ISSN 0974 - 9373

Vol. 17 No.3 (2013) **Journal of International Academy of Physical Sciences** pp. 233-244

Steady Flow between Two Porous Parallel Plates with Varying Viscosity

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(Received April 17, 2013)

Abstract: Flow between two porous parallel plates with varying viscosity is studied. Exact solutions for velocity and shearing stress are obtained. For comparative studies, expressions of velocity for particular case when viscosity is constant are also obtained. Solutions are presented and discussed graphically with the variation of viscosity and Reynolds number.

Keywords: Navier-Stokes Equation, Variable viscosity, Plate with suction and injection, Laminar flow.

Mathematics Subject Classification: 2010-58D30

1. Introduction

All the fluids are viscous. In the case where the viscous effect is minimal, fluid can be treated as an ideal fluid. It is necessary to treat the fluid as viscous fluid. Generally viscosity of the fluid is taken to be constant, but it may be change due to temperature and due to suspending solid particles. Haber and Brenner[1] and J.B.Shukla[2] studied a fluid flow by taking the viscosity as spatial varying. Eswara and Bommiah[3], Hazema[4], and Saikrishnan Roy studied the fluid flow with temperature dependent viscosity. Malik and Hooper[5], Gelu Pasa and Olivier Titaud[6] and Vanparthy and Meiburg[7] considered the fluid flow with viscosity as a exponential function of concentration. Many research papers have been published dealing with the flow of an incompressible viscous fluid through porous channels. Ahmadi and Manvi[8] derived the general equation of motion and applied the results in some basic flow problems. Gulab Ram and Mishra[9] applied these equations in to study the MHD flow of conducting fluid through porous media. This has enables us to understand the cooling process in rocket and jet. In this method a flow in the direction perpendicular to the main direction of flow is created by suction and injection of the fluid at the boundaries.

The aim of the present paper is to study the flow of a viscous, incompressible fluid between two parallel porous plates when viscosity is varying linearly on distance between two plates.

2. Mathematical Formulation and Solution of Problem

Here we consider the steady, laminar flow of viscous, incompressible fluid between two infinite parallel porous plates with injection and suction, separated by distance $\bar{h}$. In dimensional form, let $\bar{x}$ the direction of main flow, $\bar{y}$ the direction perpendicular to the flow and the width of the plates are parallel to the $\bar{z}$ direction. We take the component of velocity along $\bar{x}$ direction $\bar{u} = \bar{u}(\bar{y})$ and along $\bar{z}$ direction $\bar{w} = 0$. Then the equation of continuity $div\,\bar{\mathbf{q}} = 0$ reduces to $\frac{\partial \bar{v}}{\partial \bar{y}} = 0$. This implies the fluid flow through injection and suction are same say $\bar{v_0}$. So there is a constant velocity component along $\bar{y}$ direction.

The Navier-Stokes equations of motion for laminar, steady, viscous, and incompressible flow in dimensional form along $\bar{x}$ and $\bar{y}$ direction are given by

$$\text{(1)} \qquad \overline{\rho v_0} \frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial}{\partial \bar{y}} \left[\mu\left(\bar{y}\right) \frac{\partial \bar{u}}{\partial \bar{y}} \right]$$

$$\text{(2)} \qquad \frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial \bar{y}} = 0$$

From (2), pressure does not depend on $\bar{y}$, so $\bar{p}$ must be function of $\bar{x}$ and so (1) becomes

(3) $$\frac{d\bar{p}}{d\bar{x}}=\frac{d}{d\bar{y}}\left[\mu\left(\bar{y}\right)\frac{d\bar{u}}{d\bar{y}}\right]-\overline{\rho}v_0\frac{d\bar{u}}{d\bar{y}}$$

Differentiating (3) with respect to $\bar{x}$, we get

$$\frac{d^2\bar{p}}{d\bar{x}^2}=0$$

Therefore, $\frac{d\bar{p}}{d\bar{x}}$ = Constant = $\bar{P}$

From (3) we have,

(4) $$\bar{\mu}\frac{d^2\bar{u}}{d\bar{y}^2}+\frac{d\bar{\mu}}{d\bar{y}}\frac{d\bar{u}}{d\bar{y}}-\overline{\rho}v_0\frac{d\bar{u}}{d\bar{y}}=-\bar{P}$$

Now we express (4) in non dimensional form by following transformations

$$\bar{y}=\bar{h}y,\bar{u}=u_0u,\bar{\mu}=\bar{\mu}_0\mu\left(y\right),\bar{P}=\frac{\bar{\mu}_0u_0P}{\bar{h}^2}$$

Here u_0 is a constant velocity of upper plate along $\bar{X}$ direction. So equation (4) in non dimensional form is

(5) $$\mu\frac{d^2u}{dy^2}+\frac{d\mu}{dy}\frac{du}{dy}-\alpha\frac{du}{dy}=-P$$

Here $\alpha=\frac{\rho_0 v_0\bar{h}}{\mu_0}$ is a non dimensional Reynolds Numbers for injection and suction.

Now we take $\mu(y)=(1-\epsilon\, y);0<\epsilon<1$; where ϵ is a non dimensional variation parameter of viscosity. From (5) we get

(6) $$(1-\epsilon y)u''[y]-(\alpha+\epsilon)u'[y]=-P$$

On solving differential equation (6) we get

$$(7)\qquad u[y]=(1-y\epsilon)^{-\frac{\alpha+\epsilon}{\epsilon}}\left(\frac{1}{\alpha}-\frac{y\epsilon}{\alpha}\right)c_1+\frac{Py}{\alpha+\epsilon}+c_2$$

On applying boundary conditions, at y=1, u=1 and at y= 0, u=0 in (7) we get,

$$c_1=-\frac{\alpha(1-\epsilon)^{\alpha/\epsilon}(-P+\alpha+\epsilon)}{\left(-1+(1-\epsilon)^{\alpha/\epsilon}\right)(\alpha+\epsilon)},\quad c_2=\frac{(1-\epsilon)^{\alpha/\epsilon}(-P+\alpha+\epsilon)}{\left(-1+(1-\epsilon)^{\alpha/\epsilon}\right)(\alpha+\epsilon)}$$

On substituting the value of c_1 and c_2 in (7) we get

$$(8)\ u=\frac{-P+Py+\alpha+\epsilon+\dfrac{-P+\alpha+\epsilon}{-1+(1-\epsilon)^{\alpha/\epsilon}}-\dfrac{(1-\epsilon)^{\alpha/\epsilon}(-P+\alpha+\epsilon)(1-y\epsilon)^{-\frac{\alpha}{\epsilon}}}{-1+(1-\epsilon)^{\alpha/\epsilon}}}{\alpha+\epsilon}$$

The shearing stress is given by

$$(9)\qquad \sigma_{yx}=(1-y\epsilon)\left(\frac{P-\dfrac{\alpha(1-\epsilon)^{\alpha/\epsilon}(-P+\alpha+\epsilon)(1-y\epsilon)^{-1-\frac{\alpha}{\epsilon}}}{-1+(1-\epsilon)^{\alpha/\epsilon}}}{\alpha+\epsilon}\right)$$

Hence the skin friction on plates at y= 0and at y=1 are given by

$$(10)\qquad \left[\sigma_{yx}\right]_{\mathrm{y=1}}=\frac{(1-\epsilon)\left(P+\dfrac{\alpha(-P+\alpha+\epsilon)}{\left(-1+(1-\epsilon)^{\alpha/\epsilon}\right)(-1+\epsilon)}\right)}{\alpha+\epsilon}$$

And

(11) $$\left[\sigma_{yx}\right]_{y=0} = \frac{P+\alpha\left(-1+\frac{1}{1-(1-\epsilon)^{\alpha/\epsilon}}\right)(-P+\alpha+\epsilon)}{\alpha+\epsilon}$$

Now we consider the two particular cases

Case – (I): If there is no pressure gradient i.e. P=0 (Plane Coquette Flow) we get

(12) $$u = \frac{(1-\epsilon)^{\alpha/\epsilon}(1-y\epsilon)^{-\frac{\alpha}{\epsilon}}\left(-1+(1-y\epsilon)^{\alpha/\epsilon}\right)}{-1+(1-\epsilon)^{\alpha/\epsilon}}$$

The shearing stress is given as

(13) $$\sigma_{yx} = -\frac{\alpha(1-\epsilon)^{\alpha/\epsilon}(1-y\epsilon)^{-1-\frac{\alpha}{\epsilon}}}{-1+(1-\epsilon)^{\alpha/\epsilon}} + \frac{y\alpha(1-\epsilon)^{\alpha/\epsilon}\epsilon(1-y\epsilon)^{-1-\frac{\alpha}{\epsilon}}}{-1+(1-\epsilon)^{\alpha/\epsilon}}$$

Hence the skin friction on the plates at y= 1 and 0 are given by

(14) $$\left[\sigma_{yx}\right]_0 = \alpha\left(-1+\frac{1}{1-(1-\epsilon)^{\alpha/\epsilon}}\right)$$

And

(15) $$\left[\sigma_{yx}\right]_{+1} = \frac{\alpha}{1-(1-\epsilon)^{\alpha/\epsilon}}$$

Case-(II): When $u_0 = 0$ (Plane Poiseuille flow)

In this case boundary condition s are, at y = 1, u = 0 and at y = 0, u = 0

So we get velocity

$$(16)\quad \mathrm{u}(\mathrm{y}) = -\frac{P\left(y - y(1-\epsilon)^{\alpha/\epsilon} + (1-\epsilon)^{\alpha/\epsilon}\left(1-(1-y\epsilon)^{-\frac{\alpha}{\epsilon}}\right)\right)}{\left(-1+(1-\epsilon)^{\alpha/\epsilon}\right)(\alpha+\epsilon)}$$

And skin friction on the plates are given by

$$(17)\quad \left[\sigma_{yx}\right]_{\mathrm{y=1}} = \frac{P\left(1+\frac{\alpha}{-1+(1-\epsilon)^{\alpha/\epsilon}}-\epsilon\right)}{\alpha+\epsilon}$$

$$(18)\quad \left[\sigma_{yx}\right]_{\mathrm{y=0}} = \frac{P\left(1+\alpha+\frac{\alpha}{-1+(1-\epsilon)^{\alpha/\epsilon}}\right)}{\alpha+\epsilon}$$

Since, for $\in = 0$ we get steady, incompressible and laminar flow between two parallel plates with constant viscosity. So from (6)

$$(19)\quad u''[y] - \alpha u'[y] = -P$$

On solving the differential equation (19) and applying boundary conditions at y=1, u=1 and at y= 0, u=0 we get

$$(20)\quad \mathrm{u}(\mathrm{y}) = \frac{Py + \frac{\left(-1+\mathrm{e}^{y\alpha}\right)(-P+\alpha)}{-1+e^{\alpha}}}{\alpha}$$

The shearing stress is

$$(21)\quad \sigma_{yx} = \frac{P + \frac{\mathrm{e}^{y\alpha}\alpha(-P+\alpha)}{-1+e^{\alpha}}}{\alpha}$$

And skin friction on plates are given by

(22) $$\left[\sigma_{yx}\right]_{\mathrm{y=1}}=\frac{P}{\alpha}+\frac{\mathrm{e}^{\alpha}\left(-P+\alpha\right)}{-1+e^{\alpha}}$$

(23) $$\left[\sigma_{yx}\right]_{\mathrm{y=0}}=\frac{P}{\alpha}+\frac{-P+\alpha}{-1+e^{\alpha}}$$

For plane coquette flow, P=0, from (20) we get

(24) $$u(y)=\frac{\mathrm{e}^{y\alpha}-1}{e^{\alpha}-1}$$

and the shearing stress

(25) $$\sigma_{yx}=\frac{\mathrm{e}^{y\alpha}\alpha}{-1+e^{\alpha}}$$

Hence the skin friction on the plates at y= 1, and at y = 0 are given by

(26) $$\left[\sigma_{yx}\right]_{\mathrm{y=1}}=\frac{\mathrm{e}^{\alpha}\alpha}{-1+e^{\alpha}}$$

and

(27) $$\left[\sigma_{yx}\right]_{\mathrm{y=0}}=\frac{\alpha}{-1+e^{2\alpha}}$$

For plane Poiseuille flow we apply the boundary conditions at y = 1, u = 0 and at y = 0, u = 0 in the solution of differential equation (19).Then we get

(28) $$\mathrm{u}\left(\mathrm{y}\right)=-\frac{P\left(-1+\mathrm{e}^{y\alpha}+y-e^{\alpha}y\right)}{\left(-1+e^{\alpha}\right)\alpha}$$

and the shearing stress

(29) $$\sigma_{yx}=-\frac{P\left(1-e^{\alpha}+\mathrm{e}^{y\alpha}\alpha\right)}{\left(-1+e^{\alpha}\right)\alpha}$$

Hence the skin friction at the plates y= 1and at y = 0 are given by

(30) $$\left[\sigma_{yx}\right]_{y=1} = P\left(-\frac{e^{\alpha}}{-1+e^{\alpha}} + \frac{1}{\alpha}\right)$$

and

(31) $$\left[\sigma_{yx}\right]_{y=0} = P\left(\frac{1}{1-e^{\alpha}} + \frac{1}{\alpha}\right)$$

Result and Discussion

Graphical illustration of the velocity profiles are shown in figures (1) and (2).Figure (1) indicates that velocity u(y) is found to decrease markedly with an increase in ε parameter or decrease in viscosity of the fluid. Similarly velocity profile is decrease with an increase in Reynolds number 'α' for suction and injection. From figure (3) to (5) same results are found for velocity profile of plane Poiseuille and plane Coquette flow with the variation of 'ε' and 'α'. Figure (6) shows that shear stress decreases smoothly up to the midpoint from injection and then it increases as we move towards suction but as viscosity decrease it decreases rapidly up to the midpoint from injection and then a rapid increase found as we move towards the suction plate.

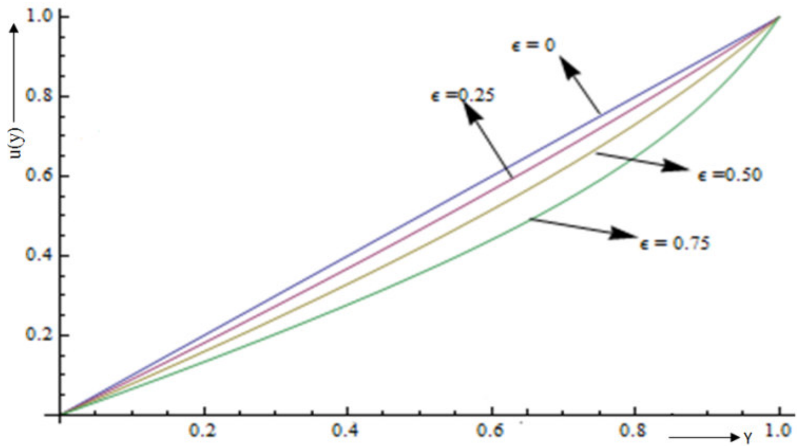

Fig.(1) Variation of u(y) For P=1, α = 1, ε =0, 0.25, 0.50, 0.75

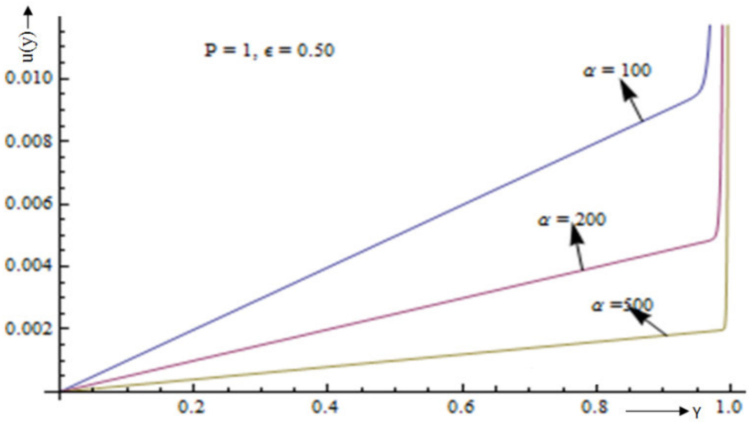

Fig.(2) Variation of u(y) For P=1, $\varepsilon = 0.50$, $\alpha = 100,200,500$

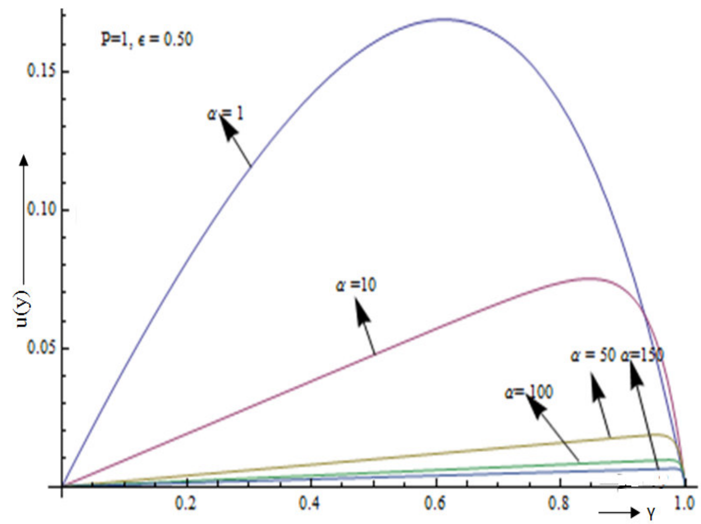

Fig.(3) Variation of u(y) For Plane Poiseuille Flow

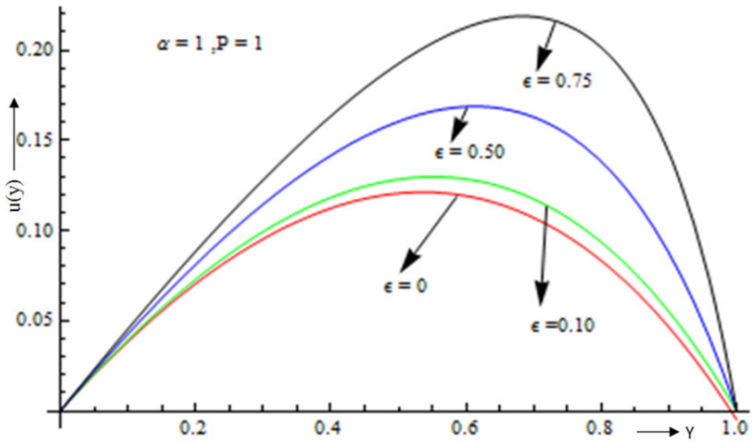

Fig.(4) Variation of u(y) For Plane Poiseuille Flow for different Value of viscosity

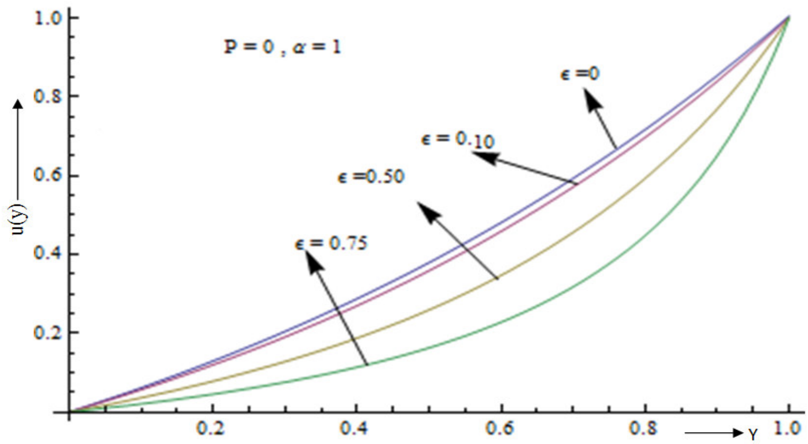

Fig.(5) Variation of u(y) For Plane Coquette Flow

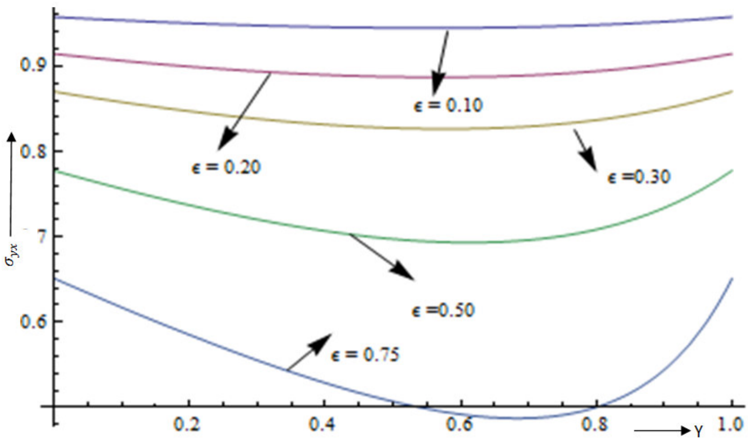

Fig.(6) Variation of Shear Stress $[\sigma_{yx}]$ for ε = 0.10, 0.20, 0.30, 0.50, 0.75, α = 1, P= 1.

Conclusions

Exact form of solutions for velocity, shearing stress and wall shear have been found with varying viscosity between porous plates. The solutions for constant viscosity are also derived for comparative study. The solutions have been presented graphically with the variation of viscosity and Reynolds number.

Acknowledgement: The first author is grateful to Dr. Satya Deo, Dept. of Mathematics, University of Allahabad for his co-operation.

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