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Analytical/Numerical Investigations of Kelvin-Helmholtz Instability of Superposed Viscous Fluids in Hydromagnetics Saturating Porous Medium

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Abstract: This paper deals with the instability of superposed, viscous fluids saturating porous medium in the presence of horizontal magnetic field. Using linear theory and normal mode technique the dispersion relation so obtained is analyzed mathematically for the stable configuration. The effects of medium porosity, medium permeability and magnetic field, on the growth rate (imaginary) of the most unstable mode have been investigated numerically. The square of the Alfven velocity and medium permeability have stabilizing effect on the system and kinematic viscosity of lower and upper fluid and medium porosity have destabilizing effect on the system. All these numerical results have been depicted graphically.

Keywords: Kelvin-Helmholtz instabilit, superposed viscous fluids, porous medium.

2010 Mathematics Subject Classification: 76S05

1. Introduction

Kelvin-Helmholtz instability occurs when we consider the character of the equilibrium of a stratified heterogeneous fluid in which different layers are in relative motion. The most important case is when two superposed fluids flow one over the other with a relative horizontal velocity, the instability of the plane interface between the two fluids when it occurs in this instance, is known as Kelvin-Helmholtz instability. The experimental demonstration of the Kelvin-Helmholtz instability has been given by Francis[1]. The effect of rotation and a general oblique magnetic field on the Kelvin-Helmholtz instability has been studied by Sharma and Srivastava[2].

Michael[3] has discussed the stability of a combined current and vortex sheet in a perfectly conducting fluid, while the effect on the Kelvin-Helmholtz instability of a magnetic field transverse to the direction of streaming has been considered by Northrop[4]. There are diverse applications of the Kelvin-Helmholtz instability like: to examine the horizontal and temporal variability of the out-of-cloud vertical velocity, the stratospheric gravity wave response to the convection to determine the vertical and spatial extent of turbulence due to gravity wave breaking, to provide a more realistic evolving background flow and convective initiation. A regional scale forecast model is used to force the cloud model the time evolution of the bulent region, effects of model resolution, wave instability and trapping. It is also used in understanding of CIT-generating mechanisms which is extremely important for commercial and other high-altitude aircraft flying above developing convection. The instability of the plane interface separating two uniform superposed streaming fluids under varying assumptions of hydrodynamics and hydromagnetics has been discussed in a treatise by Chandrasekhar[5] . Alterman[6] has studied the effect of surface tension to the Kelvin-Helmholtz instability of two rotating fluids. Reid[7] studied the effect of surface tension and viscosity on the stability of two superposed fluids. Bellman and Pennington[8] further investigated in detail illustrating the combined effects of viscosity and surface tension. Cavus and Kazkapan[9] have studied magnetic Kelvin-Helmholtz instability in the solar atmosphere and have found that the growth rate of instability increases with velocity shear, it needs higher values of magnetic field in order to stabilize as said in Lapenta and Knoll[10]. We further notice that the uniform magnetic field along the direction of shear flow parallel to interface can have stabilizing effect as given in Ofman and Thompson[11] . The medium has been assumed to be non-porous in these studies.

The flow through porous medium has been of considerable importance in recent years particularly among geophysical fluid dynamics, recovery of crude oil from the pores of reservoir rocks, chemical engineering (absorption, filtration), petroleum engineering, hydrology, soil physics and biophysics etc. The physical properties of comets, meteorites and interplanetary dust strongly suggest the significance of the effect of porosity in astrophysical context given by McDonnel[12]. The gross effect, as the fluid slowly percolates through the pores of the rock, is represented by Darcy's law which states that the usual viscous term in the equations of motion is replaced by the resistance term $-\frac{\mu}{k_1}\boldsymbol{q}$, where μ is the viscosity of the fluid, k_1 the permeability of the medium (which has the dimension of length squared), and $\boldsymbol{q}$ the filter (seepage) velocity of the fluid. Sunil and Chand[16] have investigated the effect of permeability of the porous medium on different stability problems.

Sharma et al.[13] have studied the Kelvin-Helmholtz instability through porous medium of two superposed plasmas. The instability of the plane interface between two uniform superposed and streaming fluids through porous medium has been studied theoretically and analytically by Sharma and Sapnos[14].

Sharma and Kumari[15] have studied the stability of stratified fluid in porous medium in the presence of suspended particles and variable magnetic field. Some solar activities in the solar atmosphere are created by a Kelvin-Helmholtz instability in the presence of magnetic field and subsequent reconnection processes and Kelvin-Helmholtz instability plays an important role in energy transfer mechanism in the solar atmosphere. The effect of the Kelvin-Helmholtz instability is shown to convert shear flow in compression flow that derives reconnection. Khalil Elcoot[16] has studied the new analytical approximation forms for non-linear instability of electric porous media. Asthana et al.[17] have been studied Kelvin-Helmholtz instability of two viscous fluids in porous medium for two dimensional flow. Rudraiah et al.[18] have studied the study of surface instability of Kelvin-Helmholtz type in a fluid layer bounded above by a porous layer and below by a rigid surface. The effect of porosity in astrophysical context and the plasma outflow occur in regions which are created by the Kelvin-Helmholtz vortices. We believe that the mechanism presented here opens promising possibilities of further investigation. However a clear understanding of the role of the Kelvin-Helmholtz instability in reconnection requires a fully three-dimensional flows.

Keeping in view, the diverse applications stated earlier, a study has been therefore, investigated to examine the effect of the magnetic field, medium porosity and the medium permeability on the instability of electrically conducting, streaming viscous three dimensional fluids saturating porous medium numerically using the software Mathematica version-5.2.

2. Formulation of the Problem and Perturbation Equations

The initial state whose stability we wish to examine is that of an incompressible, electrically infinitely conducting viscous fluid in which there is a horizontal streaming in the x - direction with a velocity $\boldsymbol{U}(z)$ through a homogeneous and isotropic porous medium of medium porosity ε and medium permeability k_1. A uniform horizontal magnetic field $\boldsymbol{H}$ and acceleration due to gravity $\boldsymbol{g}(0,0,,g)$ pervade the system. Then the equations of motion, continuity, incompressibility for the viscoelastic fluid and the Maxwell's equations saturating porous media are

(2.1) $$\frac{\rho}{\varepsilon}\left(\frac{\partial \boldsymbol{U}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{U}.\nabla)\boldsymbol{U}\right)=-\nabla p-\rho\boldsymbol{g}-\frac{\mu}{k_1}\boldsymbol{U}+\frac{1}{4\pi}(\nabla\times\boldsymbol{H})\times\boldsymbol{H},$$

(2.2) $$\nabla.\boldsymbol{U}=0,$$

(2.3) $$\varepsilon\frac{\partial\rho}{\partial t}+(\boldsymbol{U}.\nabla)\rho=0,$$

(2.4) $$\nabla.\boldsymbol{H}=0,$$

(2.5) $$\varepsilon\frac{\partial\boldsymbol{H}}{\partial t}=\nabla\times(\boldsymbol{U}\times\boldsymbol{H}),$$

where $\boldsymbol{U}(U(z),0,0)$, p, ρ, $\boldsymbol{g}$, μ and $\boldsymbol{H}(H,0,0)$ denote, respectively, the fluid velocity, fluid pressure, fluid density, acceleration due to gravity, viscosity and magnetic field. $\delta(z-z_s)$ denotes Dirac's delta function and the magnetic permeability is assumed to be unity. The initial stationary state solution is given by

(2.6) $$\boldsymbol{U}=(U(z),0,0),\ \rho=\rho(z), p=p(z), \boldsymbol{H}(H,0,0).$$

In other words, in the perturbed state at any point (x, y, z), we have

density = $\rho+\delta\rho$,

pressure = $p+\delta p$,

magnetic field = $\boldsymbol{H}+\boldsymbol{h}=(H+h_x,h_y,h_z)$,

velocity of the hydromagnetic fluid = $\boldsymbol{U}+\boldsymbol{u}=(U+u,\ v,\ w)$.

This initial state is given a small disturbance. As a consequence of this, let $\boldsymbol{u}(u,v,w)$, $\boldsymbol{h}(h_x,h_y,h_z)$, δp, $\delta\rho$ and $\delta z_s\ (x,y,t)$, denote, the perturbations in fluid velocity $\boldsymbol{U}(U(z),\ 0,0)$, magnetic field $\boldsymbol{H}$, pressure p and density ρ, respectively.

Using the initial stationary state solutions given by (2.6) and the linear theory (i.e. neglecting the product of perturbations and higher order perturbations), the equations (2.1) – (2.5) in the linearized perturbed form become

$$\frac{\rho}{\varepsilon}\left(\frac{\partial u}{\partial t}+\frac{U}{\varepsilon}\frac{\partial u}{\partial x}+\frac{w}{\varepsilon}\frac{\partial U}{\partial z}\right)=-\frac{\partial}{\partial x}\delta p-\frac{\mu}{k_1}(u+U), \tag{2.7}$$

$$\frac{\rho}{\varepsilon}\left(\frac{\partial v}{\partial t}+\frac{U}{\varepsilon}\frac{\partial v}{\partial x}\right)=-\frac{\partial}{\partial y}\delta p-\frac{\mu}{k_1}v+\frac{H}{4\pi}\left(\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}\right), \tag{2.8}$$

$$\frac{\rho}{\varepsilon}\left(\frac{\partial w}{\partial t}+\frac{U}{\varepsilon}\frac{\partial w}{\partial x}\right)=-\frac{\partial}{\partial z}\delta p-\frac{\mu}{k_1}w+\frac{H}{4\pi}\left(\frac{\partial h_z}{\partial x}-\frac{\partial h_x}{\partial z}\right)-g\delta\rho\,, \tag{2.9}$$

$$\frac{\partial\left(u+U\right)}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{2.10}$$

$$\left(\varepsilon\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\delta\rho=-w\frac{\mathrm{d}\rho}{\mathrm{dz}}, \tag{2.11}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0\,, \tag{2.12}$$

$$\left(\varepsilon\frac{\partial h_x}{\partial t}+U\frac{\partial h_x}{\partial x}\right)=H\frac{\partial u}{\partial x}+h_z\frac{\partial U}{\partial z}\,, \tag{2.13}$$

$$\left(\varepsilon\frac{\partial h_y}{\partial t}+U\frac{\partial h_y}{\partial x}\right)=H\frac{\partial v}{\partial x}\,, \tag{2.14}$$

$$\left(\varepsilon\frac{\partial h_z}{\partial t}+U\frac{\partial h_z}{\partial x}\right)=H\frac{\partial w}{\partial x}. \tag{2.15}$$

The disturbances are analyzed into normal modes by seeking solutions of the above equations, whose dependence on x, y and t is of the form

$$\exp i\left(k_x x+k_y y+nt\right) \tag{2.16}$$

where n is the growth rate, $k=\left(k_x{}^2+k_y{}^2\right)^{1/2}$ is the resultant wave number and $k_x,\, k_y$ are the horizontal wave numbers.

Using expression (2.16), equations (2.7)-(2.15) become

(2.17) $$\left[\frac{i\rho}{\varepsilon}\left(n+\frac{U}{\varepsilon}k_x\right)+\frac{\mu}{k_1}\right]u+\frac{\rho}{\varepsilon^2}w\frac{dU}{dz}=-ik_x\delta p\,,$$

(2.18) $$\left[\frac{i\rho}{\varepsilon}\left(n+\frac{U}{\varepsilon}k_x\right)+\frac{\mu}{k_1}\right]v=-ik_y\delta p+\frac{H}{4\pi}\left(ik_xh_y-ik_yh_x\right),$$

(2.19) $$\left[\frac{i\rho}{\varepsilon}\left(n+\frac{U}{\varepsilon}k_x\right)+\frac{\mu}{k_1}\right]w=-\frac{\partial}{\partial z}\delta p+\frac{H}{4\pi}\left(ik_xh_z-\frac{\partial}{\partial z}h_x\right)-g\delta\rho,$$

(2.20) $$ik_xu+ik_yv+Dw=0\,,$$

(2.21) $$i\left(\varepsilon n+Uk_x\right)\delta\rho=-w\frac{d\rho}{dz}\,,$$

(2.22) $$i\left(\varepsilon n+k_xU\right)h_x=ik_xHu+h_zDU\,,$$

(2.23) $$i\left(\varepsilon n+k_xU\right)h_y=ik_xHv\,,$$

(2.24) $$i\left(\varepsilon n+k_xU\right)h_z=ik_xHw\,,$$

(2.25) $$ik_xh_x+ik_yh_y+Dh_z=0\,.$$

.

Multiplying equation (2.17) by $-ik_x$ and (2.18) by $-ik_y$ and adding the resulting equations and using (2.20), we get

(2.26) $$\left[\frac{i\rho}{\varepsilon}\left(n+\frac{U}{\varepsilon}k_x\right)+\frac{\mu}{k_1}\right]Dw=-k^2\delta p+\frac{H}{4\pi}\left(k_xk_yh_y-k_y^2h_x\right).$$

.

Eliminating $u, v, h_x, h_y, h_z, \delta\rho$ and δp from equations (2.20)-(2.25) and using (2.19) and (2.26), we obtain after a little algebra,

$$D\left[\left\{\frac{i\rho}{\varepsilon}\left(n+\frac{k_xU}{\varepsilon}\right)+\frac{\rho}{k_1}\right\}Dw-\frac{ik_x\rho}{\varepsilon^2}wDU\right]+\frac{ik_x^2H^2k^2}{4\pi\left(\varepsilon n+k_xU\right)}w$$

$$-k^2\left\{\frac{i\rho}{\varepsilon}\left(n+\frac{k_xU}{\varepsilon}\right)+\frac{\rho}{k_1}\right\}w-igk^2\left[D\rho\right]\frac{w}{\varepsilon n+k_xU}$$

$$-ik_x\frac{H^2}{4\pi}D\left[\frac{1}{\left(\varepsilon n+k_xU\right)}\left\{\begin{matrix}k_xDw-\dfrac{k^2\left(DU\right)}{\left(\varepsilon n+k_xU\right)}w- \\ \dfrac{ik_y^2\left(DU\right)\left(\dfrac{\rho}{\varepsilon^2}-\dfrac{k_x^2H^2}{4\pi\left(\varepsilon n+k_xU\right)^2}\right)w}{\dfrac{i\rho}{\varepsilon^2}\left(\varepsilon n+k_xU\right)+\dfrac{\mu}{k_1}-\dfrac{ik_x^2H^2}{4\pi\left(\varepsilon n+k_xU\right)}}\end{matrix}\right\}\right] \tag{2.27}$$

$$-\frac{ik_xk_y^2H^2}{4\pi}\left[\frac{\dfrac{\mu}{k_1}\left(DU\right)w}{\dfrac{i\rho}{\varepsilon^2}\left(\varepsilon n+k_xU\right)+\dfrac{\mu}{k_1}-\dfrac{ik_x^2H^2}{4\pi\left(\varepsilon n+k_xU\right)}}\right]=0,$$

where $D=\dfrac{d}{dz}$.

3. Two Uniform Streaming Fluids Separated by a Horizontal Boundary

Let two uniform fluids of densities ρ_1 and ρ_2 be separated by a horizontal boundary at z = 0 and the density ρ_2 of the upper fluid be less than the density ρ_1 of the lower fluid so that, in the absence of streaming, the configuration is stable one. Let the two fluids be streaming with velocities U_1 and U_2. Then in each region of constant ρ, ν and U, equation (2.27) reduces to

$$\left(D^2-k^2\right)w=0. \tag{3.1}$$

The boundary conditions to be satisfied are

(i) w must be bounded both when $z\to+\infty$ (in the upper fluid) and $z\to-\infty$ (in the lower fluid).

(ii) Since U is discontinuous at $z=0$, the uniqueness of the normal displacement of any point on the interface implies, according to (16), that

(3.2) $$\frac{w}{\left(\varepsilon n+k_x U\right)}$$

must be continuous at the interface.

(iii) Integrating equation (2.27) between $z_s-\delta$ and $z_s+\delta$ passing to the limit $\delta=0,$ we obtain in view of (2.29), the jump condition

(3.3) $$\Delta_s\left[\left\{\frac{i\rho}{\varepsilon}\left(n+\frac{k_xU}{\varepsilon}\right)+\frac{\mu}{k_1}\right\}Dw-\frac{ik_x^2H^2}{4\pi}\Delta_s\frac{Dw}{\left(\varepsilon n+k_xU\right)}\right] = igk^2\left[\Delta_s\left(\rho\right)\right]\left(\frac{w}{\varepsilon n+k_xU}\right), \text{ for } z=z_s$$

while the equation valid everywhere else $z\neq 0$ is

(3.4) $$D\left[\left\{\frac{i\rho}{\varepsilon}\left(n+\frac{k_xU}{\varepsilon}\right)+\frac{\mu}{k_1}\right\}Dw\right]k^2\left\{\frac{i\rho}{\varepsilon}\left(n+\frac{k_xU}{\varepsilon}\right)+\frac{\mu}{k_1}\right\}w-\frac{ik_x^2H^2}{4\pi\left(\varepsilon n+k_xU\right)}\left(D^2-k^2\right) = igk^2\left(D\rho\right)\left(\frac{w}{\left(\varepsilon n+k_xU\right)}\right),$$

where $\Delta_{z=0}\left(f\right)=f\left(z_0+0\right)-f\left(z_0-0\right)$ is the jump which a quantity experiences at the interface $z=0$ and the subscript 0 distinguishes the value, a quantity known to be continuous at an interface, takes at $z=0$.

Since $\dfrac{w}{\left(\varepsilon n+k_xU\right)}$ must be continuous on the surface $z=0$ and w cannot increase exponentially on either side of the surface, the solutions appropriate for the two regions are

(3.5) $$w_1=A\left(\varepsilon n+k_xU_1\right)e^{+kz},\quad \left(z<0\right)$$

(3.6) $$w_2=A\left(\varepsilon n+k_xU_1\right)e^{-kz},\quad \left(z>0\right)$$

where A is a constant.

Applying the boundary condition (3.3) to the solutions (3.5) and (3.6), the general characteristic equation for frequency is obtained for magnetized case saturating a porous medium is

$$
\begin{aligned}
(3.7)\quad & n^2+\left[\frac{2k_x}{\varepsilon}\left(\alpha_1 U_1+\alpha_2 U_2\right)-\frac{i\varepsilon}{k_1}\left(\alpha_1\nu_1+\alpha_2\nu_2\right)\right]n \\
& +\left[\frac{k_x^2}{\varepsilon^2}\left(\alpha_1 U_1^2+\alpha_2 U_2^2\right)-\frac{ik_x}{k_1}\left(\alpha_1\nu_1 U_1+\alpha_2\nu_2 U_2\right)-2k_x^2V_A^2-gk\left\{\left(\alpha_2-\alpha_1\right)\right\}\right]=0,
\end{aligned}
$$

where $\nu_1\left(=\mu_1/\rho_1\right)$ and $\nu_2\left(=\mu_2/\rho_2\right)$ are the kinematic viscosities of fluids 1 and 2, respectively.

$\alpha_1=\dfrac{\rho_1}{\rho_1+\rho_2}$, $\alpha_2=\dfrac{\rho_2}{\rho_1+\rho_2}$ and $V_A{}^2=\dfrac{H^2}{4\pi\left(\rho_1+\rho_2\right)}$ is the square of Alfv¶e n velocity.

Now the special case in which the lower and upper fluids are streaming with velocities $U\left(=U_1\right)$ and $-U(=U_2)$, respectively is considered. Then the equation (3.7) reduces to

$$
\begin{aligned}
(3.8)\quad & n^2+\left[\frac{2k_x}{\varepsilon}(\alpha_1-\alpha_2)U-\frac{i\varepsilon}{k_1}\left(\alpha_1\nu_1+\alpha_2\nu_2\right)\right]n \\
& +\left[\frac{k_x{}^2U^2\left(\alpha_1+\alpha_2\right)}{\varepsilon}-\frac{ik_x}{k_1}(\alpha_1\nu_1-\alpha_2\nu_2)U-2k_x{}^2V_A{}^2-\mathrm{g}k\left\{\left(\alpha_1-\alpha_2\right)\right\}\right]=0.
\end{aligned}
$$

Since the perturbations most sensitive to Kelvin-Helmholtz instability are in the direction of streaming, so $k_x=k$. The frequency can be decomposed as $n=n_r+in_i$, where n_r and n_i are real; in equation (3.8) and equating real and imaginary parts on both sides, we get

$$
\begin{aligned}
(3.9)\quad & \left(n_r{}^2-n_i{}^2\right)+\left[\frac{2k_x}{\varepsilon}(\alpha_1-\alpha_2)U\right]n_r+\left[\frac{\varepsilon}{k_1}\left(\alpha_1\nu_1+\alpha_2\nu_2\right)\right]n_i \\
& +\left[\frac{k_x{}^2U^2}{\varepsilon}\left(\alpha_1+\alpha_2\right)-2k_x{}^2V_A{}^2-gk\left\{\left(\alpha_1-\alpha_2\right)\right\}\right]=0
\end{aligned}
$$

and

$$
(3.10)\quad \left[\frac{2k_x}{\varepsilon}(\alpha_1-\alpha_2)U\right]n_i-\left[\frac{\varepsilon}{k_1}\left(\alpha_1\nu_1+\alpha_2\nu_2\right)\right]n_r+2n_rn_i-\left[\frac{k_x}{k_1}(\alpha_1\nu_1-\alpha_2\nu_2)U\right]=0\,.
$$

On solving equations (3.9) and (3.10), we get

$$4n_r^{\,4} + 8An_r^{\,3} + \left(5A^2 + B^2 + 4C\right)n_r^{\,2} + \left(A^3 + AB^2 + 4AC\right)n_r + \left(-D^2 + ABD + A^2C\right) = 0 \tag{3.11}$$

and

$$4n_i^{\,4} - 8A^2Bn_i^{\,3} + \left(5B^2 + A^2 - 4C\right)n_i^{\,2} + \left(B^3 + A^2B + 4BC\right)n_i - \left(D^2 - ABD + B^2C\right) = 0, \tag{3.12}$$

where

$$\left.\begin{aligned} A &= \frac{2k_x}{\varepsilon}\left(\alpha_1 - \alpha_2\right)U, \\ B &= \frac{\varepsilon}{k_1}\left(\alpha_1\nu_1 + \alpha_2\nu_2\right), \\ C &= \frac{k_x^{\,2}U^2}{\varepsilon}\left(\alpha_1 + \alpha_2\right) - 2k_x^{\,2}V_A^{\,2} \\ &\quad - gk\left(\alpha_1 - \alpha_2\right), \\ D &= \frac{k_x}{k_1}\left(\alpha_1\nu_1 - \alpha_2\nu_2\right)U. \end{aligned}\right\} \tag{3.13}$$

In particular, if n is real, expression (3.12) simply represents the oscillatory waves so that the system is stable. However if n has imaginary part, it represents a perturbation which grows exponentially with time that the system is unstable.

In the remaining part of this paper, the analysis of Kelvin-Helmholtz instability mechanism is done by n_i obtained from equation (3.12) for astrophysical situation in porous medium.

4. Numerical Results and Discussion

Some solutions of equation (3.12) for astrophysical situation saturating porous medium using software Mathematica version-5.2. We have chosen the values of physical parameters from earlier studies by Chandrasekhar and many other authors while studying the Kelvin-Helmholtz instability in hydrodynamics/hydromagnetics/plasma in porous or non-porous medium. However the imaginary growth rate decreases slightly with the increase in ν_1 for a fixed wavenumber. The imaginary growth rates of the imaginary unstable mode have been examined numerically satisfying equations (3.12). In figure1, the imaginary growth rate n_i has been plotted versus wavenumber k for fixed permissible values of the parameters $k_1 = 2,\ g = 980 cm/sec^2,\ \rho_1 = 0.95,\ \rho_2 = 1.8,\ \varepsilon = 0.9,\ \nu_2 = 2,\ \ U = 300 cm/sec,$ $V_A^2 = 10,\ \alpha_1 = 0.35,\ \alpha_2 = 0.65,\ k_x = k/\sqrt{2}$ for three different values of the medium porosity $\nu_1 = 2, 8, 15$, respectively. It is clear from the graph that growth rate n_i starts to increase with increase in k, showing thereby the destabilizing effect of medium porosity on the system.

Similarly, in figure 2, the imaginary growth rate n_i has been plotted versus wavenumber k for fixed permissible values of the parameters
$k_1 = 2,\ g = 980 cm/sec^2,\ \rho_1 = 0.95,\ \ \rho_2 = 1.8,\ \ \varepsilon = 0.9,\ \nu_1 = 2, U = 300 km/sec,$ $V_A^2 = 10,\ \alpha_1 = 0.35,\ \alpha_2 = 0.65,\ k_x = k/\sqrt{2}$ for three different values of the kinematic viscosity of the lower fluid $\nu_2 = 2, 8, 14$, respectively. It is clear from the graph that growth rate n_i starts to increase as k increase, showing thereby the destabilizing effect of kinematic viscosity of the upper fluid on the system, which is exactly the same result in case of the lower fluid.

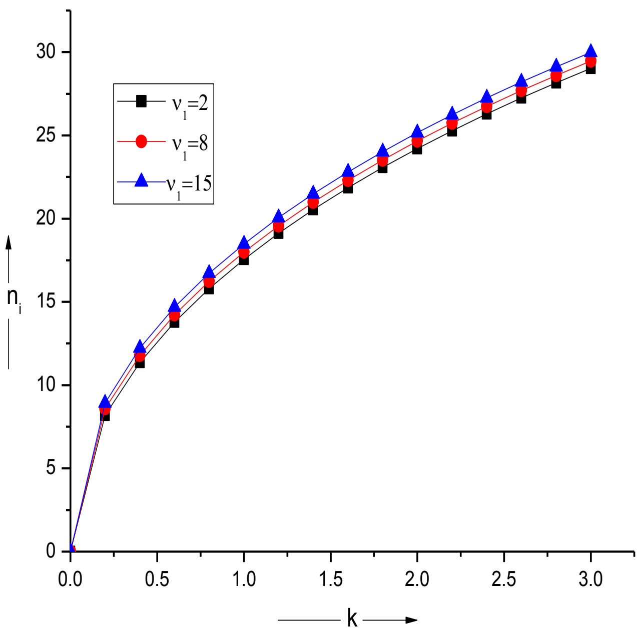

Fig. 1

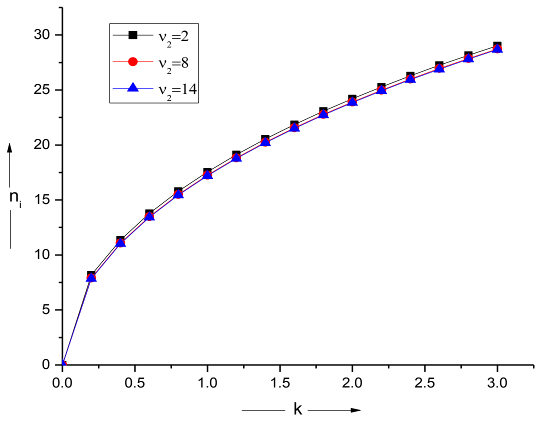

Fig. 2

The imaginary growth rate n_i has been plotted versus wavenumber k in figure 3 for three different values of the medium porosity $\varepsilon = 0.2, 0.4, 0.9$, respectively. It is clear from the graph that growth rate n_i very slightly with the increases in medium porosity ε, showing thereby the destabilizing effect of medium porosity on the system.

Figure 4 has been plotted for the imaginary growth rate n_i versus wavenumber k for the values of the medium permeability $k_1 = 2, 6, 10$, respectively. It is clear from the graph that growth rate n_i decreases as k increase, showing thereby the stabilizing effect of medium permeability on the system.

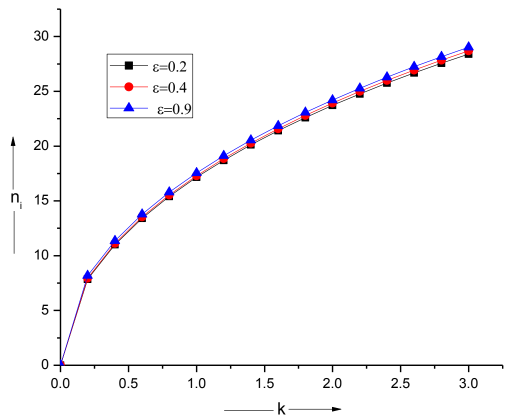

Fig. 3

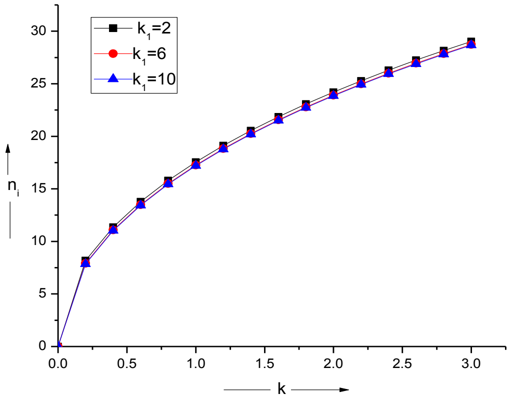

Fig. 4

The plot of n_i is given w.r.t wavenumber k in figure 5 for the values of the square of Alfvén velocity $V_A^2 = 10, 50, 100$ respectively. The graph shows that the square of Alfvén velocity has stabilizing effect on the system. The system is unstable for the small values of the wavenumber and it becomes completely stable $\left(i.e.\ n_i \to 0\right)$ at the value of the square of Alfvén velocity $V_A^2 = 100$. Thus critical wavenumber here is $k_c = 3.0$ and $k_{\max} = 1.5$.

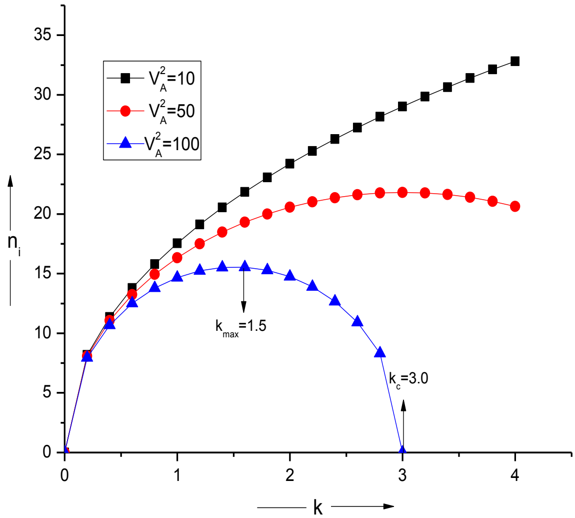

Fig. 5

5. Conclusions

A study has been made to investigate numerically the effects of kinematic viscosities of lower and upper fluid, square of the Alfvén velocity, the medium porosity and the medium permeability on the instability of superposed viscous fluids in hydromagnetics saturating porous medium. The principal conclusions drawn are as follows:

(i) The imaginary growth rate of the perturbations increases with the increase in kinematic viscosities of the lower and upper fluid implying thereby destabilizing effects on the system.

(ii) The imaginary growth rate of the perturbations decreases with the increase in the square of Alfvén velocity and medium permeability parameter, respectively. The magnetic field dissipates the energy of any disturbance more than that carried out by medium permeability. In other words, the role of medium permeability parameter shows stability on the Kelvin-Helmholtz instability problem, while the magnetic field plays the fundamental role to generate the complete stability.

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