A Computational Analysis of Friedmann Equation of Einstein Theory for Charged Perfect Fluid

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Abstract In this paper, we have computed and solved Einstein's Field Equation for distribution of charged perfect fluid with spherical symmetry, which is conformally flat by Computer program. The resulting model is found to be expanding but non-rotating and non-shearing.

Keywords: Friedmann Equations, Charged Perfect Fluid, Bianchi identities, Ricci Scalar.

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1. Introduction

The Friedmann solution then changed the concept of universe we live in the result were so striking that even Einstein at first consider it a mistake but later concept it. The knowledge of Hubble expansion made to realize the importance of Friedmann work^{1, 2}. The solution is not stationary. It is therefore we consider it to include spherically charged fluid distribution³ in order to understand the nature of this supposition. The equation for mass less field are invariant⁴ with respect to group C_g of conformal mapping where replacement of g_{ij} and the field variables $\phi_{ij,\dots,\nu}$ changes according to rule $\overline{g}_{ij} = e^{-2\sigma}g_{ij}$ and $\overline{\phi}_{ij,\dots,\nu} = e^{-\sigma(s-1)}\phi_{ij,\dots,\nu}$ where s is the spin of the field. A contravariant expression for the components of the energy momentum tensor for a charged perfect fluid is given by

(1.1)
$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} - pg_{\mu\nu} + \frac{1}{4\pi}(-F_{\mu j}F_{\nu}^{j} + \frac{1}{4}g_{\mu\nu}F_{ij}F^{ij})$$

 $T_{\mu\nu}$ is symmetric and made up from scalar fields ρ (energy density) and p (pressure), vector field u_{μ} that characterize the perfect fluid (velocity) and electromagnetic stress tensor $F_{\mu\nu}$ ⁵. The electromagnetic field tensor which satisfies

$$F_{[i\,j,k]} = 0$$
, $F_{;j}^{ij} = 4 p J^i v^j$

where J^{i} is Lorentz force density, given by another way

$$J_{\nu} = \mathcal{E}_0 F_{\mu\sigma} F_{;\mu}^{\mu\sigma}$$

with additional property that it is always conserved $J^{\mu}_{;\mu}=0$. Similarly, v^i is fundamental unit vector satisfies normalization condition

(1.2)
$$v^{\mu}v_{\mu} = -1$$

2. Field Equation

The general4-dimension spherically symmetric metric form^{6,7}

(2.1)
$$ds^{2} = \alpha(t,r)dt^{2} + 2\beta(t,r)dt dr + \gamma(t,r)dr^{2} + \delta(t,r)(d\theta^{2} + Sin^{2}\theta d\varphi^{2}).$$

Here in the consequence of the 2-spheres being subspaces of the Riemann space, the signature of $d\theta^2$ and $d\varphi^2$ must be the same. We consider time like space and put α as positive sign and β , γ , δ as negative sign. Further, variables of subspaces r and t can be carried out arbitrary nonsingular coordinate transformation

$$t = f(t', r'), \qquad r = g(t', r')$$

where f and g are arbitrary functions subject to the condition that $\frac{\partial(t,r)}{\partial(t',r')} \neq 0$. Assume $\alpha(t,r) = -\gamma(t,r) = \frac{\delta(t,r)}{r^2}$ and $\beta(t,r) = 0$. If we choose $\alpha(t,r) = a^2(t) f^2(r)$, the cosmological model could represents early universe condition, where singularity or non-singularity could be consider only in the function f(r) and as time passes on the function a will increase giving the idea of expansion of universe. So for simplification and also theoretical choice, we can assume $\alpha(t,r) = e^{l(t,r)}$.

The conformally flat metric in spherical polar coordinates

(2.2)
$$ds^{2} = e^{l} \left(dt^{2} - dr^{2} - r^{2} d\theta^{2} - r^{2} Sin^{2} \theta d\varphi^{2} \right)$$

Here, l a function of r and t, signifies both temporal and spatial disturbances.

The field equation Einstein-Maxwell taking into consideration a distribution of charged perfect fluid as

(2.3)
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8 p T_{\mu\nu}$$

(2.4)
$$T_{\mu\nu} = (\rho + p)v_{\mu}v_{\nu} - pg_{\mu\nu} + \frac{1}{4\pi} (F_{\mu j}F_{\nu}^{j} + \frac{1}{4}g_{\mu\nu}F_{ij}F^{ij})$$

Here *R* is the scalar curvature derived by the contraction of Riemannian curvature tensor and F^{ij} is electromagnetic field tensor, assuming only F^{14} is non-vanishing component and flow is only confined in two dimensional r-t plane in 4-dim space. So v^i is written as $(v^1, 0, 0, v^4)$. First two terms of equation (2.4) describe matter whereas last one shows electromagnetic field and obey Maxwell equations. The contracted Bianchi identities imply the equation of motion $T^{\mu\nu}_{;\nu} = 0$ for the whole energy momentum tensor $T_{\mu\nu}$. In the absence of matter and currents, the Maxwell equations do imply $T^{\mu\nu}_{;\nu} = 0$ but the converse is not true i.e. the Maxwell equations do not follow from the equations of motion and have to be postulated independently.

3. Solution of Field Equation

The solution of equations (2.3) and (2.4) is computed by taking

$$v^{\mu} = (v^1, 0, 0, v^4)$$

and

we get these equations

(3.1)
$$l_{;t}^{2} + 4l_{;tt} - 3l_{;r}^{2} - \frac{8}{r}l_{;r} = -512 \, p \, \pi e^{l} - 2(\pi F^{14})^{2} \, e^{-l} + 512(1+\rho) \, p \pi (v^{1})^{2}$$

(3.2)
$$l_{;t}^{2} + 4l_{;tt} - l_{;r}^{2} - \frac{4}{r}l_{;r} - 4l_{;rr} = -512 \ p \ \pi e^{l} - 2(\pi F^{14})^{2} \ e^{-l}$$

(3.3)
$$l_{;t}^{2} + 4l_{;tt} - l_{;r}^{2} - \frac{4}{r}l_{;r} - 4l_{;rr} = -512 \ p \ \pi e^{l} - 2(\pi F^{14})^{2} \ e^{-l}$$

(3.4)
$$l_{;r}^{2} + 4l_{;rr} - 3l_{;t}^{2} + \frac{8}{r}l_{;r} = 512 \ p \ \pi e^{l} + 10(\pi F^{14})^{2} \ e^{-l} + 512(1+\rho) \ p \ \pi (v^{4})^{2}$$

(3.5)
$$l_{;rt} - \frac{1}{2} l_{;r} l_{;t} = 128\pi (p + \rho) v_1 v_4$$

These equations are corresponding to G_{11} , G_{11} , G_{22} , G_{33} and G_{41} . Out of these equations, equation (3.2) and (3.3), corresponding to G_{22} and G_{33} , are identical.

The normalization condition on fundamental flow vector v^{μ} (equation (1.2)) gives

(3.6)
$$(v^4)^2 - (v^1)^2 = e^{t}$$

From equations (3.1) and (3.2)

(3.7)
$$4l_{;r\,r} - 2l_{;r}^2 - \frac{4}{r}l_{;r} = 512(1+\rho)\,p\pi(v^1)^2\,.$$

This is second order partial differential equation in r and defines static field in space. The temporal field may be obtained from equation (3.5). Suppose the flow is unidirectional. In this sense

(3.8)
$$2l_{;r\,r} - l_{;r}^2 - \frac{2}{r}l_{;r} = 0$$

(3.9)
$$l_{;rt} - \frac{1}{2} l_{;r} l_{;t} = 0.$$

The static solution of equation (3.6) for far distances

$$e^{l(r)} = (r+C_1)^{-2} e^{C_2 r}$$

where C_1 and C_2 may be a function of *t* only. Since for long distances space must be flat so $C_2=0$. From equation (3.8) and (3.9) both lead to

(3.10)
$$e^{l(r,t)} = (\alpha(r) + \beta(t))^{-2}$$

Thus, we get the general solution of metric as

(3.11)
$$ds^{2} = \frac{1}{\left[\alpha(r) + \beta(t)\right]^{2}} (dr^{2} + r^{2}d\theta^{2} + r^{2}sin^{2}\theta d^{2}\phi - dt^{2}).$$

This throws light that $e^{l(r,t)}$ becomes $\frac{1}{\left[\alpha(r)+\beta(t)\right]^2}$ i.e. α is only function of r and β is only function of $t^{8,9}$. If $\beta=0$ then this equation become

 $\frac{1}{\left[\alpha(r)\right]^2}$ *i.e.* the application of charged fluid split the function separately.

Even if β function vanishes the geometry exists. So we can think of early universe condition by taking specific function. Again if we go back to the cosmological model considering the Dual nature of Ricci scalar these function are taken as product. But in that case the cosmology is in homogenous, whereas the Friedman universe is isotropic and homogenous. The expanding nature of the universe is clear both ways. So it is applicable to tensor field in gravity.

4. Conclusion

One of important conclusion we can draw out of this distribution of charged perfect fluid is non-rotating and non-shearing. Again the acceleration, here is always radial and the flow of the fluid in time direction is uniform. Since we have used a computer program to solve the Friedmann equation, we have got a general solution which is reducible to the final result.

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