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Optimization of Traffic Signal System

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Abstract: For solving a special category of traffic problems, a method has been presented in this paper. The compatibility graph and circular arc graphs corresponding to the problem have been introduced. Compatibility graph, spanning subgraph and circular arc graphs are then utilized to reduce our problem to the solution of LP problems.
Keywords: L.P. problem, circular arc graph, compatibility graph, spanning graph, an interval graph, intersection graph, maximal clique.

1. Introduction

Now a day, increased progress of graph theory is due to its enormous utilization in a number of applications in mathematics. Graph theory has now become a proper tool for researches in different fields like encoding theory, operation research, networking and other fields. Deng X., Hell P. and Huang J. gave algorithms for proper circular-arc graphs and proper interval graphs. Here, we are utilizing the applications of circular-arc graphs in solving a special category of traffic problems. This paper is organized as follows: firstly, we describe the required definitions and preliminaries, and then proposed method has been presented in form of an example. Finally, we solve a similar problem using the offering method to prove its validity.

2. Preliminaries

A finite family of non-empty sets is considered. The intersection graph of this family is obtained by representing each set by a vertex, two vertices being connected by an edge if and only if the corresponding sets intersect. Intersection graphs have received much attention in the study of algorithmic graph theory and their applications. Well-known special classes of intersection graphs include interval graphs, chordal graphs, circular arc graphs, and so on.

Circular arc graph : A circular arc graph is the intersection graph of a family of arcs on a circle. We say that these arcs are a circular arc representation of the graph.

Clique : A clique of graph G is a complete subgraph of G.

Maximal clique : A clique of graph G is a maximal clique of G if it is not properly contained in another clique of G.

Theorem 1. *A graph G is an interval graph if, and only if, G does not contain C_4 as an induced subgraph and G^c admits a transitive orientation.*

3. Problem Statement

Here, the problem that we are dealing with is to install traffic lights at a road junction in such a way that traffic flows smoothly and efficiently at the junction. We take a specific example and explain how our problem could be tackled. *Figure 1* displays the various traffic streams, namely *a, b, c, d* that are meeting at Home science Institute crossing, Khandari, Agra ,U.P.(India).

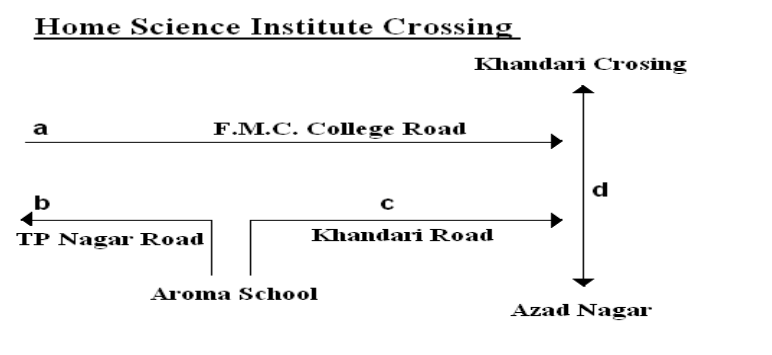

Fig. 1

Those traffic streams may be termed compatible whose simultaneous flow would not result in any accident. For instance, in *Figure 1*, streams *a* and *b* are compatible, whereas *a* and *d* are not. The phasing of lights should be such that when the green lights are on for two streams, they should be compatible. We suppose that the total time for the completion of green and red lights during one cycle is two minutes. We form a graph G whose vertex set consist of the traffic streams in question and we make two vertices of G

adjacent if and only if, the corresponding streams are compatible. This graph is the compatibility graph corresponding to the problem in question.
The compatibility graph of *Figure 1* is shown as,

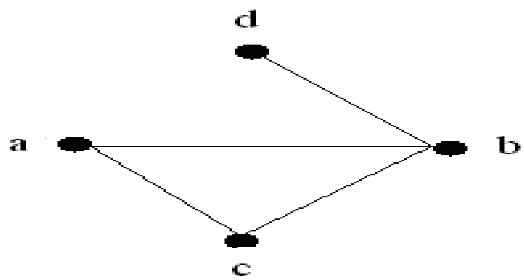

Fig. 2: Compatibility graph

Let's assume the perimeter of a circle corresponds to the total cycle period of 120 seconds. We may think that the duration when a given traffic stream gets a green light corresponds to an arc of this circle. Hence, two such arcs of the circle can overlap only if the corresponding streams are compatible. The resulting circular arc graph may not be the compatibility graph because we do not demand that two arcs intersect whenever correspond to compatible flows(There may be two compatible streams but they need not get a green light at the same time). However, the intersection graph *H* of this circular arc graph[1] will be a spanning subgraph of the compatibility graph.

The proper graph *H* for the above example is shown as,

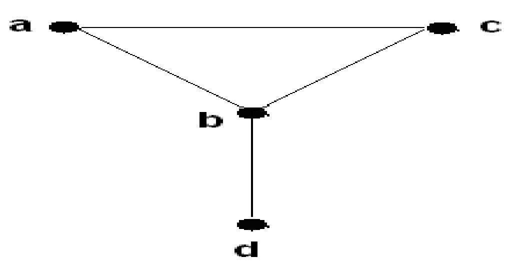

Fig. 3: Intersection Graph H

So we have to take all spanning subgraphs of *G* into account and choose from them the spanning subgraph that has the most maximal clique. The efficiency of our phasing may be measured by minimizing the total red light time during a traffic cycle,that is the total waiting time for all the traffic streams during a cycle. For the sake of concreteness, we may assume that at the time of starting, all lights are red.

The maximal clique of $H\ are\ k_1 = \{a,b,c\}\ and\ k_2 = \{b,d\}$. Each clique $k_i, 1 \le i \le 2,$ corresponds to a phase during which all streams in that clique receive green lights.

In phase 1, traffic streams *a, b* and *c* receive a green light; In phase 2, *b* and *d* receive a green light.

Suppose, we assign to each phase k_i a duration d_i. Our aim is to determine the $d_i, s(\ge 0)$ so that the total waiting time is minimum. Further, we may assume that the minimum green light time for any stream is 20 seconds.

Traffic stream *a* gets a red light when the phase k_2 receives a green light. Hence, *a*'s total red light time is d_2.

Similarly, the total red light time of traffic streams *c* and *d* , respectively, are $d_2\ and\ d_1$.

Therefore, the total red light time of all the streams in one cycle is

$$Z = d_1 + 2d_2,$$

Our aim is to minimize Z subject to[2]

$$d_i \ge 0,\quad 1 \le i \le 2;\quad d_2 \ge 20,\ \ d_1 \ge 20;\ \ and\ d_1 + d_2 \ge 20,\ \ d_1 + d_2 \ge 120.$$

The optimal solution to this problem is $d_1 = 100,\ \ d_2 = 20,\ min\ Z = 140.$

This least value would be the best phasing of the traffic lights.

Example

Now we take another specific example and explain how our problem could be tackled. *Figure 4,* displays the various traffic streams namely *a,b,c,….,g* that are meeting at the Khandari Crossing , Agra, Uttar Pradesh (India).

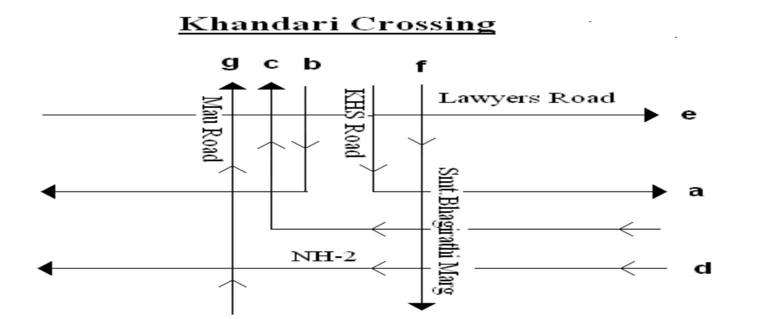

Fig. 4

Here also, those traffic streams whose simultaneous flow would not result in any accident, they may be termed as compatible streams. For instance, here in *figure 4,* streams a and d are compatible, whereas b and g are not. The phasing of lights should be such that when the green lights are on for two streams, they should be compatible. We suppose that the total time for the completion of green and red lights during one cycle is two minutes.

We form a graph G whose vertex set consists of the traffic streams in question and we make two vertices of G adjacent if, and only if, the corresponding streams are compatible. This graph is the compatibility graph corresponding to the problem in question.

The compatibility graph is shown as follows,

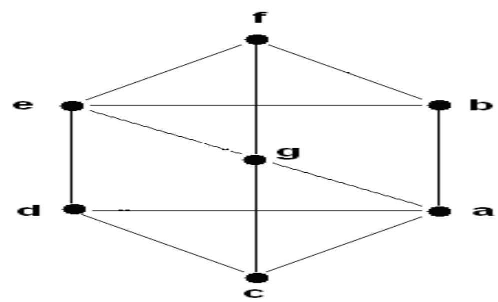

Fig. 5: Compatibility graph

We take a circle and assume that its perimeter corresponds to the total cycle period of 120 seconds. We may think that the duration when a given traffic stream gets a green light corresponds to an arc of this circle. This circle is shown as,

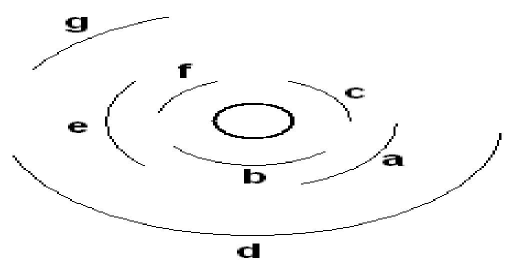

Fig. 6: A green light assignment

Hence, two such arcs of the circle can overlap only if the corresponding streams are compatible. The resulting *circular arc graph* may not be compatibility graph because we do not demand that two arcs intersect whenever they correspond to compatible flows(There may be two compatible streams but they need not get a green light at the same time).

However, the intersection graph H of this circular arc graph will be a spanning sub graph of the compatibility graph[3].

The intersection graph H is shown below,

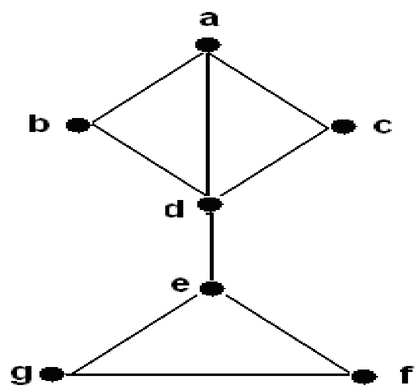

Fig. 7: Intersection Graph H

The efficiency of our phasing may be measured by minimizing the total red light time during a traffic cycle, that is, the total waiting time for all the traffic streams during a cycle. For the sake of concreteness, we may assume that at the time of starting, all lights are red. This would ensure that *graph H* is an interval graph *Figure 6* gives a feasible green light assignment whose corresponding intersection *graph H* is given in *figure 7*.

The maximal cliques [5] of *H* are

$$K_1 = \{a, b, d\},$$

$$K_2 = \{a, c, d\},$$

and
$$K_3 = \{d, e\},$$

$$K_4 = \{e, f, g\}.$$

Since *H* is an interval graph, by the Theorem 1 (*A graph G is an interval graph if, and only if, G does not contain C_4 as an induced subgraph and G^c admits a transitive orientation.*),H^c has a transitive orientation.
Since (*b,c*), (*c,e*) and (*d,f*) are arcs of H^c and since $b \in K_1$, $c \in K_2$, $d \in K_3$ and $e \in K_4$ *etc.*, we have $K_1 < K_2 < K_3 < K_4$.

In the consecutive ordering of the maximal cliques of *graph H*, each clique $K_i, 1 \le i \le 4,$ corresponds to a phase during which all streams in that clique receive green lights. We start a given traffic stream with a green light during the first phase in which it appears, and keep it green until the last phase in which it appears. Because of the consecutiveness of the ordering of the phases K_I, this gives an arc on the clock circle.

In phase 1, traffic streams *a, b* and *d* receive green light; In phase 2, *a, c,* and *d* receive green light and so on. Suppose, we assign to each phase k_i a duration d_i. Our aim is to determine the $d_i\text{'}s (\geq 0)$ so that the total waiting time is minimum. Further, we may assume that the minimum green light time for any stream is 20 seconds.

Traffic stream *a* gets a red light when the phase k_3 receives a green light.

Hence, *a*'s total red light time is $d_3 + d_4$.

Similarly, the total red light time of traffic streams *c* and *d* , respectively, are d_2 *and* d_3.

Therefore, the total red light time of all the traffic streams *b ,c, d, e, f, are g,* respectively, are $d_2 + d_3 + d_4$; $d_1 + d_3 + d_4$; d_4; $d_1 + d_2$; $d_1 + d_2 + d_3$; *and* $d_1 + d_2 + d_3$.

Therefore, the total red light time of all the streams in one cycle is

$$Z = 4d_1 + 4d_2 + 4d_3 + 3d_4.$$

Our aim is to minimize *Z* subject to

$$d_i \geq 0, 1 \leq i \leq 4; \quad d_1 + d_2 \geq 20; \quad d_1 \geq 20,$$

$$d_2 \geq 2$$

$$d_1 + d_2 + d_3 \geq 20,$$

$$d_3 + d_4 \geq 20$$

and $$d_4 \geq 20,$$

$$d_1 + d_2 + d_3 + d_4 = 120.$$

The optimal solution to this problem is

$$d_1 = 80, \ d_2 = 20, \ d_3 = 0, \ d_4 = 20, \ \textit{and min } Z = 480 \textit{ seconds}.$$

There are also other possible circular arc graphs, that give another feasible green light arrangement and their corresponding intersection graphs.

Thus, we have to exhaust all possible circular graphs and then take least of the minima thus obtained. The phasing that corresponds to this least value would then be the best phasing of the traffic lights.

4. Conclusion

Here, we present an application of interval graphs to the problem of optimization of traffic signal lights at road junction. The problem is to install traffic lights at a road junction in such a way that traffic flows smoothly and efficiently at the junction. The compatibility graph corresponding to the problem and circular arc graphs have been introduced. Illustrative examples are also included to demonstrate the validity and applicability of the technique.

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