Riess, et al., Observational evidence from supernovae for an accelerating universe and a cosmological constant, *Astron. J.*, **116** (1998) 1009-1038.
4. A. G. Riess, et al., The farthest known supernova: support for an accelerating universe and a glimpse of the epoch of deceleration, *Astrophys. J.*, **560** (2001) 49-71.
5. S. Perlmutter, et al., Discovery of a supernova explosion at half the age of the universe, *Nature*, **391** (1998) 51-54.
6. S. Perlmutter, et al., Measurements of Ω and Λ from 42 high-redshift supernova, *Astrophys. J.*, **517(2)** (1999) 565-586.
7. D. N. Spergel, et al., First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Determination of cosmological parameters, *Astrophys. J. Suppl.*, **148** (2003) 175-194.
8. Yu. L. Bolotin, et al., Cosmological models in terms of the deceleration parameter, part 1, arXiv:1502.00811v1 [gr-qc] (2015).
9. M. S. Berman and F. de Mello Gomide, Cosmological models with constant deceleration parameter, *Gen. Relativ. Gravitation*, **20(2)** (1998) 192-198.
10. A. Pradhan, et al., FRW universe with variable G and Lambda-terms, *Rom. J. Phys.*, **52(3-4)** (2007) 445-458.
11. O. Akarsu and T. Dereli, Cosmological models with linearly varying deceleration parameter, *Int. J. Theor. Phys.*, **51(2)** (2012) 612-621.
12. O. Akarsu, et al., Probing kinematics and fate of the Universe with linearly time-varying deceleration parameter, *Eur. Phys. J. Plus,* **129(2)** (2014) 22.
13. C. Chawla, R. K. Mishra and A. Pradhan, String cosmological models from early deceleration to current acceleration phase with varying G and Λ, *Eur. Phys. J. Plus.*, **127(11)** (2012) 137.

14. R. K. Mishra, et al., Anisotropic Viscous Fluid Cosmological Models from Deceleration to Acceleration in String Cosmology, *Int. J. of Theo. Physics.,* **52(8)** (2013) 2546–2559.

15. C. Chawla , R. K. Mishra and A. Pradhan, A new class of accelerating cosmological models with variable G & Λ in Sáez and Ballester theory of gravitation, *Rom. Journ. Phys*., **59** (2014) 12-25.

16. R. K. Mishra, A. Chand and A. Pradhan, Dark Energy Models in f(R; T) Theory with Variable Deceleration Parameter, *Int. J. Theor. Phys*., **55(2)** (2016) 1241-1256.

17. A. Chand, R. K. Mishra, A. Chand and A. Pradhan, FRW cosmological models in Brans-Dicke theory of gravity with variable q and dynamical Λ-term, *Astrophys Space Sci.*, **361(2)** (2016) 81.

18. R. K. Mishra and A. Chand, Cosmological models in alternative theory of gravity with bilinear deceleration parameter, *Astrophys Space Sci*., **361(8)** (2016) 259.

19. D. Sáez and V. J. Ballester, A simple coupling with cosmological implications, *Phys. Lett. A,* **113(9)** (1986) 467-47-.

20. P. A. M. Dirac, The cosmological constants, *Nature*, **139(3512)** (1937) 323.

21. P. S. Wesson, *Gravity, Particle and Astrophysics*, D. Reidel, Dordrecht (Holland), 1980.

22. V. M. Canuto and J. V. Narlikar, Cosmological tests of the Hoyle-Narlikar conformal gravity, *Astrophys. J.*, **236** (1980) 6-23.

23. A. I. Arbab, Cosmological Models with Variable Cosmological and Gravitational "Constants" and Bulk Viscous Models, *Gen. Relativ. Gravit*., **29(1)** (1997) 61-74.

24. K. S. Thorne, Primordial Element Formation, Primordial Magnetic Fields, and the Isotropy of the Universe, *Astrophys. J.*, **148** (1967) 51.