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Cosmological Model with Bilinear Deceleration Parameter

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Abstract: The study of cosmological models with time dependent DP **(**deceleration parameter**)** is one of the important topics of research in these days**.** After publishing the research findings regarding the accelerating expansion of the universe by Perlmutter and Riess et al**.**[1,2], the subject is getting huge attention by the researchers**.** The variable DP has much importance to measure the expansion rate and it provides the opportunities to analyze the results**.** Due to the importance of the subject, we have decided to study the cosmological models with bilinear DP**.** We have assumed DP as a bilinear function of proper cosmic time as suggested by Mishra et al**.**[3]and investigated the various cosmological aspects of the model presented in pictorial form in this paper**.**

Keywords: Bianchi type-I space-time, Bilinear varying deceleration parameter, Cosmological constant.

2000 Mathematics Subject Classification: 83F05.

1. Introduction

The cosmological constant was introduced by Albert Einstein in 1917 to describe the static nature of the universe in accordance with the accepted theory of that time. Since then the problem of the cosmological constant is one of the unsettled problems in cosmology. The "cosmological constant problem" may be expressed as the discrepancy between the negligible value of cosmological constant (Λ) for the present universe[4]and the values 10^{50}larger expected by the Glashow-Salam-Weinberg model[5] or by the Grand Unified Theory (GUT) where it should be 10^{107}larger[6].Recent

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observations of Supernova type Ia(SNeIa) by two independent groups[1,7-8] and measurements of cosmic microwave background (CMB)[9] indicate that the universe is in an accelerating expansion phase. Both the cited above research groups obtain $\Omega_m \approx 0.3$ and $\Omega_\Lambda \approx 0.7$ andruled out $(\Omega_m, \Omega_\Lambda) = (1,0)$ universe. This value of the density parameter (Ω_Λ) corresponds to the cosmological constant which is small, non-zero and positive ie. $\Lambda \approx 10^{-52} m^{-2} \approx 10^{-35} s^{-2}$. As on date, the intense research is going on to know the true nature of the acceleration.

One of the most important problems in cosmology concerning to the research community is understanding of structure formation in the universe. The existing theories for the structure formation fall in two categories, based either upon the amplifications of quantum fluctuations in a scalar field during inflation or upon symmetry breaking phase transition in the early universe which leads to the formation of topological defects such as cosmic strings, monopoles and other hybrid creatures. Many cosmologists have already constructed several cosmological models by taking variable cosmological constant under different assumptions. The authors have also published their research findings by taking variable Λ with the following assumptions $\Lambda \propto H^2, \Lambda \propto \rho$ and $\Lambda \propto R^{-n}$,$n$is a positive constant[10–12], here H, ρ and R represent the Hubble's parameter, energy density and average scale factorrespectively. The Hubble's parameter (H) and dimensionless deceleration parameter (q) are the most significant parameters to understand the dynamicity of the universe. It is also noticed that time dependence of cosmic scale factor $R(t)$ has great importance as H and q are the functions of scale factor $R(t)$. Therefore, the study of various models with time dependent DP indicate new sector in theoretical cosmology. Many authors /cosmologists proposed their cosmological models with Berman's law (constant DP)[13] i.e.

$$q = m - 1. \tag{1.1}$$

Some others including our peer research group have also published several results by assuming the time dependent DP[3,14–17] along with suitable physical variation of scale factor. During investigation, we observed that Akarsu and Dereli[18] have proposed a special linear varying law for DP as:

(1. 2) $$q = -kt + m - 1.$$

Here $k > 0$ is a constant with the dimension of the inverse of time and m is a dimensionless constant. By using the law one can generalize the cosmological solutions. From the literature, we know that the universe would exhibit different nature of expansion as given below in brief:

- Super exponential expansion $(q < -1)$
- Exponential expansion $(-1 \le q < 0)$
- Expansion with constant rate $(q = 0)$
- Accelerating power expansion $(-1 < q < 1)$
- Decelerating expansion $(q > 0)$

We know that DP is defined as

(1. 3) $$q = \frac{-R\ddot{R}}{\dot{R}^2},$$

which is a dimensionless time-variable quantity. This variable deceleration parameter has much importance to predict and analyze the expansion of the universe. Now we wish to assume the above deceleration parameter as bilinear varying deceleration parameter (BVDP) which is given by

(1. 4) $$q = \frac{\alpha(1-t)}{1+t}, \alpha > 0,$$

as suggested by Mishra et al[3]. Here the present paper is divided into four different sections including introduction presented in section 1, the metric and field equations are described in section 2 while the solution of field equations and the physical and geometrical properties of the model are presented in section 3.At the end of the paper, results and concluding remarks are summarized in section 4.

2. The Metric and Field Equations

We wish to consider the space time of totally anisotropic Bianchi type-I line element given by

(2. 1) $$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2,$$

where the metric potentials A, B and C are the functions of t only.This ensures that model is spatially homogeneous.The Einstein's field equations (EFE) with the time varying Λ read as

(2. 2) $$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G T_{ij} + \Lambda(t) g_{ij},$$

where T_{ij} is the energy momentum tensor for a perfect fluid distribution, given by

(2. 3) $$T_{ij} = (\rho + p) u_i u_j - p g_{ij},$$

where $u_i = (0,0,0,1)$ is the four-velocity vector of fluid particles satisfying the condition $u_i u^i = -1$. Here p is the pressure and ρ is the energy density of cosmic fluid. The field equations (2.2) for the line element (2.1) and the energy distribution (2.3) lead to the following system of differential equations

(2. 4) $$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda,$$

(2. 5) $$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G p + \Lambda,$$

(2. 6) $$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -8\pi G p + \Lambda,$$

(2. 7) $$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = 8\pi G \rho + \Lambda.$$

The energy conservation equation $T^{ij}_{,j} = 0$ leads

(2. 8) $$\dot{\rho} + 3H(\rho + p) = 0,$$

where H is the Hubble's parameter, which for Bianchi type-I is defined as

(2. 9) $$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{1}{3}(H_x + H_y + H_z).$$

Here H_x, H_y and H_z are the directional Hubble's parameters in the direction of x, y and z respectively.

The spatial volume for model (2.1) is given by

(2. 10) $$V = R^3 = ABC.$$

The expansion scalar (θ), the shear scalar (σ) and the anisotropic parameter (A_m) are defined as

(2. 11) $$\theta = 3\frac{\dot{R}}{R} = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right),$$

(2. 12) $$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2,$$

(2. 13) $$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2.$$

The field equations (2.4) to (2.7) involves six unknowns viz. A, B, C, p, ρ and Λ. Inorder to solve the field equations explicitly, we require two additional constraints which we shall consider in our next section.

3. Solution of Field Equations

Firstly, we wish to consider the bilinear form of the DP as suggested by Mishra et al[3].

(3.1) $$q = \frac{\alpha(1-t)}{1+t}, \alpha > 0.$$

The motivation to choose such time-dependent DP is that the universe has an accelerated expansion at present as observed in the recent observations of SNeIa[8,19–22] and CMB anisotropies[2,23- 24] and a decelerated expansion in the past. Now for the Universe which was decelerating in the past and accelerating in the present time, the DP must show signature flipping[25,26]. So in general, we may say that the DP is not a constant but a time variable. Now further to calculate the Hubble's parameter, we write the expression of q as

(3. 2) $$q(t) = \frac{d}{dt}\left(\frac{1}{H} - t\right).$$

On integrating above equation, we get

(3. 3) $$H = \frac{1}{\int q(t)dt + t + k}.$$

On substituting the value of $q(t)$ in Eq.(3.3), we get

(3. 4) $$H = \frac{1}{\int\left(1 + \frac{\alpha(1-t)}{1+t}\right)dt + k}.$$

Here k is a constant of integration. For physical viability, $k = 0$ because $t \to 0$, $H \to \infty$. As at early inflationary stage the expansion rate is very-very large.

(3. 5) $$H = \frac{1}{t(1-\alpha) + 2\alpha \log(1+t)}.$$

Since $H = \frac{\dot{R}(t)}{R(t)}$, so on simplification from above expression, we have

(3. 6) $$R(t) = R_0 t^{\frac{1}{1+\alpha}}.\exp(F_1(t)),$$

where

(3. 7) $$F_1(t) = \frac{\alpha}{(1+\alpha)^2}t + \frac{-2\alpha + \alpha^2}{6(1+\alpha)^3}t^2 + \frac{3\alpha - 2\alpha^2 + \alpha^3}{18(1+\alpha)^4}t^3$$

$$+ \frac{-18\alpha + 11\alpha^2 - 14\alpha^3 + 2\alpha^4}{180(1+\alpha)^5}t^4 + O(t^5).$$

and R_0 is a constant of integration.

Secondly we assume that the component σ_1^1 of the shear tensor $\left(\sigma_i^j\right)$ is proportional to the expansion scalar (θ) ie. $\sigma_1^1 \propto \theta$.[27] This condition leads to

(3. 8) $$\frac{1}{3}\left(3\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=\beta\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right),$$

which yields

(3. 9) $$\frac{\dot{A}}{A}=l\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right),$$

where $l=\dfrac{1+3\beta}{3(1-\beta)}$ with β being the constant of proportionality.

Above equation, after integration reduces to

(3. 10) $$A=k_1(BC)^l.$$

Without any loss of generality, we assume $k_1 = 1$ for simplicity.Hence we have

(3. 11) $$A=(BC)^l.$$

Subtracting (2.6) from (2.4) and taking the integral of resultant equation two times, we get

(3. 12) $$\frac{B}{C}=k_2\exp\left(k_3\int\frac{1}{R^3}dt\right),$$

where k_2 and k_3 are constants of integration.By solving (3.11) and (3.12), we get

(3. 13) $$A(t)=R^{\frac{3l}{l+1}},$$

(3. 14) $$B(t)=\sqrt{k_2}R^{\frac{3}{2(l+1)}}\exp\left(\frac{k_3}{2}\int\frac{1}{R^3}dt\right),$$

$$C(t) = \frac{1}{\sqrt{k_2}} R^{\frac{3}{2(l+1)}} \exp\left(-\frac{k_3}{2} \int \frac{1}{R^3} dt \right). \qquad (3.15)$$

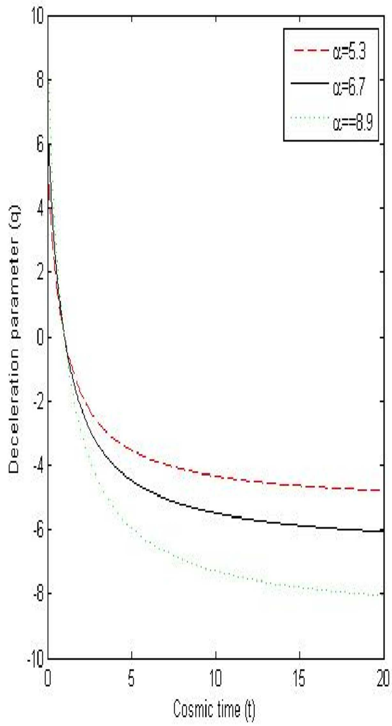

Fig 1: The plot of deceleration parameter (q) vs. time (t)

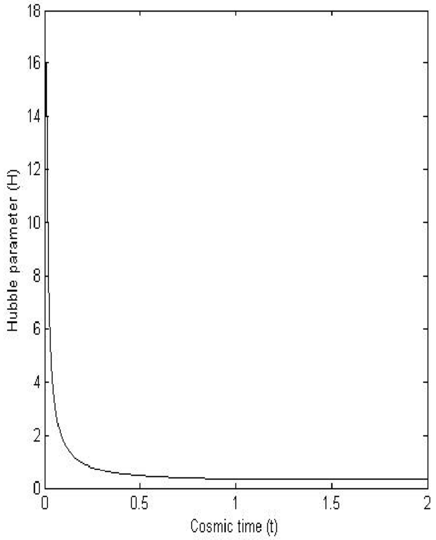

Fig 2: The plot of Hubble's parameter (H) vs. time (t) for α = 5.3

Using Eq.(3.6) in (3.13)-(3.15), we will obtain the following expressions for metric potentials

$$A(t) = \left(R_0 t^{\frac{1}{1+\alpha}} \exp(F_1(t)) \right)^{\frac{3l}{l+1}}, \tag{3.16}$$

$$B(t) = \sqrt{k_2} \left(R_0 t^{\frac{1}{1+\alpha}} .\exp(F_1(t)) \right)^{\frac{3}{2(l+1)}} \exp(F_2(t)), \tag{3.17}$$

$$C(t) = \frac{1}{\sqrt{k_2}} \left(R_0 t^{\frac{1}{1+\alpha}} .\exp(F_1(t)) \right)^{\frac{3}{2(l+1)}} \exp(-F_2(t)), \tag{3.18}$$

where $F_2(t) = \frac{k_3}{2} \int R_0^{-3} t^{\frac{-3}{1+\alpha}} \exp(-3F_1(t))dt.$ For the derived model, expressions for observational physical quantities such as spatial volume (V), Hubble's parameter (H), directional Hubble's parameters (H_i), expansion scalar (θ), shear scalar (σ), and anisotropic parameter (A_m) are given by

(3. 19) $$V = ABC = R^3 = R_0^3 t^{\frac{3}{1+\alpha}} \exp(3F_1(t)),$$

(3. 20) $$H = \frac{1}{t(1-\alpha) + 2\alpha \log(1+t)},$$

(3. 21) $$H_x = \frac{\dot{A}}{A} = \frac{3l}{l+1} H,$$

(3. 22) $$H_y = \frac{\dot{B}}{B} = \frac{3}{2(l+1)} H + F_3(t),$$

(3. 23) $$H_z = \frac{\dot{C}}{C} = \frac{3}{2(l+1)} H - F_3(t),$$

(3. 24) $$\theta = 3H = \frac{3}{t(1-\alpha) + 2\alpha \log(1+t)},$$

(3. 25) $$\sigma^2 = \frac{3}{4}\left(\frac{2l-1}{l+1}\right)^2 H^2 + \left(F_3(t)\right)^2,$$

(3. 26) $$A_m = \frac{1}{2}\left(\frac{2l-1}{l+1}\right)^2 + \frac{2}{3}\left[\left\{t(1-\alpha) + 2\alpha \log(1+t)\right\} F_3(t)\right]^2.$$

From the above equations (3.19)-(3.25), it can be observed that at $t = 0$, the spatial volume (V)vanishes while the other parameters such as Hubble's parameter (H), directional Hubble's parameters (H_i), expansion scalar (θ) and shear scalar (σ) are infinite, which is a big bang scenario. As $t \to \infty$, V diverges to ∞, whereas H, H_i, θ and σ approach to zero.

(3. 27) $$\frac{\sigma^2}{\theta^2} = \frac{1}{12}\left(\frac{2l-1}{l+1}\right)^2 + \left[\frac{\{t(1-\alpha)+2\alpha\log(1+t)\}F_3(t)}{3}\right]^2.$$

Clearly $\frac{\sigma^2}{\theta^2} \to \frac{1}{12}\left(\frac{2l-1}{l+1}\right)^2$ as $t \to \infty$ indicating that $\frac{\sigma^2}{\theta^2}$ does not approach to zero as long as $l \neq \frac{1}{2}$. It means we may say that our universe does not reach isotropy throughout the evolution of the universe unless $l = \frac{1}{2}$. Therefore, the dynamic of isotropic condition depends upon the value of l and for $l = \frac{1}{2}$, $\frac{\sigma^2}{\theta^2} \to 0$ as $t \to \infty$ ie. the model approaches isotropy at late times.Using the (2.8), we get the following expression for ρ

(3. 28) $$\rho = \exp\left(-3(1+\omega)\int H dt + k_4\right).$$

Here k_4 is a constant of integration which may be taken as zero for simplicity.

(3. 29) $$\rho = t^{\frac{-3(1+\omega)}{1+\alpha}} \exp(-3(1+\omega)F_1(t)),$$

(3. 30) $$p = \omega t^{\frac{-3(1+\omega)}{1+\alpha}} \exp(-3(1+\omega)F_1(t)).$$

From (2.7), we get the following expression for $\Lambda(t)$

(3. 31) $$\Lambda(t) = \frac{9}{4}\frac{(4l+1)}{(l+1)^2}H^2 - (F_3(t))^2 - 8\pi G\left[t^{\frac{-3(1+\omega)}{1+\alpha}} \exp(-3(1+\omega)F_1(t))\right],$$

where $\omega = \frac{p}{\rho}$ is an EoS parameter.

Here $F_3(t)=\dot{F}_2(t)=\left(\dfrac{k_3}{2R_0^3 t^{\frac{3}{1+\alpha}}\exp(3F_1(t))}\right).$

4. Results and Discussions

The present study deals with the homogeneous and anisotropic Bianchi type-I space time in general theory of relativity. Here we have obtained the solution of the EFE by assuming DP as a bilinear function of the cosmic time. For better understanding of model, some physical and geometrical parameters have also been determined. The major outcomes of this study are listed below

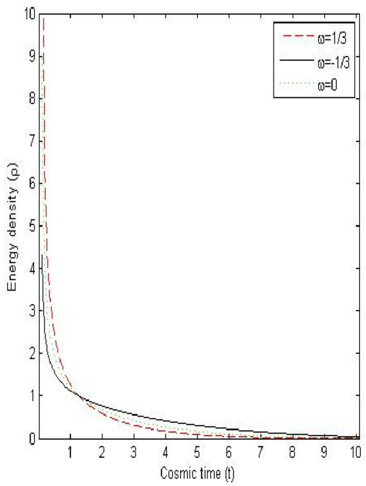

Fig 3: The plot of energy density (ρ) vs. time (t) with α = 5.3.

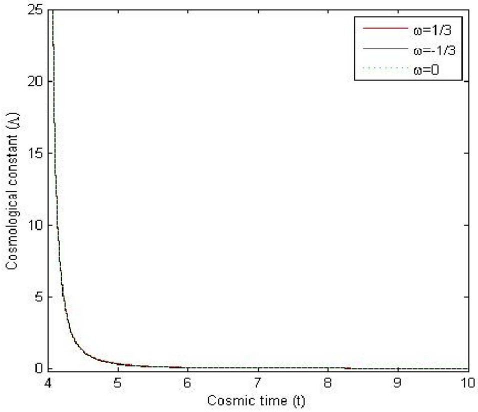

Fig 4:The plot of cosmological constant vs. (Λ) time (t) with $\alpha = 5.3$, $R_0 = 1$, $l = 2.345$ and $k_3 = 1$.

- It can be observed from the (1.4) that $q > 0$ if $0 < t < 1$, $q = 0$ at $t = 1$ and $q < 0$ if $t > 1$. Hence we may say that the universe was decelerating at early time $t < 1$, constant expansion for a moment at $t = 1$ and for $t > 1$ expanding with an accelerating rate. Figure 1 shows the variation of DP q with cosmic time t for the chosen values of α (i.e. $\alpha = 5.3, 6.7, 8.9$). The behaviour of DP shows the transitional phase of the universe (i.e.from early time deceleration to present accelerating phase).

- It can be seen from the Eq(3.19) that the spatial volume is zero at t=0 and thereafter increasing continuously with time t. Hence our constructed model shows similar behaviour as Big Bang model of the universe.

- Figure 2 depicts the variation of Hubble's parameter versus time t. The value of $H \to 0$ as $t \to \infty$. Such type of the behaviour of the universe validate by recent cosmological observations.[1,2,8,19]
- The variation in energy density parameter ρ against time t is presented in figure 3. The figure indicates that energy density is a

positive valued and decreasing function of time. It is also approaching zero with the evolution of time.

- In figure 4, we have seen the variation of Λ against time t. It has been found that cosmological constant Λ is a decreasing function of time and it approaches to a small positive value at the present which is in good agreement with the recent observations.[8,19]

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