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Mathematical Model for Malaria Transmission and Chemical Control

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Abstract: In this paper, we have studied the effect of insecticides for the control of vector borne diseases, like malaria. A non-linear SIS mathematical model for transmission of Malaria caused by infected mosquito population on susceptible human population and using chemicals to control the disease is proposed and analyzed [1-3]. It is further assumed that susceptible human population become infected by direct contact with vector mosquitos . The mosquito population density is assumed to be governed by general logistic model. It is found that model exhibits three non-negative equilibria viz. mosquito and disease free equilibrium E_0 Mosquito persistence and disease free equilibrium E_1 and endemic equilibrium E^* . The model is analyzed by using local and global stability theory of differential equations and numerical simulation [3, 4] .

Keywords: Mathematical model, human population, mosquito population, insecticide concentration.

1. Introduction

Mosquito are the major public health pest throughout the tropical and subtropical region in the world. Out of 3,492 species of mosquitoes recorded worldwide, more than a hundred species are able of transmitting various diseases in human and other vertebrates. In India, malaria is one of the major cause of direct or indirect infant, child, and adult, mortality. About two million confirmed malarial cases and 1,000 deaths are reported annually [5-7] . The control of mosquito larva worldwide depends primarily on continued applications of organophosphates such as temephos and insect growth regulators such as diflubenzuron and methoprene. In tropical and subtropical region, mosquito borne diseases are a major problem and are responsible for many life- threatening diseases, e.g. malaria, yellow fever, dengue fever, and Chikungunya, etc. Out of these mosquito borne life-threatening diseases, the malaria is a serious illness in human caused by several species of mosquito-borne parasite (Plasmodium falciparum, vivax, knowlesi and ovale) and it is endemic in many parts of the World . In the past, several efforts have been made to reduce the incidence of malaria focusing on reducing the number of mosquitoes and preventing mosquito bites [9]. However, most of these efforts, especially the ones that use pesticides, have been banned by the environmental protection agencies as they harm non-targeted population . Also due to continuous application of pesticides, mosquitoes have developed resistance to these chemicals and now they are not so effective. Chemical control means introduction or manipulation of chemical to suppress vector population. Chemical control, particularly using DDT plays a very positive role in controlling mosquitoes. The method of control of mosquito using DDT is not new, it has been implemented since 1937 in many parts of the world. But control of mosquitoes using pesticides was fast so it suppressed this conventional method of control of mosquitoes. Now again this method of chemical control is accepted and successfully implemented in many part of world [10-13]

2. The Mathematical Model

Let $N(t)$ be the total human population density in the region under consideration at any time t, which is divided into two subclasses namely, $X(t)$ as susceptible class and infective class with density as $Y(t)$. $M_s(t)$ as density of susceptible mosquitoes, $M_i(t)$ as density of infective mosquitoes. $C_h(t)$ is the concentration of insecticides at time t. A is the constant immigration rate, the constant β represents the transmission rate of susceptible to the infective class, constant d and ν denote the natural death rate and recovery rate of human population respectively. Constant α represents disease induced death rate of human population, b_m and d_m are the natural birth rate and natural death rate of mosquito respectively. K is the carrying capacity of mosquito. θ_0 is death rate of mosquito and θ_1 is the depletion rate of mosquito due to insecticides λ is the transmission rate of mosquito. ϕ is the spraying rate of chemical and constant ϕ_0 is the natural decay rate of insecticides and ϕ_1 decay rate of due to uptake by mosquito of the chemicals.

(1)
$$\begin{aligned}
\frac{dx}{dt} &= A - \beta X M_i - dX - \nu Y \\
\frac{dy}{dt} &= \beta X M_i - (d + \nu + \alpha) Y \\
\frac{dM_s}{dt} &= b_m M - \frac{rM^2}{K} - \theta_1 C_h M_s - \theta_0 M_s - \lambda M_s Y - d_m M_s \\
\frac{dM_i}{dt} &= \lambda M_s Y - \theta_0 M_i - \theta_1 C_h M_i - d_m M_i \\
\frac{dC_h}{dt} &= \phi M - \phi_0 C_h - \phi_1 M C_h
\end{aligned}$$

Using $N = X + Y, M = M_s + M_i$ and $r = b_m - d_m$, the reduced model is

(2)
$$\begin{aligned}
\frac{dY}{dt} &= \beta (N - Y) M_i - (d + \nu + \alpha) Y \\
\frac{dN}{dt} &= A - dN - \alpha Y \\
\frac{dM_i}{dt} &= \lambda (M - M_i) Y - \theta_0 M_i - \theta_1 C_h M_i - d_m M_i \\
\frac{dM}{dt} &= rM - \frac{rM^2}{K} - (\theta_0 + \theta_1 C_h) M \\
\frac{dC_h}{dt} &= \phi M - \phi_0 C_h - \phi_1 M C_h .
\end{aligned}$$

It suffices to study the model system (2) The region of attraction for all solutions initiating in the
positive orthant is given by

$$\Omega := \left\{ (Y, N, M_i, M, C_h) : 0 \le Y \le N \le \frac{A}{d}, 0 \le M_i \le M \le K, 0 \le C_h \le \frac{\phi K}{\phi_0} \right\}$$

3. Equilibrium Analysis

The model system (2) has the following three non-negative equilibria:

- Mosquito and Disease Free Equilibrium $E_0\left(0, \frac{A}{d}, 0, 0, 0\right)$ which always exists.
- Mosquito persistence and Disease Free Equilibrium $E_1\left(0, \frac{A}{d}, 0, M_1, C_{h1}\right)$.
- $E^*(Y^*, N^*, M_i^*, M^*, C_h^*)$ as an endemic equilibrium.

3.1 Existence of E_1: If $M = 0$, then equilibrium point $E_0(0, \frac{A}{d}, 0, 0, 0)$ exists without any condition but this is not a feasible Equilibrium as mosquito population is zero. So next we assume that $M \neq 0$. Now Equilibrium will be obtained by solving the following equations:

(3) $$\beta(N - Y)M_i - (d + \nu + \alpha)Y = 0,$$

(4) $$A - dN - \alpha Y = 0,$$

(5) $$\lambda(M - M_i)Y - \theta_0 M_i - \theta_1 C_h M_i - d_m M_i = 0,$$

(6) $$r(1 - \frac{M}{K}) - (\theta_0 + \theta_1 C_h) = 0,$$

(7) $$\phi M - \phi_0 C_h - \phi_1 M C_h = 0$$

from eqn.(4), we have

(8) $$N = \frac{A - \alpha Y}{d}$$

now

(9) $$N - Y = \frac{A - \alpha Y}{d} - Y = \frac{A - (\alpha + d)Y}{d}$$

from eqn. (7), we have

(10) $$M_i = \frac{\lambda MY}{\lambda Y + \theta_0 + d_m + \theta_1 C_h}$$

now using (9) and (10) in (3), we get,

(11) $$\beta\left[\frac{A - (\alpha + d)Y}{d}\right]\left[\frac{\lambda MY}{\lambda Y + \theta_0 + d_m + \theta_1 C_h}\right] - (\nu + \alpha + d)Y = 0$$

Which gives, either, $Y = 0$

(12) $$\beta\left[\frac{A - (\alpha + d)Y}{d}\right]\left[\frac{\lambda M}{\lambda Y + \theta_0 + d_m + \theta_1 C_h}\right] - (\nu + \alpha + d) = 0$$

Now, if $Y = 0$ then $N = \frac{A}{d}$, $M_i = 0$ M and C_h are given by following two equations,

(13) $$r(1 - \frac{M}{K}) = (\theta_0 + \theta_1 C_h)$$

(14) $$\phi M - \phi_0 C_h - \phi_1 M C_h = 0$$

(15) $$C_h = \frac{\phi M}{\phi_0 + \phi_1 M},$$

using this value of C_h in (13), we get

(16) $$(r-\theta_0) = r\frac{M}{K} + \frac{\theta_1\phi M}{\phi_0+\phi_1 M},$$

(17) $$(r-\theta_0)\phi_0 + \phi_1(r-\theta_0)M = \frac{r}{K}\phi_0 M + \frac{r}{K}\phi_1 M^2 + \theta_1\phi M.$$

(18) $$\frac{r}{K}\phi_1 M^2 + (\frac{r}{K}\phi_0 + \theta_1\phi - \phi_1(r-\theta_0))M - \phi_0(r-\theta_0) = 0$$

As

(19) $$(r-\theta_0) > 0,$$

This is because of the fact that the maximum intrinsic growth rate of mosquito population must be positive otherwise mosquito population will extinct. Now clearly one root of above equation is positive which gives $M = M_2^*$ After getting $M = M_2^*$ we get $C_h = C_{h2}^*$ from equation (15) Thus the equilibrium $E_2(0, \frac{A}{d}, 0, M_2^*, C_{h2}^*)$ exists provided $(r-\theta_0) > 0$

3.2 Existence of E^* : Now we assume $Y \neq 0$ In this case

(20) $$\beta(\frac{A-(\alpha+d)Y}{d})\frac{\lambda M}{\lambda Y + \theta_0 + d_m + \theta_1 C_h} = (\nu+\alpha+d)$$

now from equation (9) we have,

(21) $$C_h = \frac{\phi M}{\phi_0 + \phi_1 M}$$

From eqn.(6) and (21) we get

(22) $$r(1-\frac{M}{K}) - (\theta_0 + \frac{\theta_1\phi M}{\phi_0+\phi_1 M}) = 0,$$

(23) $$(r-\theta_0) - \frac{r}{K}M - \frac{\theta_1\phi M}{\phi_0+\phi_1 M} = 0$$

Again using eqn (21) in (20), we get,

(24) $$\beta(\frac{A-(\alpha+d)Y}{d})\lambda M=(\nu+\alpha+d)[\lambda Y+\theta_0+d_m+\frac{\theta_1\phi M}{\phi_0+\phi_1 M}]$$

(25) $$\beta\lambda M(\frac{A-(\alpha+d)Y}{d})=(\nu+\alpha+d)[\lambda Y+\theta_0+d_m+\frac{\theta_1\phi M}{\phi_0+\phi_1 M}]$$

now we prove the existence of M^* and Y^* by analyzing the isoclines (23) and (25),

Analysis of isocline (23): As eqn. (23), $(r-\theta_0)-\frac{r}{K}M-\frac{\theta_1\phi M}{\phi_0+\phi_1 M}=0$ Now we at $M=0, Y=\frac{d}{K}M-\frac{\theta_1\phi M}{\phi_0+\phi_1 M}>0$ At $Y=0$,

$$(r-\theta_0)-\frac{r}{K}M-\frac{\theta_1\phi M}{\phi_0+\phi_1 M}=0$$

(26) $$\frac{r}{K}\phi_1 M^2+(\frac{r}{K}\phi_0+\theta_1\phi-\phi_1(r-\theta_0))M-\phi_0(r-\theta_0)=0$$

It has always a positive root and moreover this positive root will be M_2^* differentiating (26) w.r.t. M, we get,

(27) $$-\frac{r}{K}-\frac{\theta_1\phi_0\phi}{(\phi_0+\phi_1 M)^2}=0$$

Analysis of isocline(25): At $M=0$, $Y=-\frac{\theta_0+d_m}{\lambda}<0$, $Y=\frac{A}{\alpha+d}$ is an asymptote. At $Y=0$

(28) $$\frac{\beta\lambda A}{d}M=(\nu+\alpha+d)(\theta_0+d_m+\frac{\theta_1\phi M}{\phi_0+\phi_1 M})$$

(29) $$\frac{\beta\lambda A}{d(\nu+\alpha+d)}M=\theta_0+d_m+\frac{\theta_1\phi M}{\phi_0+\phi_1 M},$$

put $a = \dfrac{\beta\lambda A}{d(\nu+\alpha+d)}, b = \theta_0 + d_m$, then above equations reduces to,

(30) $$aM - b = \frac{\theta_1 \phi M}{\phi_0 + \phi_1 M}$$

(31) $$a\phi_0 M + a\phi_1 M^2 - b\phi_0 - b\phi_1 M = \phi_1 \phi M\ ,$$

(32) $$a\phi_1 M^2 + (a\phi_0 - b\phi_1 - \theta_1\phi)M - b\phi_0 = 0$$

(33) $$M = -\frac{(a\phi_0 - b\phi_1 - \theta_1\phi) \pm \sqrt{(a\phi_0 - b\phi_1 - \theta_1\phi)^2 + 4ab\phi_0\phi_1}}{2a\phi_1}$$

Now,

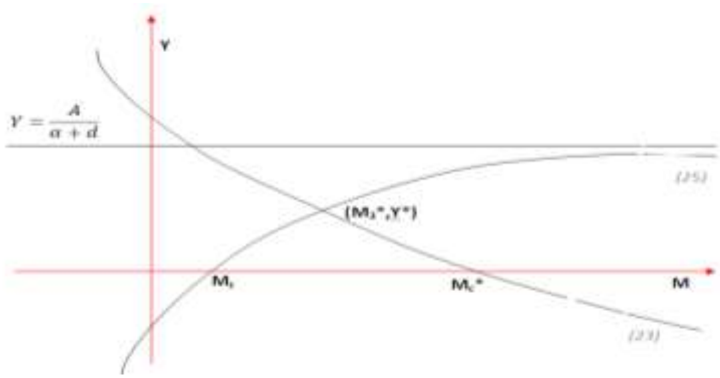
(34) $$(a\phi_0 - b\phi_1 - \theta_1\phi)^2 + 4ab\phi_0\phi_1$$
$$= a^2\phi_0^2 + b^2\phi_1^2 + \theta_1^2\phi^2 + 2ab\phi_0\phi_1 - 2a\theta_1\phi\phi_0 + 2b\theta_1\phi\phi_1$$

let $M = M_c$ be the positive root of above eqn. Now differentiating equation (25) w.r.t. M, we get,

(35) $$\beta\lambda M(-\frac{\alpha+d}{d})\frac{dY}{dM} + \beta\lambda(\frac{A-(\alpha+d)Y}{d})$$
$$= (\nu+\alpha+d)(\lambda\frac{dY}{dM} + \frac{\phi_0\phi\theta_1}{(\phi_0+\phi_1 M)^2})$$

$$\frac{dY}{dM} = \frac{(\nu+\alpha+d)[\lambda Y + \theta_0 + d_m + \dfrac{\theta_1\phi M}{\phi_0+\phi_1 M} - \dfrac{\theta_1\phi_0\phi M}{(\phi_0+\phi_1 M)^2}]}{\lambda M[\beta(\dfrac{\alpha+d}{d})M + (\nu+\alpha+d)]}$$

$$\frac{(\nu+\alpha+d)[\lambda Y + \theta_0 + d_m + \dfrac{\theta_1\phi\phi_1 M^2}{(\phi_0+\phi_1 M)^2 M}]}{\lambda M[\beta(\dfrac{\alpha+d}{d})M + (\nu+\alpha+d)]} > 0$$



Thus the two isoclines (23) and (25) will intersect in the interior of first quadrant, provided $M_c < M_2^*$

4. Stability Analysis

4.1 Local Stability: Let J_0, J_1 and J^* be the variational matrices evaluated at E_0, E_1 and E^* respectively, The general variational matrix of the model system (2) is given by:

$$J = \begin{pmatrix} -(\beta M_i + d + \nu + \alpha) & \beta M_i & \beta(N-Y) & 0 & 0 \\ -\alpha & -d & 0 & 0 & 0 \\ \lambda(M - M_i) & 0 & -(\lambda Y + \theta_0 + \theta_1 C_h + d) & \lambda Y & -\theta_1 M_i \\ 0 & 0 & 0 & r - \frac{2rM}{K} - (\theta_0 + \theta_1 C_h)) & 0 \\ 0 & 0 & 0 & (\phi - \phi_1 C_h) & -(\phi_0 + \phi_1 M) \end{pmatrix},$$

E_0 is unstable if $r - \theta_0 > 0$, If E_1 exists,

$$(\nu + d + \alpha + a)(\theta_0 + \theta_1 C_{h1} + d + a) - \frac{\beta A \lambda M_1}{d} = 0 .$$

$$a^2 + (d + \nu + \alpha + \theta_0 + \theta_1 C_{h1} + d)a + (d + \nu + \alpha)(\theta_0 + \theta_1 C_{h1} + d) = 0 ,$$

$$a^2 + ba \pm c = 0$$

If $c<0$ atleast $\lambda>0, R_0=\dfrac{\beta A\lambda M_1}{d(d+\alpha+\nu)(\theta_0+\theta_1 C_{h1}+d)}$

Theorem 1: *Let the following inequality holds,*

$$(36)\quad \beta^2\lambda^2(N^*-Y^*)^2(M^*-M_i^*)^2<\frac{4}{9}(\lambda Y^*+\theta_0+\theta_1 C_h^*+d_m)^2(\beta M_i^*+d+\nu+\alpha)^2,$$

then E^* is locally asymptotically stable.

Proof : By Liapunov Linearization Method, let

$$Y=Y^*+y,\ N=N^*+n,\ M_i=M_i^*+m_i,\ M=M^*+m \text{ and } C_h=C_h^*+c_h.$$

The linearized system corresponding to E^* is

$$\dot{y}=-(\beta M_i^*+d+\nu+\alpha)y+\beta M_i^* n+\beta(N^*-Y^*)m_i,$$

$$\dot{n}=-\alpha y-dn,$$

$$\dot{m}_i=\lambda(M^*-M_i^*)y-(\lambda Y^*+\theta_0+\theta_1 C_h^*+d_m)m_i+\lambda Y^* m-\theta_1 M_i^* c_h,$$

$$\dot{m}=(r-\frac{rM^*}{K}-\theta_1 C_h^*-\theta_0)m-\theta_1 M^* c_h,$$

$$\dot{c}_h=(\phi-\phi_1 C_h^*)m-(\phi_0+\phi_1 M^*)c_h$$

To establish the local stability of the equilibrium E^*, we consider the following positive definite function

$$(37)\qquad V=\frac{1}{2}y^2+\frac{k_1}{2}n^2+\frac{k_2}{2}m_i^2+\frac{k_3}{2M^*}m^2+\frac{k_4}{2}c_h^2,$$

where $k_i\,(i=1,2,3,4)$ are positive constants to be chosen appropriately. Differentiating (\ref{eqn52}), with respect to 't' using the linearized system corresponding to E^*, we get,

$$(38)\quad \dot{V}=-(\beta M_i^*+d+\nu+\alpha)y^2+\beta M_i^* ny+\beta(N^*-Y^*)m_i y-k_1\alpha ny-k_1 dn^2$$

$$+k_2\lambda(M^*-M_i^*)ym_i-k_2(\lambda Y^*+\theta_0+\theta_1C_h^*+d_m)m_i^2+k_2\lambda Y^*mm_i-k_2\theta_1M_i^*C_hm_i$$

$$+\frac{k_3}{M^*}\left[r-\frac{rM^*}{K}-\theta_1C_h^*-\theta_0\right]m^2-\theta_1k_3c_hm+k_4(\phi-\phi_1C_h^*)mc_h-k_4(\phi_0+\phi_1M_i^*)c_h^2$$

Choose, $k_1=\dfrac{\beta M_i^*}{\alpha}$ and $k_4=\dfrac{\theta_1}{(\phi-\phi_1C_h^*)}k_3$, where, $(\phi-\phi_1C_h^*)>0.$ we have,

(39) $$\dot V=-(\beta M_i^*+d+\nu+\alpha)y^2+\beta(N^*-Y^*)m_iy-k_1dn^2+k_2\lambda(M^*-M_i^*)ym_i$$

$$-k_2(\lambda Y^*+\theta_0+\theta_1C_h^*+d_m)m_i^2+k_2\lambda Y^*mm_i-k_2\theta_1M_i^*C_hm_i$$

$$+\frac{k_3}{M^*}\left[r-\frac{rM^*}{K}-\theta_1C_h^*-\theta_0\right]m^2-k_4(\phi_0+\phi_1M_i^*)c_h^2$$

$\dot V$ will be negative definite if the following conditions are satisfied

(40a) $$\beta^2(N^*-Y^*)^2<\frac{2}{3}k_2(\lambda Y^*+\theta_0+\theta_1C_h^*+d_m)(\beta M_i^*+d+\nu+\alpha),$$

(40b) $$k_2\lambda^2(M^*-M_i^*)^2<\frac{2}{3}(\lambda Y^*+\theta_0+\theta_1C_h^*+d_m)(\beta M_i^*+d+\nu+\alpha),$$

(40c) $$k_2\lambda^2Y^{*2}<\frac{2k_3}{3M^*}(r-\frac{rM^*}{K}-\theta_1C_h^*-\theta_0)(\lambda Y^*+\theta_0+\theta_1C_h^*+d_m)$$

From inequalities (40a) and (40b), we can get a positive value of k_2 provided the condition (\ref{eqn2e}) holds. Further from the inequality (40c), we can get a positive values of k_3 .Hence the proof.

4.1 Global Stability: Furthermore, to establish the non-linear stability o the equilibrium E^* we employ Liapunov's stability theory. Thus, we obtain following results regarding the global stability of equilibrium E^*.

Theorem 2: *The equilibrium* E^* *if exists, is globally asymptotically stablein* Ω *, provided,* *(41)* $$\beta^2\lambda^2(N_m - Y_m)^2 < \frac{1}{4}\left[\beta M_i^* + (d+\nu+\alpha)^2\right]\left[\lambda Y^* + \theta_0 + \theta_1 C_h^* + d_m\right]^2,$$

(42) $$\left[\frac{2m_2\lambda^2 Y^{*2} K\theta_1}{r\phi(\lambda Y^* + \theta_0 + \theta_1 C_h^* + d_m)}\right] < m_4$$

$$< min\left[\frac{2r\theta_1\beta d M_i^*}{\phi K\alpha}, \frac{K(\phi_0 + \phi_1 M_i^*)(\lambda Y^* + \theta_0 + \theta_1 C_h^* + d_m)}{\phi_1^2 C_{h_{max}}^2}\right].$$

Proof: Let us consider the positive definite function

(43) $$W = \frac{1}{2}(Y - Y^*)^2 + \frac{m_1}{2}(N - N^*)^2 + \frac{m_2}{2}(M_i - M_i^*)^2$$
$$+ m_3\left(M - M^* - M^*\ln\frac{M}{M^*}\right) + \frac{m_4}{2}(C_h - C_h^*)^2$$

(44) $$\dot{W} = (Y - Y^*)\frac{dY}{dt} + m_1(N - N^*)\frac{dN}{dt} + m_2(M_i - M_i^*)\frac{dM_i}{dt}$$
$$+ m_3\frac{(M - M^*)}{M}\frac{dM}{dt} + m_4(C_h - C_h^*)\frac{dC_h}{dt}.$$

(45) $$\dot{W} = \left[\beta N - \beta Y + m_2\lambda M - \lambda m_2 M_i\right](Y - Y^*)(M_i - M_i^*)$$

$$+\left[\beta M_i^* - m_1\alpha\right](Y - Y^*)(N - N^*) + (m_2\lambda Y^*)(M_i - M_i^*)(M - M^*)$$

$$+\left[(-\lambda Y^* - \theta_0 - \theta_1 C_h^* - d_m)m_2\right](M_i - M_i^*)^2 - \frac{r}{K}m_3(M - M^*)^2$$

$$+\left[-m_3\theta_1 + m_4\phi\right](C_h - C_h^*)(M - M^*) + \left[-\phi_0 m_4 - \phi_1 M_i^* m_4\right](C_h - C_h^*)^2$$

$$-m_4\phi_1 C_h(C_h - C_h^*)(M_i - M_i^*).$$

Choosing $m_1 = \frac{\beta M_i^*}{\alpha}$, $m_3 = \frac{m_4\phi}{\theta_1}$, $m_4 = 1$ and $m_3 = \frac{\phi}{\theta_1}$. $\dot{W}$ can be made negative definite inside the region of attraction Ω if the following conditions are satisfied

(46a) $$\beta^2(N-Y)^2 < \frac{m_2}{2}\left[\beta M_i^* + (d+\nu+\alpha)\right]\left[\lambda Y^* + \theta_0 + \theta_1 C_h^* + d_m\right]$$

(46b) $$m_2\lambda^2(M-M_i)^2 < \frac{1}{2}\left[\beta M_i + (d+\nu+\alpha)\right]\left[\lambda Y^* + \theta_0 + \theta_1 C_h^* + d_m\right]$$

(46c) $$m_2\lambda^2 Y^{*2} < \frac{rm_4\phi}{2K\theta_1}\left[\lambda Y^* + \theta_0 + \theta_1 C_h^* + d_m\right]$$

(46d) $$m_4\phi_1^2 C_h^2 < m_2(\phi_0 + \phi_1 M_i^*)(\lambda Y^* + \theta_0 + \theta_1 C_h^* + d_m)$$

Now From inequalities (46a)and (46b), we can get a positive value of m_2 provided the condition (\ref{eqn1f}) holds. Further, from the inequality (46c) and (46d), we can get the positive value of m_4 provided(\ref{eqn1g}) holds. Hence the proof.

5. Numerical Simulation

To confirm the analytically obtained results and to illustrate the dynamical behavior of the system, numerical simulation has been carried out using MATLAB 7.0.5. We have taken the following set of parameter values in model system (2):

$$\begin{aligned}
&A = 10, \beta = 0.00000001, d = 0.00004, \nu = 0.2,\\
&\alpha = 0.0001, \lambda = 0.000001, \theta_0 = 0.01, r = 0.1,\\
&L = 10000000, \theta_1 = 0.00001, \phi = 0.00001,\\
&d_m = 0.098, \phi_0 = 0.0000001, \phi_1 = 00000001
\end{aligned}$$

For the above set of parameter values it may be checked that the condition of existence of endemic equilibrium E^* is satisfied. The equilibrium components are found as follows:

$$\begin{aligned}
&Y^* = 849.69908,\ N^* = 247875.7523,\\
&M_i^* = 68842.446,\ M^* = 8900000.011 C_h^* = 9999
\end{aligned}$$

The eigenvalues of the Jacobian matrix corresponding to the equilibrium E^* for the model system (2) are -0.3098851638, -0.0007590279143,

-0.00007393177048, -0.08900000021and -0.8900001010. We note that all the five eigen values of J_{E^*} are negative. Hence, for the above set of parameter values the endemic equilibrium E^* is locally asymptotically stable. With these parameter values, the solution trajectories of the model system (2) have been drawn in figure 1 with different initial starts. From this figure, we may see that all the trajectories initiating inside the region of attraction are approaching towards the equilibrium point (M^*, C_h^*) This shows the non-linear stability behavior of the endemic equilibrium E^* in $M^* - C_h^*$ plane.

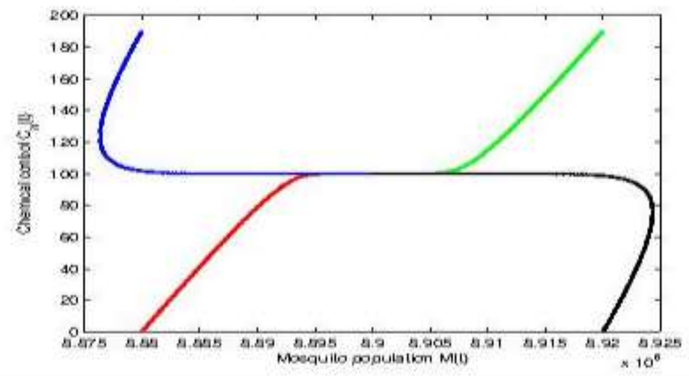


Figure 2. Nonlinear stability of (M^*, C_h^*) in $M - C_h$ plane.

The variation of infective human population $Y(t)$ and infective mosquito population $M_i(t)$ with respect to time t for different values of rate of transmission of susceptible human population to infective human population 'β' and the rate of transmission of susceptible mosquito population to infective mosquito population 'λ' are shown in figures 3 and 4, respectively.

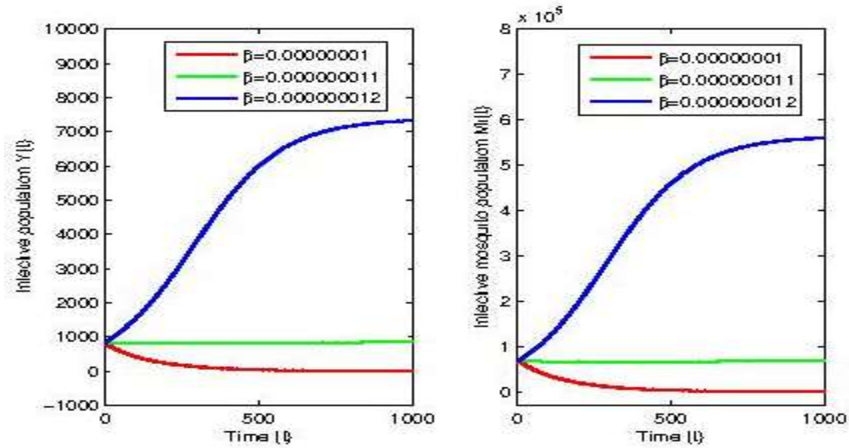


Figure 3. Variation of infected human population and infected mosquito population with time for different values of β.

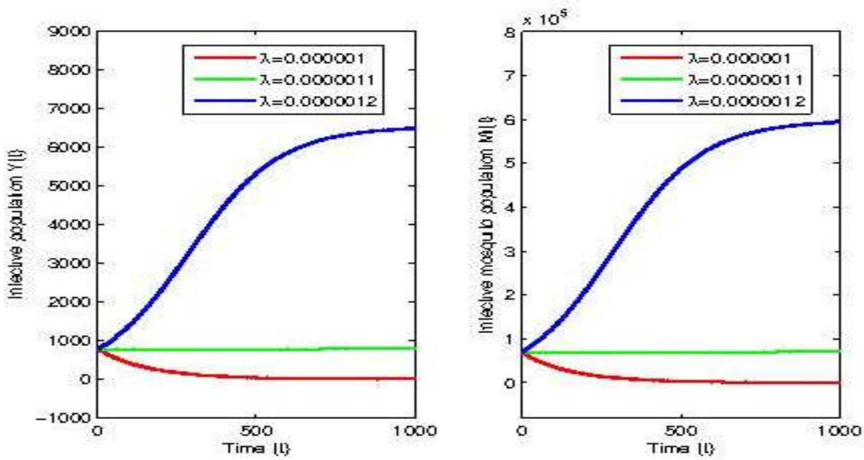


Figure 4. Variation of infected human population and infected mosquito population with time for different values of λ

These figures, illustrate that as the rate of transmission of susceptible human population to infective human population and the rate of transmission of susceptible mosquito population to infective mosquito population increase, infective human population $Y(t)$ and infective mosquito population `$M_i(t)$' both increase. Further, the variations of infective human population `$Y(t)$' and infective mosquito population $M_i(t)$

with respect to time `t' for different values of growth rate coefficient of insecticides due to mosquito population 'ϕ' and decay rate due to uptake by mosquito of the chemicals ϕ_1 are shown in figures 5 and 6, respectively. From these figures, it is apparent that as the depletion rate coefficient of mosquito population due to insecticides 'ϕ' and growth rate coefficient of insecticides due to mosquito population ϕ_1 increase, infective human population $Y(t)$ and infective mosquito population $M_i(t)$ decrease.

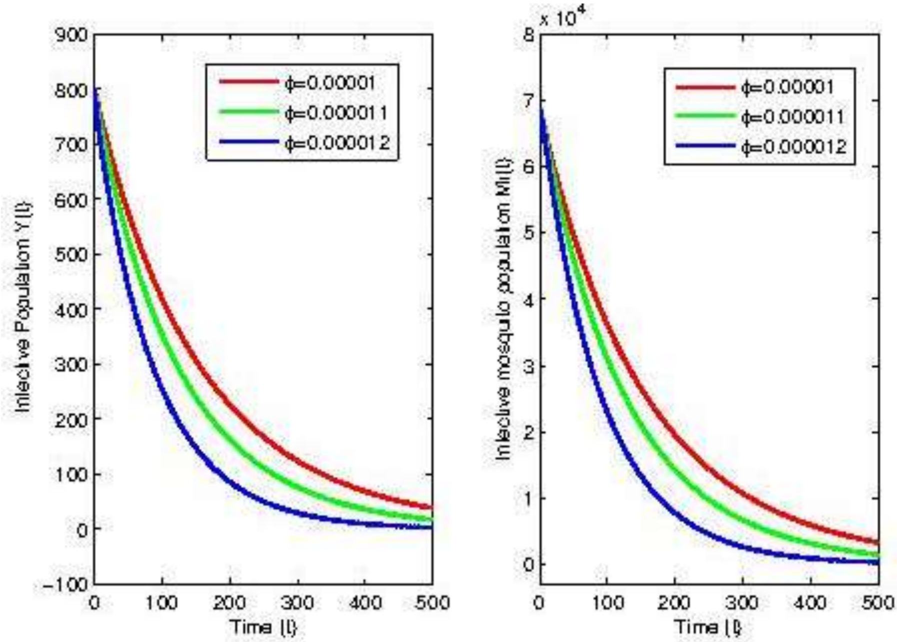


Figure 5. Variation of infected human population and infected mosquito population with time for different values of ϕ

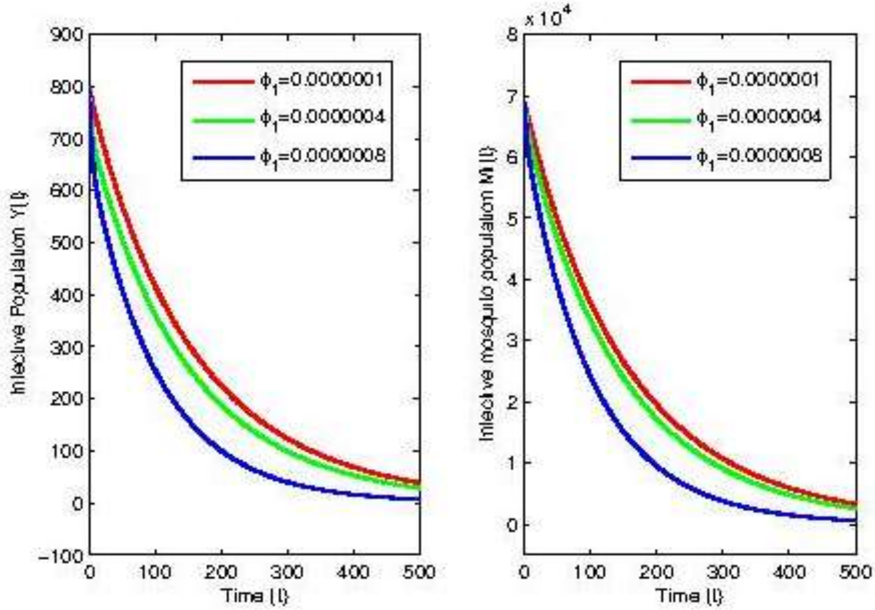


Figure 6. Variation of infected human population and infected mosquito population with time for different values of ϕ_1

6. Conclusion

In this paper, A nonlinear mathematical model for malaria is proposed and analyzed. Equilibria of the model are found and stability behavior of these equilibria are discussed using variational matrix method. It is found that under some conditions the mosquito population may present in the atmosphere but the infected human population is zero. This suggest that under some conditions,the malaria can be eradicated from the community. It is also found that the number of infected individuals decreases as the rate of introduction of chemicals increases. Furthermore it is found that immigration rate of susceptible class makes disease more endemic. Further, numerical simulation is performed to demonstrate the analytical results.

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