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MHD Stagnation Point Flow and Heat Transfer of a Micropolar Fluid in a Porous Medium

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Abstract: In the present paper, an analysis is carried out to study the steady laminar flow of an incompressible electrically conducting micropolar fluid impinging on a permeable flat plate in a porous medium in the presence of transverse magnetic field. The effect of viscous dissipation is taken into account. The governing boundary layer equations are transformed to a system of nonlinear ordinary differential equations by taking suitable similarity variables. The resulting equations are solved numerically using Runge-Kutta method accompanied with Shooting Technique. The effects of various parameters such as porosity, magnetic, material, prandtl number and the Eckert number for corresponding velocity and temperature field have been discussed in detail through graphical representation.

Keywords: Micropolar fluid, MHD, Heat transfer, Stagnation point, Porous medium.

1. Introduction

Concept of micropolar fluid deals with a class of fluids that exhibit microscopic effects arising from the local structure and micromotions of the fluid elements. Micropolar fluids are those which contain dilute suspensions of rigid macro-molecules with individual motions that support stress and body moments and are influenced by spin inertia. Eringen[1] proposed a theory of molecular fluids taking into account the local effects arising from the microstructure and the intrinsic motion of the fluid elements. Physically micropolar fluid can consist of a suspension of small, rigid cylindrical elements such as large dumbbell-shaped molecules. The theory of micropolar fluid has many practical applications e.g. analyzing the behaviour of exotic lubricants, liquid-crystels, additive suspensions, polymeric fluids, turbulent shear flow etc. Wilson[2] studied the micropolar boundary layer flow near a stagnation point with the aid of Karman-Polhausen approximate integral method. Peddieson and McNitt[3] applied the micropolar boundary layer theory to the problems of steady flow over a semi-infinite flat plate. Eringen[4] extended the theory of micropolar fluids to the theory of thermo-micropolar fluids. An application of the micropolar fluid model to the calculation of turbulent shear flow was investigated by Peddieson[5]. Ahmadi[6] studied the self-similar solution of incompressible micropolar boundary layer flow over a semi-infinite plate. Jena and Mathur[7] investigated the similarity solutions for laminar free-convection flow of a thermo-micropolar fluid past a non-isothermal flat plate. Guram and Smith[8] studied the steady stagnation flows of micropolar fluids with strong and weak interactions. Ramchandran and mathur[9] studied the thermal boundary layer of micropolar fluids in the vicinity of a stagnation point. Jena and Mathur[10] studied the free-convection in the laminar boundary layer flow of thermo-micropolar fluids past a non-isothermal vertical flat plate with succession and injection.Gorla[11] investigated the problem of stagnation point on a moving wall and discussed the dependence of the important flow characteristics on the material parameter. Dey and Nath[12] studied micropolar fluid flow over a semi-infinite plate using parabolic coordinates to consider the flow regime including the leading edge. Takhar and Soundalgekar[13] studied the effects of suction & injection on the flow of a micropolar fluid past a continuously moving semi-infinite porous plate. Sharma and Gupta[14] studied the effects of medium permeability on the flow of micropolar fluids. The effects of magnetic field on the laminar boundary layer mixed convection flow of a micropolar fluid over a horizontal plate were studied by Mohammadein and Gorla[15]. Rees and Bassom[16] studied the blasius boundary layer flow over a flat plate and this study suggests that much more information about the solution of boundary layer flows of a micropolar fluid can be obtained. A thorough study of micropolar fluids and its applications was done by Lukaszewicz[17].

In recent years the flow problems of fluid saturated porous media has become increasingly more attractive to researchers due to its many different practical applications in industries and environmental issues such as geothermal power plants, building thermal insulators, grain storage etc. Youn and Lee[18] studied the problem of oscillatory two dimensional laminar flow of a viscous incompressible electrically conducting micropolar fluid over a semi-infinite vertical moving porous plate ìn the presence of a transverse magnetic field. Zakaria[19] studied the effect of a transverse magnetic field on the motion of an electrically conducting micropolar fluid through a porous medium in one-dimension. Hassanien et al.[20] studied the natural convection flow of micropolar fluid from a permeable uniform heat flux surface in porous media. The problem of stagnation point flow of a micropolar fluid towards a stretching sheet was studied by Nazar et al.[21]. Attia[22, 23] studied the problems of stagnation point flow and heat transfer of a micropolar fluid. Rahman and Sattar[24] studied the MHD convective flow of a micropolar fluid past a continuously moving vertical porous plate in the presence of heat generation/absorption. Lok et al.[25] investigated unsteady mixed convection flow of a micropolar fluid near the stagnation point on a vertical surface. Zaimi et al.[26] studied the boundary layer flow due to a stretching sheet in a porous medium filled with a micropolar fluid. Recently the problems of micropolar fluid through the porous medium were studied by Sultana et al.[27], Islam et al.[28] and Mekheimer et al.[29].

The purpose of the present paper is to study the effect of the porosity of the medium on the steady laminar flow of an incompressible electrically conducting micropolar fluid at a two-dimensional stagnation point with heat transfer in the presence of a uniform transverse magnetic field. The flow in the porous media deals with the study in which the differential equation governing the fluid motion is based on the Darcy's law. Numerical solutions are obtained for the governing momentum and energy equations by standard techniques.

2. Mathematical Formulation

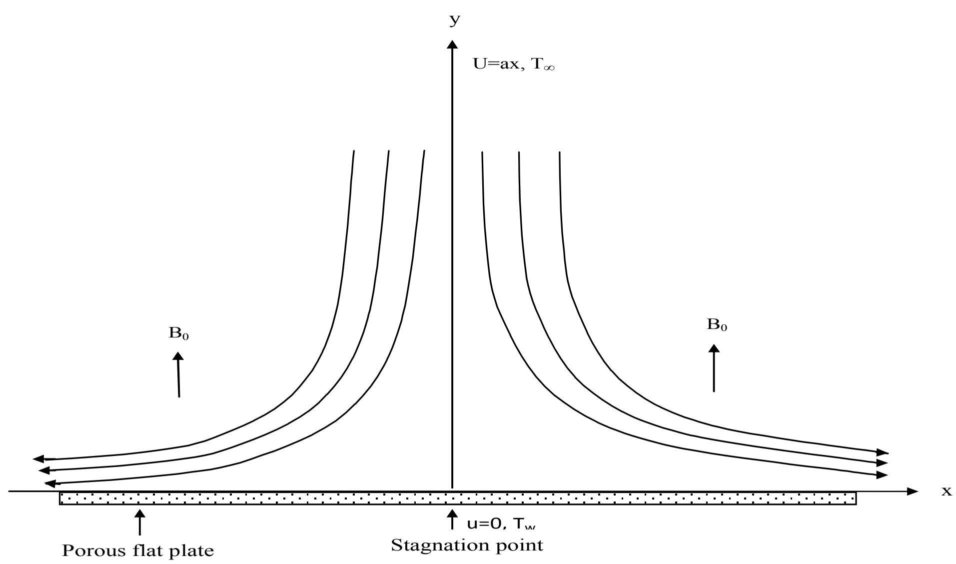

Figure 1. Physical model and coordinate system

Consider a steady two-dimensional flow of an incompressible electrically conducting micropolar fluid impinging normally near the stagnation point on a permeable horizontal porous flat plate placed at $y = 0$ and flowing away along the x-axis, divides into two streams on the plate in both directions (Figure 1). Let (u, v) be the velocity components at any point (x,y). A uniform transverse magnetic field of strength B_0 is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected as compared to the applied field. We further assume that there is no polarization voltage, so the induced electric field is zero. The potential flow velocity components in the vicinity of the stagnation point is given by $U(x) = ax$ and $V(y) = -ay$ where the constant a (>0) is proportional to the free stream velocity far away from the plate. The fluid properties are assumed to be constant. Under the usual boundary layer approximation including the viscous dissipation, the governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \frac{(\mu+h)}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{h}{\rho}\frac{\partial N}{\partial y} - \frac{1}{\rho}\sigma B_0^2 u + \frac{\mu}{\rho D1}[U(x) - u] \tag{2.2}$$

$$\rho\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \frac{\gamma}{j}\frac{\partial^2 N}{\partial y^2} - \frac{h}{j}\left(2N + \frac{\partial u}{\partial y}\right) \tag{2.3}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} + (\mu + h)\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 u^2 \tag{2.4}$$

Subject to the boundary conditions:

$$\begin{aligned} y = 0 \ &: \ u = 0,\ v = 0,\ N = -m\frac{\partial u}{\partial y},\ T = T_w \\ y \to \infty &: \ u = U(x) = ax,\ v \to 0,\ N \to 0,\ T = T_\infty \end{aligned} \tag{2.5}$$

where N is the component of the microrotation vector normal to the xy-plane, T and T_w are the temperatures of the fluid and the plate, respectively whereas the temperature of the fluid far away from the plate is T_∞, μ is the viscosity, ρ is the density, σ is the electrical conductivity, κ is the thermal conductivity, c_p is the specific heat at constant pressure, j is the micro-inertia density, γ is the spin gradient viscosity, h is the vortex viscosity, D1 is the Darcy permeability and m ($0 \le m \le 1$) is the boundary parameter. The case $m = 0$, which indicates $N = 0$ at the surface represents concentrated particle flows in which the microelements close to the wall surface are unable to rotate (Jena and Mathur[7]). This case is also known as strong concentration of microelements (Guram and Smith[8]). The case $m = 1/2$ indicates the vanishing of anti-symmetric part of the stress tensor and denotes weak concentration (Ahmadi[6]). The case $m = 1$ is used for the modeling of turbulent boundary layer flows (Peddieson[5]). In the present problem γ, j and h are assumed to be constants and γ is assumed to be given by Nazar et al.[21]

(2.6) $$\gamma = (\mu + \frac{h}{2}) j$$

We take $j = \frac{\upsilon}{a}$ as a reference length, where υ is the kinematic viscosity.

3.Analysis

The equation of continuity (2.1) is identically satisfied by stream function $\Psi(x, y)$ defined as

(3.1) $$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

For the solution of the equations (2.2) to (2.4), the following dimensionless similarity variables are defined:

(3.2) $$\psi(x, y) = x\sqrt{a\upsilon} f(\eta), \quad \eta = y\sqrt{\frac{a}{\upsilon}}, \quad N(x, y) = ax\sqrt{\frac{a}{\upsilon}} g(\eta),$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}$$

Thus the equations (2.2) to (2.4) reduce to (after some simplifications):

(3.3) $$(1 + K) f''' + ff'' - f'^2 + S(1 - f') + Kg' + 1 - Mf' = 0$$

(3.4) $$(1+\frac{K}{2})g''+fg'-f'g-K(2g+f'')=0$$

(3.5) $$\theta''+\Pr f\theta'+(1+K)\Pr Ecf''^2+MEc\Pr f'^2=0$$

The corresponding boundary conditions (2.5) are:

(3.6) $$\eta=0: f=0,\ f'=0,\ g=-mf'',\ \theta=1$$

$$\eta\to\infty: f'\to 1,\ g\to 0,\ \theta\to 0$$

where primes denote differentiation with respect to η, $K=\frac{h}{\mu}$ (+ve) is the material parameter, $M=\frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter, $S=\frac{\upsilon}{aD1}$ is the porosity parameter, $\Pr=\frac{\mu c_p}{\kappa}$ is the prandtl number and $Ec=\frac{U^2}{c_p(T_w-T_\infty)}$ is the Eckert number. Equation (3.4) is same as that obtained by Attia [23] for non-magnetic case whereas the equation (3.3) and (3.5) are differential equations with values prescribed at two boundaries (3.6) which can be converted to initial value problem by known technique. These equations are solved numerically on the computer for different values of the various parameters.

The physical quantities of interest are the local skin-friction coefficient C_f and the local Nusselt number Nu which are defined, respectively, as

(3.7) $$C_f=\frac{\tau_w}{\rho U^2},\quad Nu=\frac{xq_w}{\kappa(T_w-T_\infty)}$$

where τ_w is the wall shear stress which is given by

$$\tau_w=\left\{(\mu+h)\frac{\partial u}{\partial y}+hN\right\}_{y=0}$$

and $q_w=-\kappa\left\{\frac{\partial T}{\partial y}\right\}_{y=0}$ is the heat transfer from the plate.

Thus, we get

(3.8) $$C_f = \frac{(1+\frac{K}{2})}{\sqrt{\text{Re}}} f''(0) \quad \text{and} \quad Nu = -\sqrt{\text{Re}}\,\theta'(0)$$

where $\text{Re} = \frac{xU}{\upsilon}$ is the local Reynolds number.

3. Results and Discussion

The non linear ordinary differential equations (3.3) and (3.5) subject to the boundary conditions (3.6) were solved numerically. The computations were done by a programme which uses a symbolic and computational language Matlab. The velocity and temperature profiles for various values of the parameters involved are plotted against η in Figures 2 to 4 and Figures 5 to 8 respectively. It is observed from these graphs of velocity profiles that the velocity increases with the increasing values of the parameter S while it decreases with the increase of the parameters K and M. It is evident from the graphs of temperature profiles that the temperature increases with the increasing values of the parameters K, M and Ec while it decreases with the increasing values of the parameters S and Pr. Numerical values of the functions $f''(0)$ and $-\theta'(0)$, which are proportional to skin friction and nusselt number respectively for various values of parameters are presented in tables 1 and 2 respectively.

Table 1: Numerical values of $f''(0)$ for various values of parameters M and S when K=1.

M	S=0	S=0.1	S=0.2	S=0.5	S=1
0	1.0064	1.0387	1.0701	1.1592	1.2944
0.1	0.9670	1.0007	1.0334	1.1257	1.2650
0.2	0.9299	0.9650	0.9988	1.0941	1.2370
0.5	0.8321	0.8701	0.9066	1.0089	1.1604
1	0.7067	0.7471	0.786	0.8944	1.0540

Table 2: Numerical values of $-\theta'(0)$ for various values of parameters M, Ec, S and Pr when K=1.

		Pr=0.05					Pr=0.1				
S	Ec	M=0	M=0.1	M=0.2	M=0.5	M=1	M=0	M=0.1	M=0.2	M=0.5	M=1
0	0	0.198	0.197	0.195	0.192	0.188	0.231	0.229	0.226	0.219	0.210
	0.1	0.192	0.190	0.189	0.185	0.181	0.221	0.217	0.214	0.206	0.196
	0.5	0.167	0.164	0.161	0.156	0.154	0.177	0.170	0.165	0.152	0.141
	1	0.135	0.131	0.127	0.119	0.118	0.122	0.112	0.103	0.085	0.073
	2	0.073	0.065	0.058	0.047	0.045	0.013	0.004	-0.02	-0.049	-0.06
0.5	0	0.201	0.200	0.198	0.195	0.191	0.234	0.232	0.230	0.224	0.215
	0.1	0.194	0.193	0.191	0.187	0.182	0.222	0.218	0.215	0.208	0.198
	0.5	0.169	0.165	0.162	0.155	0.147	0.172	0.165	0.158	0.144	0.129
	1	0.138	0.131	0.126	0.114	0.104	0.109	0.097	0.087	0.063	0.044
	2	0.074	0.063	0.053	0.032	0.017	-0.01	-0.036	-0.055	-0.096	-0.12
1	0	0.200	0.199	0.198	0.196	0.193	0.236	0.234	0.232	0.227	0.220
	0.1	0.192	0.190	0.189	0.185	0.183	0.222	0.219	0.216	0.209	0.199
	0.5	0.159	0.155	0.152	0.145	0.142	0.167	0.159	0.152	0.136	0.118
	1	0.117	0.111	0.106	0.094	0.090	0.097	0.084	0.072	0.044	0.017
	2	0.034	0.023	0.013	-0.007	-0.01	-0.04	-0.065	-0.087	-0.137	-0.18
		Pr=0.5					Pr=1				
S	Ec	M=0	M=0.1	M=0.2	M=0.5	M=1	M=0	M=0.1	M=0.2	M=0.5	M=1
0	0	0.411	0.403	0.396	0.376	0.374	0.535	0.526	0.517	0.492	0.454
	0.1	0.360	0.351	0.342	0.318	0.288	0.441	0.430	0.419	0.390	0.352
	0.5	0.160	0.142	0.125	0.087	0.052	0.063	0.043	0.026	-0.014	-0.05
	1	-0.08	-0.119	-0.144	-0.200	-0.24	-0.40	-0.439	-0.465	-0.521	-0.56
	2	-0.59	-0.642	-0.686	-0.778	-0.83	-1.35	-1.405	-1.448	-1.535	-1.57
0.5	0	0.420	0.414	0.408	0.392	0.367	0.549	0.542	0.535	0.514	0.483
	0.1	0.362	0.353	0.345	0.323	0.293	0.440	0.429	0.419	0.392	0.355
	0.5	0.132	0.111	0.092	0.046	0.00	-0.01	-0.020	-0.043	-0.097	-0.15
	1	-0.15	-0.191	-0.223	-0.298	-0.37	-0.54	-0.584	-0.621	-0.709	-0.79
	2	-0.73	-0.797	-0.855	-0.990	-1.11	-1.63	-1.710	-1.777	-1.934	-2.07
1	0	0.427	0.422	0.417	0.403	0.382	0.561	0.554	0.548	0.531	0.504
	0.1	0.363	0.354	0.346	0.324	0.295	0.438	0.427	0.417	0.390	0.353
	0.5	0.104	0.081	0.060	0.008	-0.05	-0.05	-0.081	-0.107	-0.173	-0.25
	1	-0.21	-0.258	-0.295	-0.386	-0.48	-0.66	-0.717	-0.763	-0.878	-1.00
	2	-0.86	-0.939	-1.008	-1.177	-1.35	-1.89	-1.990	-2.076	-2.288	-2.51

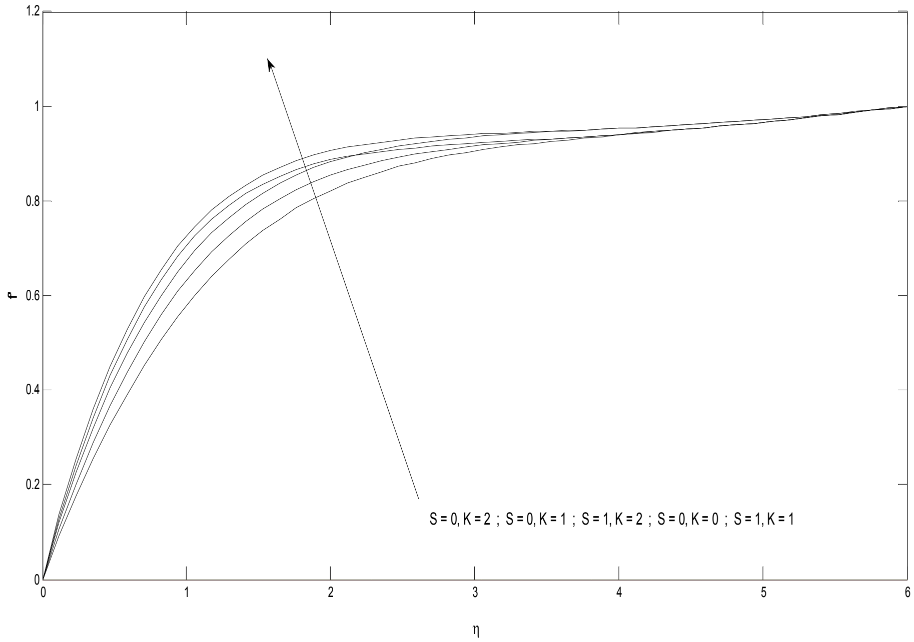

Figure 2. Velocity profile against η for various values of parameters S and K when M=0.2

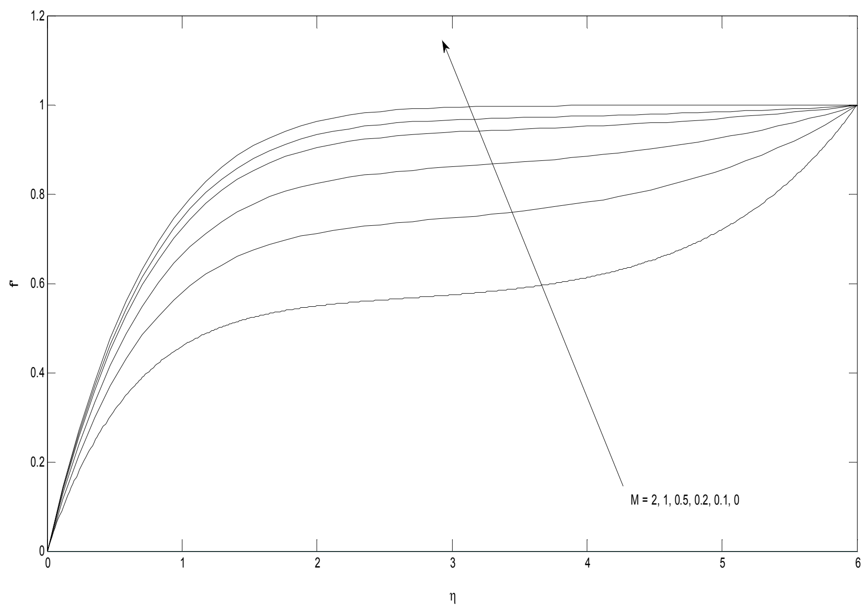

Figure 3. Velocity profile against η for various values of parameter M when S=1 and K=1

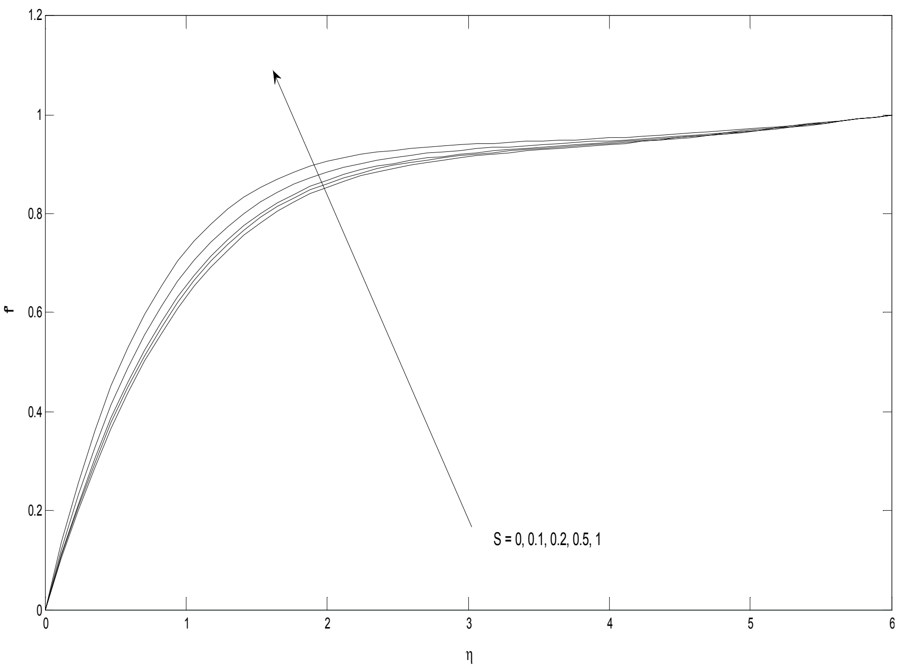

Figure 4. Velocity profile against η for various values of parameter S when K=1 and M=0.2

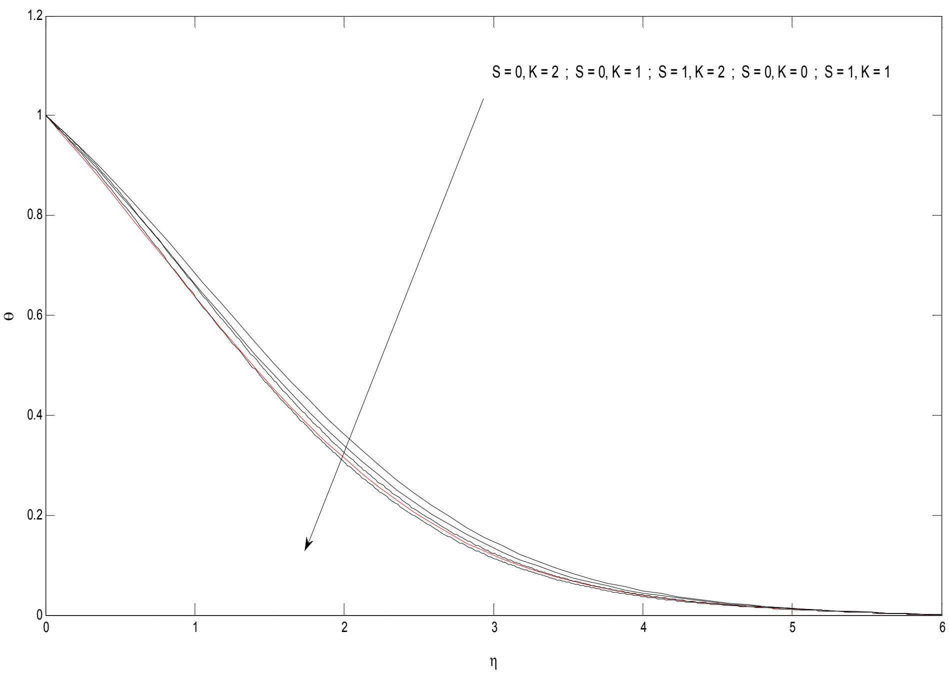

Figure 5. Temperature profile against η for various values of parameters S and K when Pr=0.5, Ec=0.2 and M=0.2

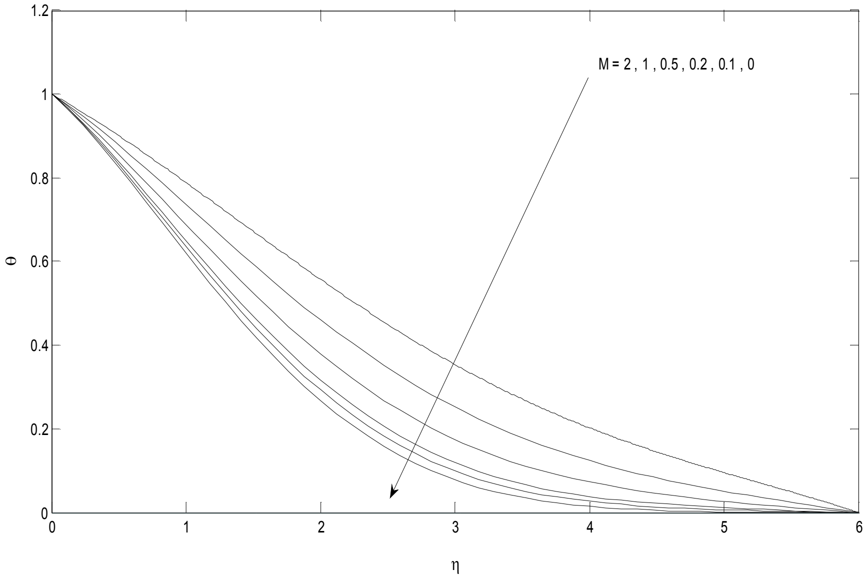

Figure 6. Temperature profile against η for various values of parameter M when Pr=0.5, Ec=0.2, K=1 and S=1

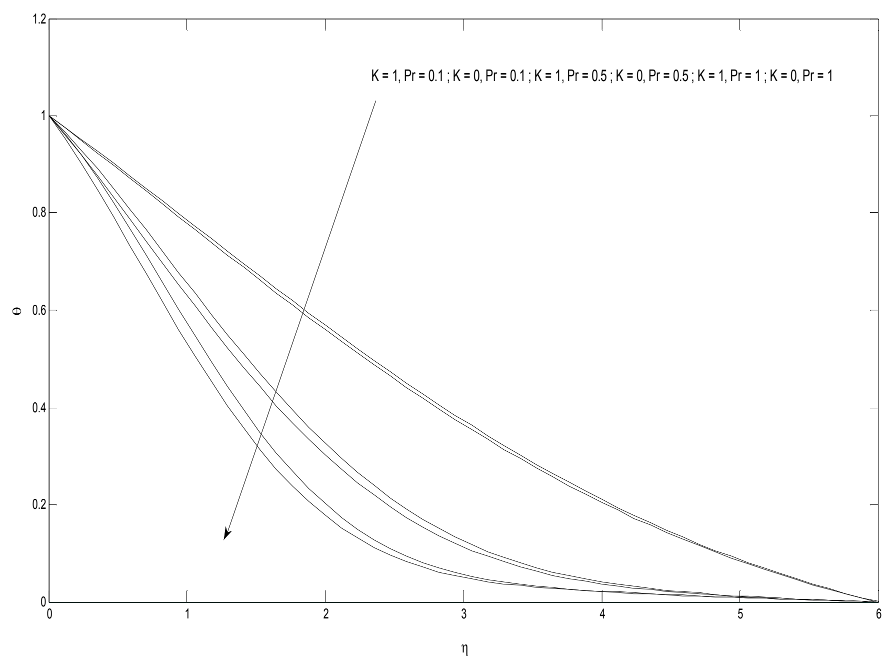

Figure 7. Temperature profile against η for various values of parameters K and Pr when S=0.5, M=0.2 and Ec=0.2

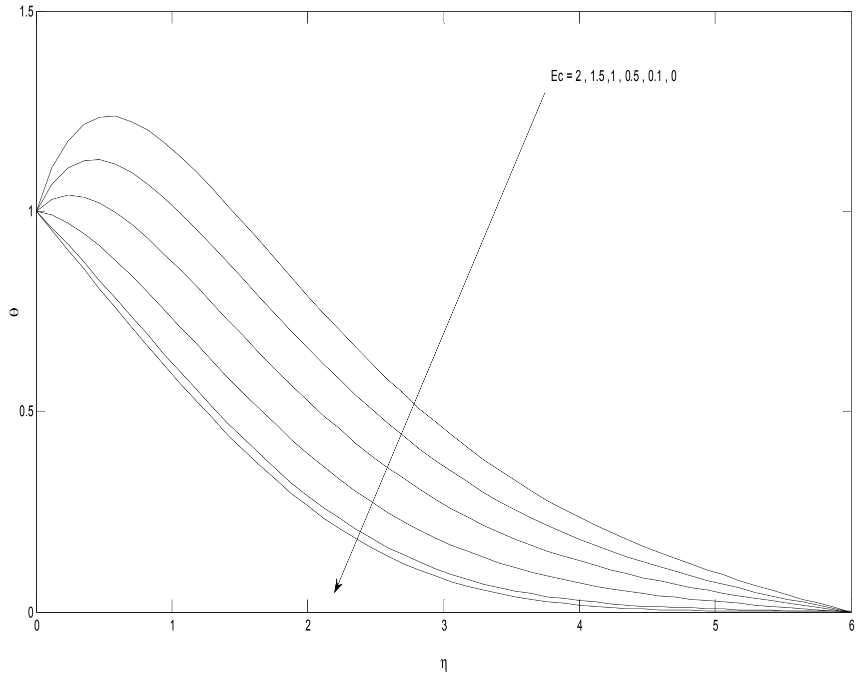

Figure 8. Temperature profile against η for various values of parameter Ec when S=1, K=1, Pr=0.5 and M=0.2

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