Image Quality Estimation by Entropy and Redundancy Calculation for Various Wavelet Families

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Abstract: We implement image compression using various wavelet filter banks and measure performance with rate distortion characterizations. Various separable filter banks are chosen and compared. Coefficients in the subbands obtained by wavelet decomposition are quantized. The image is then reconstructed from the quantized coefficients, and distortion is measured. Three distortion measures are used: Entropy of reconstructed image, energy retained (ER) and redundancy.

Keywords: Image compression, wavelets, entropy, energy retained, and redundancy.

1. Introduction

Compression methods are being rapidly developed to compress large data files such as images, where data compression in multimedia applications has lately become more vital [1]. With the increasing growth of technology and the entrance into the digital age, a vast amount of image data must be handled to be stored in a proper way using efficient methods usually succeed in compressing images, while retaining high image quality and marginal reduction in image size [2].

Recently, discrete wavelet transform (DWT) has emerged as a popular technique for image coding applications [3-4]. In wavelet based image coding, the coding performance depends on the choice of wavelets. Several wavelets which provide suboptimal coding performance have been proposed in the literature [5]. Recently, a few approaches for selecting the optimal filter bank in an image coder have been proposed in the literature. Generally, a wavelet providing optimal performance for the whole image is selected. Finding the optimal wavelet for a particular image is a computationally intensive task. In the proposed work optimal basis is provided on the basis of entropy

calculation and it has been concluded that biorthogonal wavelets perform better as compare to orthogonal ones. The viability of symmetric extension with biorthogonal wavelets is the primary reason cited for their superior performance.

2. Distortion Characterizations

2.1 Entropy

The image entropy can be estimated as [6]

$$(2.1) \qquad H(X) = -\sum_{i=0}^{255} p_i \log p_i,$$

$$(2.2) \qquad p_i = \frac{N_i}{N},$$

where the number of pixels with grey level is N_i; the total number of pixels in the image is N; p_i is the probability of occurrence of one gray level intensity, and

$$(2.3) \qquad \sum_{i=0}^{255} p_i = 1, \qquad\qquad 0 \le p_i \le 1.$$

The entropy of a given source is affected by the number of elements in X. Thus a normalized measure, redundancy, is better for comparing multiple sources.

2.2 Energy Retained

When compressing with orthogonal wavelets the energy retained is :

$$(2.4) \qquad \frac{100*(vector-norm(coeff\_of\_the\_current\_decomposition))^2}{(vector-norm(riginal\_signal))^2}.$$

2.3 Redundancy

The common characteristics of most of the images is that, the neighboring pixels are correlated, and image contains redundant information[7]. Therefore the most important task in image compression is to find a less correlated representation of the image. The fundamental component of image compression is reduction of redundancy and irrelevancy. Redundancy reduction aims at removing duplication from

image, and irrelevancy reduction omits parts of the signal that will not be noticed by Human Visual System (HVS). The redundancies in an image can be identified as spatial redundancy, spectral redundancy and temporal redundancy. Image compression research aims at reducing the number of bits needed to represent an image by removing the spatial and spectral redundancies as much as possible. Since the focus is only on still natural image compression, the temporal redundancy is not considered as it is used in motion picture [7]. Information redundancy, r, is

(2.5) $r = b - He,$

where b is the smallest number of bits for which the image quantization levels can be represented.

3. Entropy and Histogram

From Equation (2.1), a histogram of an image is measured using 256 bins, which correspond to the 256 quantize levels, and the count of each level is divided by N to give the probability p_i. The maximum entropy value is achieved when the gray level values are distributed uniformly, and the minimum entropy value is achieved when the image consists of only one gray level value and the histogram shows one bin. Therefore, the entropy of a gray scale image changes with the distribution of the histogram of the image. When an image is natural image[8], the histogram typically consists of combinational Gaussian distributions in the range (0, 255). In image compression, low image entropy suggests that a high compression ratio can be achieved using efficient coding. The image with lower entropy implies lower image quality for lower information content. In Figure 3 and 4, histograms of test images lena and cameraman are shown. The normalized histogram of Lena has more rise and fall; it has a more uniform distribution with more consistent numbers in each bin. Thus, the entropy of Lena image is higher than the entropy of the cameraman image.

4. Materials and Methods

The following tasks are identified to accomplish efficient image compression.
1. A bitmap image (raw image) of varying sizes 256X256 and 512X512 are taken.
2. Compression is performed by apply 2D DWT over the whole image.
3. Image data entropy is estimated from the gray level histogram.
4. The pseudo program for the ER and redundancy is also depicted.

Figure 1: Lena Image
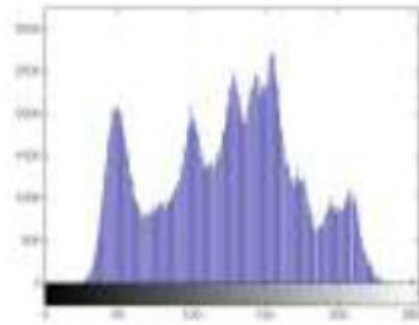
Figure 2: Histogram of lena image

Figure 3: Cameraman Image
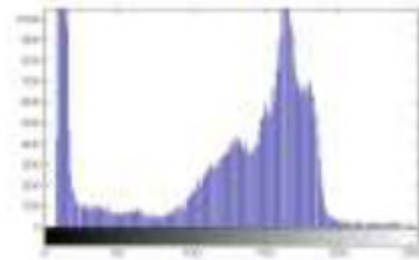
Figure 4: Histogram of cameraman image

5. Experimental Results

The proposed algorithm is used to measure objective quality of compressed images. The quality parameters are energy retained, entropy and redundancy. Importance of these parameters has been stated earlier. The algorithm is applied on images, Lena, Cameraman, Papper, Wbarb, and Mandrill using wavelet toolbox. However, experimental results for Lena image are shown here.
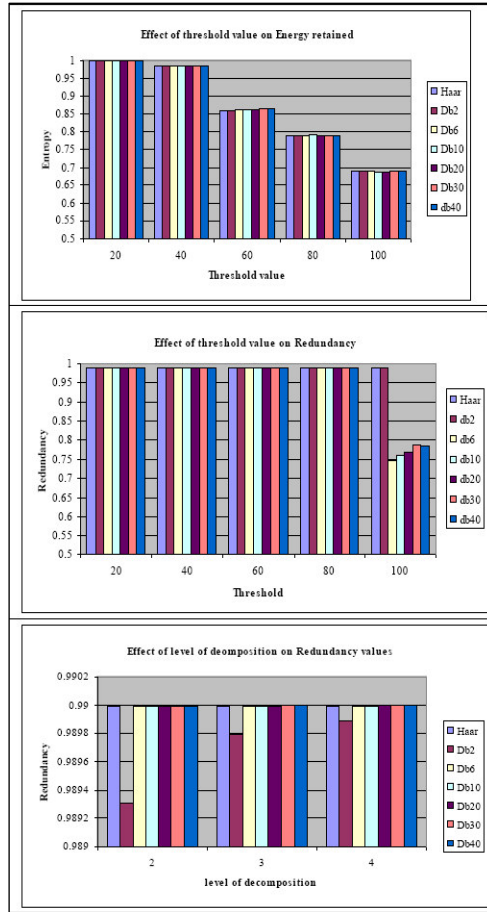
Fig. 5: Experimental results of
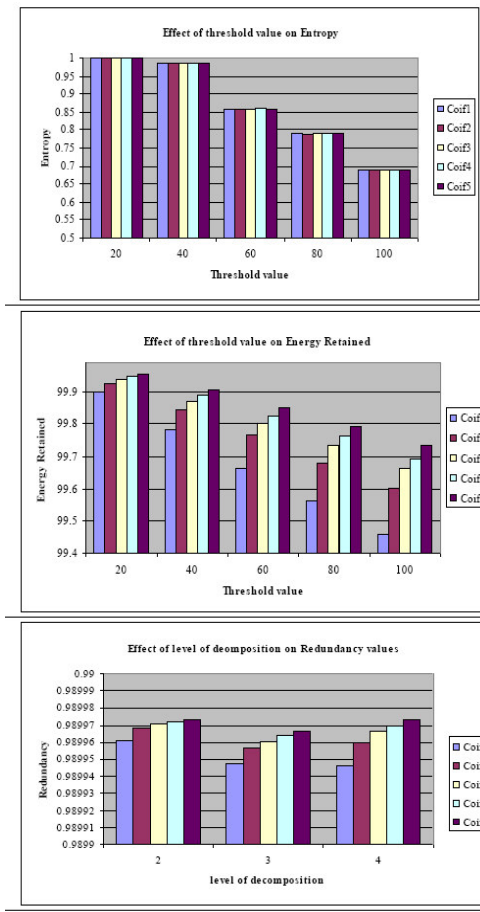Daubechies wavelet family

Fig.6: Experimental results of Coiflet
wavelet family.

6. Results and Discussions

Here analysis of results for image compression for different wavelet families is presented. Results are represented for energy retained, average entropy and redundancy. The average entropy of any source symbols decides the maximum codeword length. It means if the entropy of the compressed image is larger, more no of bits are required to present it. All the wavelets do not have the same properties so the compression for different wavelets will have to be different. On the basis of above tables it is clear that average entropy is minimum for Biorthogonal wavelets for all the test images used. Results indicate that maximum energy in the compressed image is retained in Sym 20 at level of decomposition4 whereas the average entropy found

minimum for Biorto 2.6. Also, the probability of occurrence of events can be used to calculate the coding redundancy. For reducing the redundancy the 2D pixel array that is normally used for human viewing and interpretation must be transformed into a more efficient format. This transformation is called reversible if the original image elements can be reconstructed from the transformed image. Thus the visual redundancies can be reduced using the fact that the human eye does not respond with equal sensitivity to all visual information. Using this fact redundancy calculations have been made and coding redundancy is found minimum for Biortho 2.6 wavelet. This shows that the objective quality of the compressed image is better for Biortogonal wavelet family. Reason behind this performance is that Biorthogonal wavelets can use filters with similar or dissimilar order for decomposition (Nd) and reconstruction (Nr). Therefore Biorthogonal wavelet is parameterized by two numbers and filter length is {max (2Nd, 2Nr) +2}. Higher filter orders give higher degree of smoothness. Wavelet based image compression prefers smooth functions of relatively short support and so the Biorthogonal wavelets perform better.
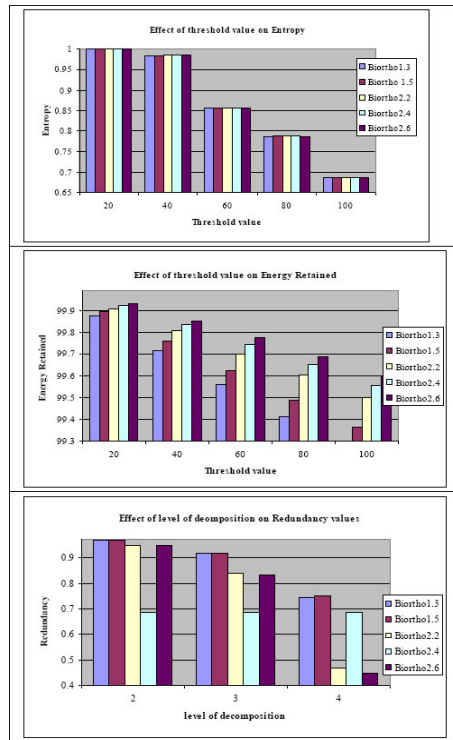

Fig. 7: Experimental results of   Fig. 8: Experimental results of Biorthogonal
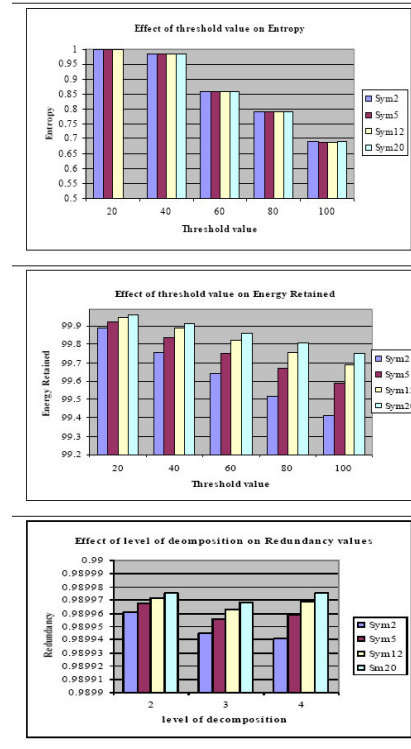   Symlet wavelet family   wavelet family

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References

1. M. J. Nadenau, J. Reichel and M. Kunt, Wavelet Based Color Image Compression: Exploiting the Contrast Sensitivity Function, *IEEE Transactions Image Processing*, **12(1)** (2003) 58-70.
2. Y. K. Jain and S. Jain, Performance Analysis and Comparision of wavelet families Using for Image Compression, *International Journal of Soft Computing*, **2(1)** (2007) 161-171.
3. K. H. Talukder and K. Harada, Wavelet Based Approach for Image Compression and Quality Assessment of Compressed Image, *IAENG International Journal of Applied Mathematics*, 2007.

4. H. Z. Mohamadian Ye and Y. Ye, Information Measures for Biometric Identification via 2D Discrete Wavelet Transform, *Proceedings of the 3rd Annual IEEE Conference on Automation Science and Engineering, CASE'2007* (2007) 835-840.
5. D. Salomon, *Data compression*, Springer, 2nd edn. 2000.
6. M. Misiti, Y. Misiti, G. Oppenheim and J. M. Poggi, *Wavelet Toolbox User's Guide, Version 2.1*, *The Mathworks, Inc.,* 2000.
7. Subhasia Saha, Image Compression – from DCT to Wavelets: A review, *ACM Cross words student's magazine*, **6** No.3, Springer, 2000.
8. Sukanesh R. et al, Analysis of Image Compression by Minimum Relative Entropy (MRE) and Restoration through Weighted Region Growing Techniques for Medical Images, *Engineering Letters*, 14:1, 2007.