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The Propagation of Flare Generated Shockwaves in Self-Gravitating Solar Atmosphere

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Abstract: The effects of an interplanetary magnetic field on the propagation of the flare generated shock wave in a non-uniformly distributed solar self-gravitating atmosphere are investigated by using an approximate analytic method of Whitham. The expressions for the velocity of shock wave and for the other flow variables behind weak and strong shocks are derived and discussed.

Key Words: Shockwaves, Solar Atmosphere, Fundamental equation for spherically symmetric flow of a self graviting atmosphere in the presence of magnetic field.

Mathematics Subject Classification: 2010-76L05.

1. Introduction

Recent observations have confirmed that a large amount of coronal mass is ejected into the interplanetary space during the solar flare activity. As a result of this interplanetary shock waves that propagate outwards from the sun into the interplanetary space are generated (Dryer and Wu[1], Smithetal[2], Yeh[3], Hundhausen[4], Maxwell & Dryer[5], Berlaga[6]).The theoretical studies of the flare generated shock wave propagation into the interplanetary space are treated by Parker[7], Korobeinikov[8], Summers[9], Rosenau and Frankenthal[10], Low[11-14], Ojha and Tiwari[15] and many others by using the method of self- similarity. Numerical solutions have also been obtained to discuss the propagation of flare associated shock waves with (Steinolfsonetal[16]) or without (Hundhassen and Gentry[17]), magnetic field.

Bird[18] constructed a model of the outer solar atmosphere regarding it as a shock heated atmosphere based on the hypothesis that any equilibrium state of such an atmosphere would be such that the shock strength remained constant and determine the effects produced by the variation in properties of a gravitational atmosphere on a spherical shock wave propagating within it by using Whitham[19]) approximate analytic method. Expressions are also presented for the rate of change of the shock strength with the distance with and without steady mass motion and with ordinary distribution of temperature and composisons. The special cases of weak and strong shocks in adiabatic and isothermal atmosphere are treated in some detail. Ojha and Singh[20] have extended the analysis of Bird by taking into account the rotation and non-uniform distribution of ambient solar atmosphere respectively. In all the above studies the effects due to interplanetary magnetic field are neglected.

Tam and Yousefian [21] investigated the effect of the interplanetary magnetic field on the propagation of solar flare generated interplanetary shock waves and found that the effect of the magnetic field on the shock speed is unimportant for strong shocks in comparison to the weak shocks and suggested that the model without magnetic field gives sufficiently accurate numerical results for the propagation of the shock waves.Since the magnetic field pressure and plasma pressure are comparable on the earth's orbit, the consideration of the effect of magnetic field is important.Therefore our aim here is to investigate the shock wave propagation through non-uniformly distributed self- gravitating solar atmosphere in presence of interplanetary magnetic field.

On propagating outwards from the sun, the interplanetary shock will experience a changing environment and as a result of this it generates unsteady disturbances behind it. Under these circumstances Whitham[19] method for treating the propagation of shock waves through the regions of non-uniform flow is applicable. Expressions for the velocity of the shock,fluid pressure, density and fluid velocity are derived behind the weak and strong shock considering the cases of weak and strong magnetic field separately.

2. Equations of Motion And Shock Conditions

Fundamental equation for spherically symmetric flow of a self gravitating atmosphere in the presence of magnetic field are (c.f. Lee and Chen [22], Tam and Yousefian [21], Rosenau and Frankenthal[10].

(2.1) $$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\left(\frac{\partial u}{\partial r}+\frac{2u}{r}\right)=0$$

(2.2) $$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mathrm{H}}{\rho}\frac{\partial \mathrm{H}}{\partial r}+\frac{\mathrm{H}^2}{\rho r}+\frac{G\mathrm{M}}{r^2}=0$$

(2.3) $$\frac{\partial \mathrm{H}}{\partial t}+u\frac{\partial \mathrm{H}}{\partial r}+\mathrm{H}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)=0$$

(2.4) $$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}-a^2\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)=0$$

(2.5) $$\frac{\partial \mathrm{M}}{\partial r}=4\pi\rho r^2$$

where r is the radial co-ordinate at time t and u, p, ρ, H and M are the radial component of the partical velocity, pressure, density, magnetic field and the mass inside a sphere of radius r respectively, a is the speed of sound.

Suppose the distribution of the ambient density and magnetic field of the solar atmosphere are given as $\rho_c r^{-w}$ and $h_c r^{-m}$ respectively. Thus in case of hydrostatic equilibrium;

(2.6) $$p_0=\left\{\frac{2\pi\rho_c^2 G}{(w-1)(3-w)}-\left(\frac{w-2}{w-1}\right)\frac{h_c{}^2}{2}\right\}r^{-2m}$$

or

(2.7) $$p_0=p_c r^{-2m},\ p_c=\left\{\frac{2\pi\rho_c^2 G}{(w-1)(3-w)}-\left(\frac{w-2}{w-1}\right)\frac{h_c^2}{2}\right\}$$

where ρ_c, h_c *and* p_c are constants and $m=w-1$.

(2.8) $$\mathrm{M}_0=\frac{4\pi\rho_c}{(3-w)}r^{(3-w)}$$

which gives $w<3$ and therefore $m<2$, in particular, when $w=2, m=1$.

The speed of sound a_0 is given by

(2.9) $$a_0=a_c r^{\frac{2-w}{2}},\ a_c^2=\frac{\gamma p_c}{\rho_c}$$

The magnetohydrodynamic shock condition may be written as (Whitham[19])

(2.10) $$\rho = \rho_0 \mathrm{N}$$

(2.11) $$H = H_0 \mathrm{N}$$

(2.12) $$u = \left(\frac{\mathrm{N}-1}{\mathrm{N}} \right) U$$

(2.13) $$p = p_0 \left\{ \frac{(\mathrm{N}+1)+\gamma(\mathrm{N}-1)}{(\mathrm{N}-1)^3(\gamma-1)} + \frac{1}{\beta_0} \right\} \left(\frac{(\mathrm{N}-1)^2(\gamma-1)}{(\gamma+1)-(\gamma-1)\mathrm{N}} \right)$$

(2.14) $$U^2 = \frac{2\mathrm{N}\left\{ \left(\frac{2-\gamma}{\gamma} \right)\mathrm{N}+1 \right\} a_0^2}{(\gamma+1)-\mathrm{N}(\gamma-1)} \left[\frac{\gamma}{(2-\gamma)\mathrm{N}+\gamma} + \frac{1}{\beta_0} \right]$$

where the quantities with suffix 0 and without suffix define values of the quantities immediately ahead and behind the shock, U is the shock velocity, a_0 is the sound speed ahead of the shock and β_0 is defined as the parameter for the strength of magnetic field ahead of the shock where

(2.15) $$\beta_0 = \frac{2p_0}{\mathrm{H}_0^2} = \frac{2p_c}{h_c^2}$$

With the help of (2.7) and (2.15), we get

(2.16) $$2\pi\rho_c \, G\!\big/_{3-w} = \frac{p_c}{\rho_c} \left(\frac{\beta_0(w-1)+(w-2)}{\beta_0} \right)$$

3. Discussions

For diverging shocks, the characteristic form of the system of equations (2.1) – (2.4) are (Whitham[19]).

(3.1) $$dp + \mathrm{H}d\mathrm{H} + \rho c du + \frac{2\rho c u}{(u+c)} \frac{dr}{r} + \frac{\rho G M c}{(u+c)} \frac{dr}{r^2} = 0$$

where

(3.2) $$c^2 = a^2 + b^2 = \frac{\gamma p}{\rho} + \frac{H^2}{\rho}$$

Weak Shocks

For very weak shock we can take N as

(3.3) $$\frac{\rho}{\rho_0} = \mathrm{N} = 1 + \alpha \qquad \text{where } \alpha << 1$$

For weak magnetic field i.e. where for very high ρ_0, the shock conditions take the forms

(3.4) $$\rho = \rho_0(1+\alpha),\ \mathrm{H} = \mathrm{H}_0(1+\alpha),\ u = a_0\alpha,\ p = p_0(1+\gamma\alpha)$$

and

$$U = \left(1 + \left(\frac{\gamma+1}{4}\right)\alpha\right)a_0$$

Substituting (3.4) in (3.1), we have

(3.5) $$\left(2 + \frac{2}{\gamma\beta_0}\right)d\alpha + \left(\frac{dp_0}{p_0} + \frac{da_0}{a_0} + \frac{2dr}{r}\right)\alpha = 0$$

Substituting dp_0 and da_0 in (3.5)

(3.6) $$\frac{d\alpha}{\alpha} = -\left(\frac{10-5w}{4}\right)\left(\frac{\gamma\beta_0}{1+\gamma\beta_0}\right)\frac{dr}{r}$$

or

(3.7) $$\alpha = K_1 r^{-n_1},\ n_1 = \left(\frac{10-5w}{4}\right)\left(\frac{\gamma\beta_0}{1+\gamma\beta_0}\right) \approx \frac{5}{4}(2-w)$$

Thus, the flow variables and velocity of the shock are not affected by weak magnetic field. When $w = 2$, α is propotional to r .

For strong magnetic field i.e. for very small β_0 the shock conditions take the forms

(3.8) $$\rho = \rho_0(1+\alpha),\ H = H_0(1+\alpha),\ u = \left(\frac{2}{\gamma\beta_0}\right)^{\frac{1}{2}} a_0\alpha,\ p = p_0(1+\gamma\alpha)$$

and

$$U = \left(1 + \frac{3}{4}\alpha\right) a_0 \sqrt{\frac{2}{\gamma\beta_0}}$$

Substitution of (3.8) in (3.1) gives

$$\left(1 + \frac{4}{\gamma\beta_0}\right)\frac{d\alpha}{\alpha} + \left(\frac{dp_0}{p_0} + \frac{2}{\gamma\beta_0}\left(\frac{da_0}{a_0} + \frac{2dr}{r}\right)\right) = 0 \tag{3.9}$$

Substituting the value of $\frac{dp_0}{p_0}$ and $\frac{da_0}{a_0}$ we get

$$\frac{d\alpha}{\alpha} = -\left\{\frac{(6-w) + 2\gamma\beta_0(w-1)}{(\gamma\beta_0 + 4)}\right\}\frac{dr}{r} \tag{3.10}$$

Integrating we get

$$\alpha = \mathrm{K}_2 r^{-n_2}, \quad n_2 = \frac{(6-w) + 2\gamma\beta_0(w-1)}{(\gamma\beta_0 + 4)} \tag{3.11}$$

where K_2 be a constant

Thus, in this case,

$$\rho = \rho_0\left(1 + \mathrm{K}_2 r^{-n2}\right), \quad u = \sqrt{\frac{2}{\gamma\beta_0}}\mathrm{K}_2 r^{-n2}.a_0 \tag{3.12}$$

$$p = p_0\left(1 + \gamma\mathrm{K}_2 r^{-n2}\right), \quad \mathrm{H} = \mathrm{H}_0\left(1 + \mathrm{K}_2 r^{-n2}\right)$$

$$U = a_0\left(1 + \frac{3\mathrm{K}_2 r^{-n2}}{4}\right)\sqrt{\frac{2}{\gamma\beta_0}}$$

Thus, the strong magnetic field affects significantly the velocity of the shock and the flow variables behind it.

Strong Shocks

When the gas pressure is much greater than magnetic pressure i.e. when β_0 is very high,

$\mathrm{N} = \frac{\gamma+1}{\gamma-1}$ For strong shocks and the solar atmosphere may be regarded as independent of magnetic field.

When magnetic pressure is much greater than gas pressure i.e. when β_0 is very small, the shock conditions in terms of N and Shock velocity may be written as,

(3.13) $$\rho = \rho_0 \mathrm{N}$$

(3.14) $$\mathrm{H} = \mathrm{H}_0 \mathrm{N}$$

(3.15) $$u = \left(\frac{\mathrm{N}-1}{\mathrm{N}}\right) U$$

(3.16) $$p = \mathrm{X}\frac{U^2}{a_0^2} p_0 + L p_0$$

where

(3.17) $$\mathrm{X} = \frac{\gamma(\gamma-1)(\mathrm{N}-1)^3}{2\mathrm{N}((2-\gamma)\mathrm{N}+\gamma)}$$

and

(3.18) $$L = \left\{\frac{(\gamma+1)\mathrm{N}-(\gamma-1)}{(\gamma+1)-(\gamma-1)\mathrm{N}} - \frac{\gamma(\gamma-1)(\mathrm{N}-1)^3}{(2\mathrm{N}-\gamma(\mathrm{N}-1)\ )(\ (\gamma+1)\mathrm{N}-(\gamma-1))}\right\}$$

Now

(3.19) $$c^2 = \frac{\gamma p}{\rho} + \frac{\mathrm{H}^2}{\rho} = \left\{\frac{\mathrm{X}}{\mathrm{N}} + \left(\frac{L}{\mathrm{N}} + \frac{2\mathrm{N}}{\gamma\beta_0}\right)\frac{a_0^2}{U^2}\right\} U^2$$

Since for strong shocks $U^2 >> a_0^2$, we may take

(3.20) $$c = U\sqrt{\frac{\mathrm{X}}{\mathrm{N}}}$$

(3.21) $$u + c = \left(\frac{\mathrm{N}-1}{\mathrm{N}}\right) U + U\sqrt{\frac{\mathrm{X}}{\mathrm{N}}}$$

(3.22) $$dp = \mathrm{X}U^2 \frac{dp_0}{a_0^2} + \frac{\mathrm{X}p_0}{a_0^2} dU^2 - 2\mathrm{X}p_0 U^2 \frac{da_0}{a_0^3} + L dp_0$$

(3.23) $$\mathrm{H}d\mathrm{H} = \mathrm{N}^2 \frac{dp_0}{\beta_0}$$

(3.24) $$\rho c du = \rho_0 (\mathrm{N}-1)\sqrt{\frac{\mathrm{X}}{\mathrm{N}}} U du$$

(3.25) $$\rho c G\mathrm{M} = \rho_0 \mathrm{N} U \sqrt{\frac{\mathrm{X}}{\mathrm{N}}} G\mathrm{M}$$

(3.26) $$\mathrm{M} = \frac{4\pi\rho_c r^{3-w}}{(3-w)}$$

Substituting (3.20) – (3.26) in (3.1) we have

$$(3.27)\ \left(\frac{\mathrm{X}}{\gamma} + \left(\frac{\mathrm{N}-1}{2}\right)\sqrt{\frac{\mathrm{X}}{\mathrm{N}}}\right) dU^2 + \left(\frac{\mathrm{X}}{\gamma}\frac{dp_0}{p_0} - \frac{2\mathrm{X}}{\gamma}\frac{da_0}{a_0} + \frac{2(\mathrm{N}-1)}{(\mathrm{N}-1)+\sqrt{\mathrm{XN}}}\frac{dr}{r}\right) U^2$$
$$+ \left\{\left(L + \frac{\mathrm{N}^2}{\beta_0}\right)\frac{dp_0}{\rho_0} + \frac{G\mathrm{MN}\sqrt{\mathrm{XN}}}{(\mathrm{N}-1)+\sqrt{\mathrm{XN}}}\frac{dr}{r^2}\right\} = 0$$

Substituting the values of $\frac{dp_0}{p_0}, \frac{da_0}{a_0}, \frac{dp_0}{\rho_0}$ and M_0 we get

(3.28) $$\frac{d}{dr}U^2 + P.\frac{U^2}{r} + Qr^{(1-\omega)} = 0$$

where

(3.29) $$P = \frac{\left(2(\mathrm{N}-1) - \frac{\omega}{\gamma}\right)\mathrm{X}}{\left((\mathrm{N}-1) - \sqrt{\mathrm{XN}}\right)\left\{\frac{\mathrm{X}}{\gamma} + \left(\frac{\mathrm{N}-1}{2}\right)\sqrt{\frac{\mathrm{X}}{\mathrm{N}}}\right\}}$$

(3.30) $$Q = \left[\frac{2\left(L + \frac{\mathrm{N}^2}{\beta_0}\right)(1-w)\frac{p_c}{\rho_c} + \frac{G4\pi\rho_c\mathrm{N}\sqrt{\mathrm{XN}}}{\left((\mathrm{N}-1)+\sqrt{\mathrm{XN}}\right)(3-w)}}{\left(\frac{\mathrm{X}}{\gamma} + \left(\frac{\mathrm{N}-1}{2}\right)\sqrt{\frac{\mathrm{X}}{\mathrm{N}}}\right)}\right]$$

Integrating (3.28) we get

$$U^2 = -\left(\frac{Q}{P-w+2}\right) r^{2-w} + Kr^{-P} \tag{3.31}$$

where K is integration constant.

Expressions for the velocity density and pressure immediately behind the shock may be written as

$$u = \left(\frac{\mathrm{N}-1}{\mathrm{N}}\right)\left\{-\left(\frac{Q}{P-w+2}\right) r^{2-w} + \mathrm{K} r^{-P}\right\}^{\frac{1}{2}} \tag{3.32}$$

$$\rho = \rho_0 \mathrm{N} \tag{3.33}$$

$$p = \left[\frac{\mathrm{X}\rho_c}{\gamma}\left\{\mathrm{K} r^{-(P+w)} - \left(\frac{Q}{P-w+2}\right) r^{2(1-w)}\right\} + Lp_c r^{2(1-w)}\right] \tag{3.34}$$

4. Conclusions

It is obvious from the above expressions that the density distribution and magnetic field distribution in ambient solar atmosphere affect significantly the velocity of shock and flow variables behind the shock waves.

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