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# Nanofluid Flow and Heat Convection in a Channel Filled with Porous Medium

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**Abstract:** The nanofluid flow and heat convection in a horizontal parallel-plates channel filled with porous medium is investigated. The upper plate of the channel is considered porous, subjected with a constant suction while there is no cross-flow at the lower plate. Flow and heat convection in the channel is influenced by a static transverse magnetic field by means of Lorenzian force and Joule heating effect. The velocity and temperature profiles are derived by differential transform method (DTM), computed and discussed through graphs. Convection of heat in the nanofluids $AL_2O_3/H_2O$, $Cu/H_2O$ and $Ag/ H_2O$ are compared in terms of Nusselt number and investigated the effects of various physical parameters Reynolds number, Hartmann number and Prandtl number. The skin-friction coefficients at the plates are also calculated and discussed through graphs.

**Keywords:** Nanofluid, MHD, DTM, Porous media, Horizontal channel.

**AMS Mathematics Subject Classification:** 76Dxx.

## Nomenclature

u: velocity component along x direction fluid

$\mu_f$: Dynamic viscosity of base fluid

v: velocity component along y direction

p: fluid pressure

$\beta$: Coefficient of thermal expansion

$\mu_{nf}$ : viscosity of nano particles

$\rho_s$: Density of solid

$\rho_{nf}$ : Density of nano fluid

$\rho_f$: Density of fluid

$T_w$ : Free stream tempareture

$\kappa_{nf}$ : Thermal conductivity of nanoparticals

$\kappa_f$ : Thermal conductivity of base fluid

$\theta$ : Non dimensional temperature

$B_0$ : Applied magnetic field

$\phi$ : volume fraction of solid particles

$\kappa_s$ : thermal conductivity of fluid

$U_0$ : characterstic velocity

$(\rho Cp)_s$ : heat capacity of nanoparticles

$(\rho Cp)_{nf}$ : heat capacity of nano fluid

$\eta$ similarity variable

M: Hartman number

Ec: Eckert number

Pr : Prandtl number

Nu: Nusselt number

$(\rho Cp)_f$ : heat capacity of fluid

$\nu_f$ : kinematic viscosity of fluid

Da: Darcy number

## 1. Introduction

Nanofluid is the mixture of solid-liquid in which metallic or non-metallic nanoparticles are suspended in base fluid with some special requirements of even and stable suspension, durable suspension, negligible agglomeration of particles and no chemical change of fluid. Generally, the size of nanoparticles lies between 1-100 nm. Nanofluid, a name conceived by Choi (Argonne National Laboratory at 1995). Saidus et al[1] give their excellent contribution to the application and challenges of the nanofluids. Sarit et al[2] reviewed the works done on heat transfer in nanofluids. Soundalgekar and Patil[3] studied Stoke's problem for a vertical infinite plate with variable temperature. Raptis and Tzivanidis[4] studied mass transfer and free convective MHD flow past and accelerated vertical porous plate under consideration of transverse magnetic field. The governing equations are solved with the help of power series.

The differential transformation method (DTM) was first applied in the engineering domain by Zhou. DTM obtains an analytical solution in the form of a polynomial by means of an iterative procedure. DTM is an alternative procedure for obtaining analytic Taylor series solution of the differential equations. Chen and Liu[5] used DTM to solve two point boundary value problems, Ayaz[6] applied DTM to solve differential-algebraic equation, Kangalgil and Ayaz[7] used DTM to solve the KdVand mKdV equations, Kanth and Aruna[8] applied DTM to the Schrdinger equations and Arikoglu and Ozkol[9] solve fractional differential equations by using DTM. Yahyazadeh et al[10] evaluated natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field by the differential transformation method.

A wide range of applications of porous media in practical problems can be found in[11-13]. The role of MHD flow is very important because the influence of a magnetic field on the viscous flow of electrically conducting fluid which is applicable in many industrial processes, such as in magnetic materials processing, purification of crude oil, magneto hydrodynamic electrical power generation, glass manufacturing, geophysics and paper production, etc. The heat transfer by natural convection withCu-$H_2O$ nanofluid in a three dimensional annulus enclosure filled with porous media (silica sand) between two horizontal concentric cylinders is analyzed by Saleh[14]. Chamkha and his co-workers have carried out a number of investigations relating to a variety of convective flows (free, forced or mixed convection) involving nanofluids in the presence of porous media[15-18] or without permeable media [19-21]. Wang et al[22] conducted a study on natural convection in nanofluid filled vertical and horizontal enclosures. Polidori et al.[23] analyzed the heat transfer enhancement in natural convection using nanofluids.

The purpose of present study is to investigate the effect of different physical parameters on flow profile of velocity and temperature. The variation in Nusselt number and skin friction coefficient for different nanofluids is calculated.

## 2. Formulation of the Problem

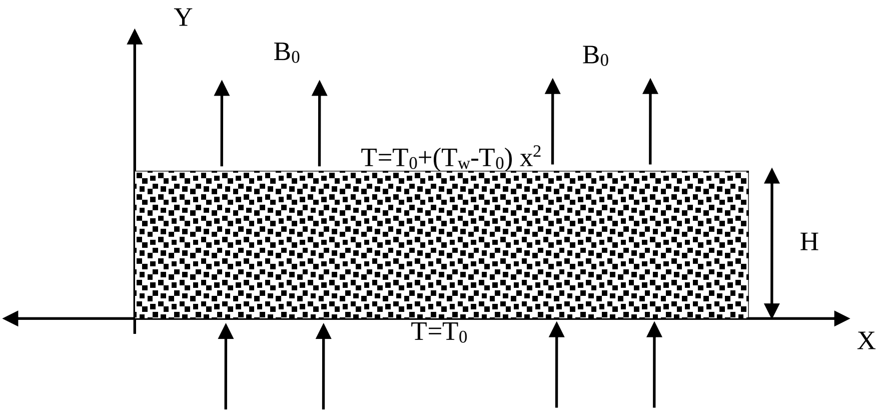


**Fig.1**. Physical model of the problem

The flow and heat convection in nanofluid flowing through parallel plates horizontal channel filled with porous media is considered. The lower plate is taken along x-axis and upper plate is at a distance H apart. The axis of y is taken normal to the channel. The upper plate is permeable while lower one is non-permeable. A nanofluid is flowing in the influence of state transverse magnetic field B $(0,B_0,0)$ and suction at the upper plate. The upper plate is kept at constant temperature while the lower is subjected to variable temperature.

Under these conditions the governing equations of motion are given by Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

The governing equation of motion in x- direction

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma B_0^2 u-\mu_{nf}\,\frac{u}{k} \tag{2.2}$$

The governing equation of motion in y- direction

$$\rho_{nf}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\mu_{nf}\,\frac{v}{k} \tag{2.3}$$

The equation of energy

$$(\rho Cp)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\sigma B_0^2 u^2 \tag{2.4}$$

Boundary conditions are

$$\begin{cases} y=0;\ u=0,\ v=0, \\ T=T_0, \qquad y=h; \\ u=0,\ \ v=-U_0 \\ T=T_0+\left(T_w-T_0\right)x^2 \end{cases} \tag{2.5}$$

## 3. Mathematical Analysis

Using the following non-dimensional parameters

(3.1) $$\begin{cases}\bar{u}=\dfrac{u}{U_0},\bar{v}=\dfrac{v}{U_0},\bar{x}=\dfrac{x}{H},\bar{y}=\dfrac{y}{H},\bar{p}=\dfrac{p}{\rho_f U_0^2},\theta=\dfrac{T-T_0}{T_w-T_0}\end{cases}$$

And following expressions of nanofluid given by Oztop and Nada[24]

(3.2) $$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s$$

(3.3) $$(\rho Cp)_{nf}=(1-\phi)(\rho Cp)_f+\phi(\rho Cp)_s$$

(3.4) $$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}\ \text{[Maxwell model]}$$

The effective thermal conductivity of the nano-fluid is approximated by Hamilton-Crosser[25] model as:

(3.5) $$\kappa_{nf}=\kappa_f\,\frac{\kappa_s+2\kappa_f-2\phi(\kappa_f-\kappa_s)}{\kappa_s+2\kappa_f+\phi(\kappa_f-\kappa_s)}$$

Into the equation (2.2), (2.3) and (2.4), we get

(3.6) $$A\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{B}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\left(\frac{M^2}{\mathrm{Re}}+\frac{B}{\mathrm{Re}\,Da}\right)u$$

(3.7) $$A\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{B}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\left(\frac{B}{\mathrm{Re}\,Da}\right)v$$

(3.8) $$D\left(u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}\right)=\frac{E}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial y^2}\right)+\left(\frac{M^2\,Ec}{\mathrm{Re}}\right)u^2$$

where $A=(1-\phi)+\phi\dfrac{\rho_s}{\rho_f}$, $B=\dfrac{1}{(1-\phi)^{2.5}}$ $D=(1-\phi)+\phi\dfrac{(\rho Cp)_s}{(\rho Cp)_f}$

$$E=\frac{\kappa_s+2\kappa_f-2\phi(\kappa_f-\kappa_s)}{\kappa_s+2\kappa_f+\phi(\kappa_f-\kappa_s)},\ Da=\frac{K}{H^2},\ \mathrm{Re}=\frac{U_0 H}{\mu_f},\quad Ec=\frac{U_0^2}{(Cp)_f(T_w-T_0)}$$

$$\Pr = \frac{(\rho Cp)_f \nu_f}{\kappa_f}, \quad M^2 = \frac{\sigma B_0^2 H^2}{\mu_f}.$$

Using $u = \frac{\partial \psi}{\partial y}, \;\; v = -\frac{\partial \psi}{\partial x}$ the equation of motion and differentiating equation of motion in x and y direction w. r. t. 'y' and 'x' respectively to eliminate pressure term, resulting equation is given by

$$(3.9) \quad \begin{aligned} &A\left(\frac{\partial \psi}{\partial y}\frac{\partial^3 \psi}{\partial x \partial y^2} + \frac{\partial \psi}{\partial y}\frac{\partial^3 \psi}{\partial x^3} - \frac{\partial \psi}{\partial x}\frac{\partial^3 \psi}{\partial y \partial x^2} - \frac{\partial \psi}{\partial x}\frac{\partial^3 \psi}{\partial y^3}\right) \\ &= \frac{B}{\mathrm{Re}}\left(\frac{\partial^4 \psi}{\partial y^4} + 2\frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial y^4}\right) - \frac{B}{\mathrm{Re}\, Da}\left(\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial x^2}\right) \\ &\quad - \frac{M^2}{\mathrm{Re}}\frac{\partial^2 \psi}{\partial y^2} \end{aligned}$$

Energy equation reduces to

$$(3.10) \qquad D\left(\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}\right) = \frac{E}{\mathrm{Re}\Pr}\left(\frac{\partial^2 \theta}{\partial y^2}\right) + \left(\frac{M^2\, Ec}{\mathrm{Re}}\right)\left(\frac{\partial \psi}{\partial y}\right)^2.$$

Corresponding boundary conditions are reduced to

$$(3.11) \qquad \begin{cases} y = 0; \frac{\partial \psi}{\partial y} = 0, \frac{\partial \psi}{\partial x} = 0, \theta = 0, \\ y = 1; \frac{\partial \psi}{\partial y} = 0, \frac{\partial \psi}{\partial x} = 1, \theta = x^2 \end{cases}$$

Using the similarity transformations into equation (3.9) and (3.10)

$$(3.12) \qquad y = \eta, \;\; \psi = x\, f(\eta), \theta = x^2 \widehat{\theta}(\eta)$$

we get ODEs as

$$(3.13) \qquad A\left(f'\, f'' - f\; f'''\right) = S\; f^{iv} - R\, f''$$

(3.14) $$D\left(2f'\widehat{\theta}-f\widehat{\theta}'\right)=G\widehat{\theta}''+\frac{Ec\,M^2}{\mathrm{Re}}(f')^2,$$

where $S=\dfrac{B}{\mathrm{Re}}, R=\dfrac{B}{\mathrm{Re}\,Da}+\dfrac{M^2}{\mathrm{Re}}, G=\dfrac{E}{\Pr \mathrm{Re}}$.

Corresponding boundary conditions are reduced to

(3.15) $$\eta=0;\ f=0,\ f'=0, \widehat{\theta}=0,\ \ \eta=1;\ f=1,\ f'=0,\ \widehat{\theta}=1$$

Now applying DTM on (3.13) and (3.14) with (3.15)

where $F(k)$ and $\tilde{\theta}(k)$ is DTM of $f(\eta)$ and $\theta(\eta)$ respectively.

(3.16) $$F(k+4)=\frac{1}{S\,(k+1)(k+2)(k+3)(k+4)}\left(A\left(\begin{array}{l}\sum_{h=0}^{k}(h+1)(k-h+1)(k-h+2)F(h+1)F(k-h+2)\\ -\sum_{h=0}^{k}(k-h+1)(k-h+2)(k-h+3)F(h)F(k-h+3)\end{array}\right)\begin{array}{l}\\ \\ +R(k+1)(k+2)F(k+2)\end{array}\right)$$

(3.17) $$D\left(2\sum_{h=0}^{k}(h+1)F(h+1)\widehat{\theta}(k-h)-\sum_{h=0}^{k}(k-h+1)F(h)\widehat{\theta}(k-h+1)\right)$$
$$=G(k+1)(k+2)\widehat{\theta}(k+2)+\frac{M^2Ec}{\mathrm{Re}}\left(\sum_{h=0}^{k}(h+1)(k-h+1)F(h+1)F(k-h+1)\right)$$

corresponding boundary conditions are given by

(3.18) $$\eta=0;\ F(0)=0,\ \ F(1)=0,\ \ \widehat{\theta}(0)=0$$

Suppose that

(3.19) $$F(2)=b,\ F(3)=c,\ \widehat{\theta}(1)=d$$

Now applying inversion of DTM, we get

(3.20) $$f(\eta)=\sum_{k=0}^{\infty}\eta^{k}F(k)=F(0)+\eta F(1)+\eta^{2}F(2)+\eta^{3}F(3)+\eta^{4}F(4)+\eta^{5}F(5)+\eta^{6}F(6)+\eta^{7}F(7)+\eta^{8}F(8)$$

Using the (3.15) we can find the value of unknown i.e. a and b. Now applying inversion of DTM, we get

(3.21) $$\theta(\eta)=\sum_{k=0}^{\infty}\eta^{k}\widehat{\theta}(k)=\widehat{\theta}(0)+\eta\widehat{\theta}(1)+\eta^{2}\widehat{\theta}(2)+\eta^{3}\widehat{\theta}(3)+\eta^{4}\widehat{\theta}(4)+\eta^{5}\widehat{\theta}(5)+\eta^{6}\widehat{\theta}(6)+\eta^{7}\widehat{\theta}(7)+\eta^{8}\widehat{\theta}(8)$$

**Velocity Components:**

(3.22) $$u=\frac{\partial\psi}{\partial y}\quad\therefore\quad u=x\,f'(\eta)$$

(3.23) $$v=-\frac{\partial\psi}{\partial x}\quad\therefore\ v=-f(\eta)$$

**Heat Transfer Coefficient:** Local heat transfer coefficient ($Nu_x$) in dimensionless is defined as

(3.24) $$Nu_x=-\left(\frac{\partial\theta}{\partial x}+\frac{\partial\theta}{\partial y}\right)=-(2x\widehat{\theta}(\eta)+x^{2}\widehat{\theta}'(\eta))$$

For $x=1$;

(3.25) $$Nu=-(2\widehat{\theta}(\eta)+\widehat{\theta}'(\eta)\Big|_{\eta=1}$$

Coefficient of local skin friction is given by

(3.26) $$C_{fx}=\frac{2\tau_w}{\rho U_0^2}=\frac{2B}{\mathrm{Re}}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=\frac{2xBf''(\eta)}{\mathrm{Re}}$$

For $x=1$; At upper plate

(3.27) $$C_f = \frac{2Bf''(\eta)}{\mathrm{Re}}\bigg|_{\eta=1}$$

**Table 1:** Thermodynamics Properties of Water and nanoparticles

| | $\rho$ [kg m$^{-1}$] | $Cp$ [J kg$^{-1}$K$^{-1}$] | $k$ [Wm$^{-1}$K$^{-1}$] | $\beta$·105 [K$^{-1}$] |
|---|---|---|---|---|
| Pure water | 997.1 | 4179 | 0.613 | 21 |
| Copper (Cu) | 8933 | 385 | 401 | 1.67 |
| Silver (Ag) | 10500 | 235 | 429 | 1.89 |
| Alumina (Al2O3) | 3970 | 765 | 40 | 0.85 |
| Titanium oxide | 4250 | 686.2 | 8.9538 | 0.9 |

**Table 2:** Computation showing skin friction for $\phi$=0.04, Ec=0.01=Da, Re=1,Pr=5.784

| M | $C_f(H_2O\text{-}Al_2O_3)$ | $C_f$ $(H_2O\text{-}Cu)$ | $C_f$ $(H_2O\text{-}Ag)$ |
|---|---|---|---|
| 1 | -125.981798 | -125.981565 | -125.981491 |
| 2 | -126.746463 | -126.746299 | -126.746247 |
| 3 | -127.939156 | -127.939072 | -127.939046 |
| 4 | -129.458423 | -129.458405 | -129.458399 |
| 5 | -131.194755 | -131.194778 | -131.194785 |
| 6 | -133.047027 | -133.047067 | -133.047079 |

**Table 3:** Computation showing skin friction for Re=M=1, Ec=0.01=Da, Pr=5.784

| $\phi$ | $C_f(H_2O\text{-}Al_2O_3)$ | $C_f$ $(H_2O\text{-}Cu)$ | $C_f$ $(H_2O\text{-}Ag)$ |
|---|---|---|---|
| 0 | -113.784389 | -113.784389 | -113.784389 |
| 0.01 | -116.672889 | -116.672831 | -116.672813 |
| 0.02 | -119.665337 | -119.665221 | -119.665185 |
| 0.03 | -122.766593 | -122.766419 | -122.766364 |
| 0.04 | -125.981798 | -125.981565 | -125.981491 |
| 0.05 | -129.31639 | -129.316098 | -129.316006 |
| 0.06 | -132.776132 | -132.775781 | -132.77567 |
| 0.07 | -136.367127 | -136.366716 | -136.366586 |
| 0.08 | -140.095849 | -140.095378 | -140.095229 |
| 0.09 | -143.969163 | -143.968631 | -143.968463 |
| 0.1 | -147.994359 | -147.993767 | -147.99358 |

## 4. Results and Discussion

We study the velocity and temperature profiles for Cu-Water type nanofluid. The velocity profiles in horizontal direction for different parameters like Solid volume fraction parameter, Hartmann number, Reynolds number, Eckert number and Darcy number and effect of same parameters including Prandtl number on temperature are depicted by figures. The values of the parameters considered are the solid volume fraction parameter ($\phi$ )=0, 0.02,0.04, Reynold number(Re)= 0.1,0.5,1,2 and Hartman number( M)=0,3,6 ,Eckert number( Ec)=0.01,0.02,0.04 .

**4. 1. Velocity Profiles with Parameter Variation:** In figure(2) when Reynold number (Re) is increased, keeping all other parameters constant the horizontal velocity distribution $f'(\eta)$ is slightly increasing after four decimal points. The result shows a good agreement with Fakour et al.[26]. Figure(3) shows the effects of Hartman number on the velocity distribution $f'(\eta)$. It is observed by figure that velocity is gradually decreasing for increasing value of M and changes are more between limits 0.85-0.95. It is noteworthy from figure (4) that $f'(\eta)$ is increasing with growing amount of Da upto $\eta$=0.657(approx.) after that start decreasing. The effect of solid volume fraction on $f'(\eta)$ is depicted in figure (5). Different type nanofluids have almost same velocity profile as demonstrated by figure (6).

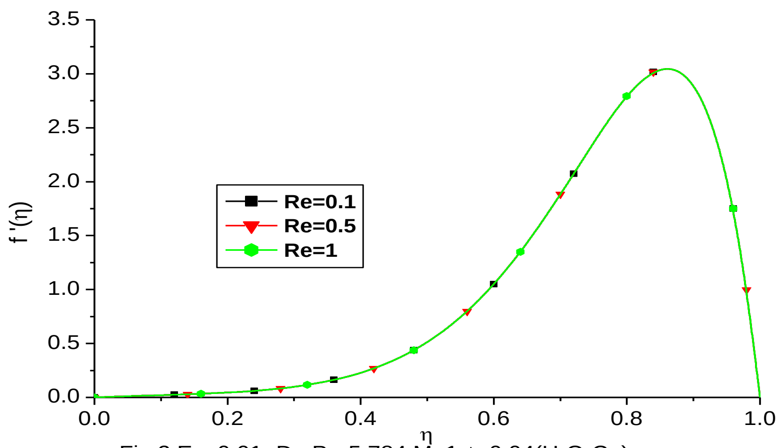

Fig.2:Ec=0.01=Da,Pr=5.784,M=1,$\phi$=0.04($H_2O$-Cu)

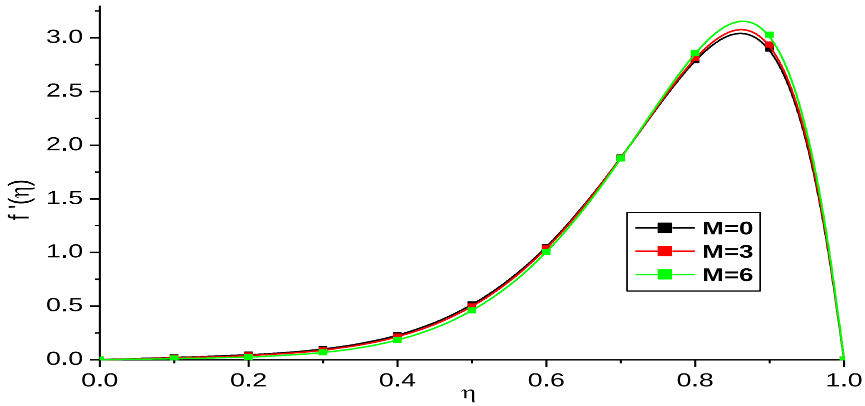


Fig.3:Re=M=1,Da=Ec=0.01,Pr=5.784,$\phi$=0.04($H_2O$-Cu)

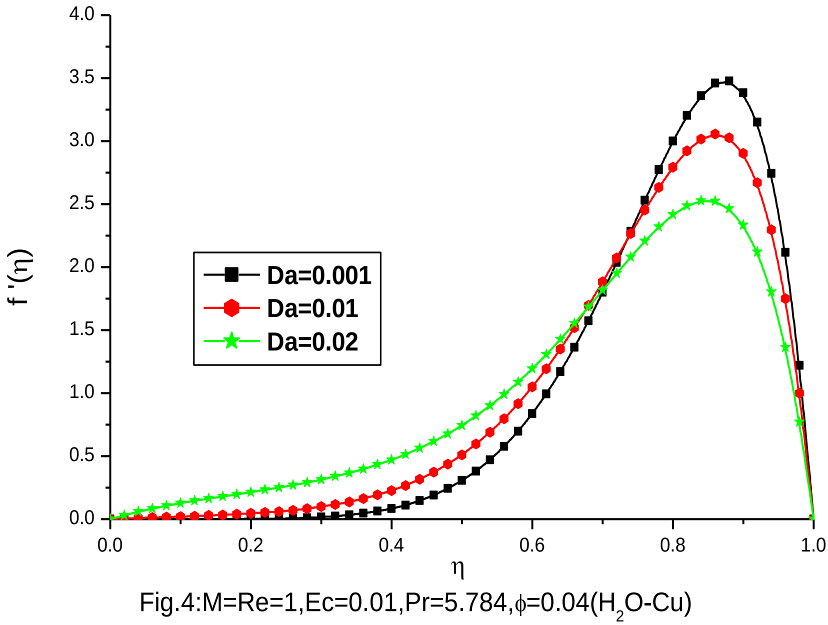


Fig.4:M=Re=1,Ec=0.01,Pr=5.784,$\phi$=0.04($H_2O$-Cu)

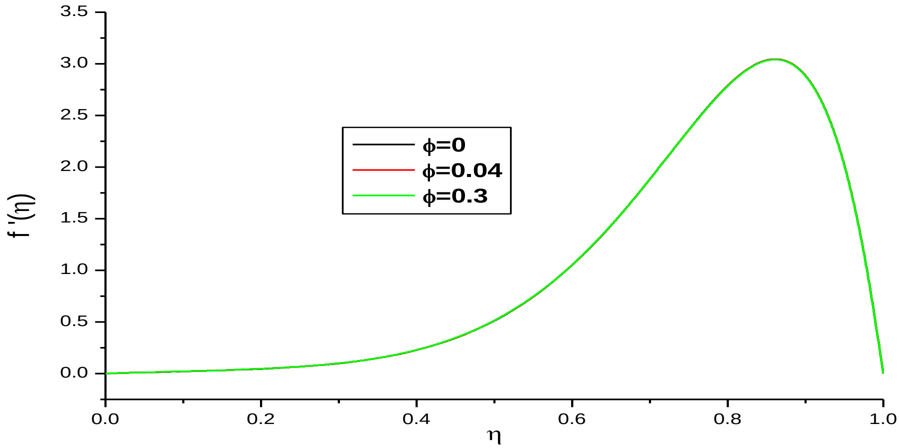


Fig.5:Re=M=1,Ec=Da=0.01,Pr=5.784,$H_2O$-Cu

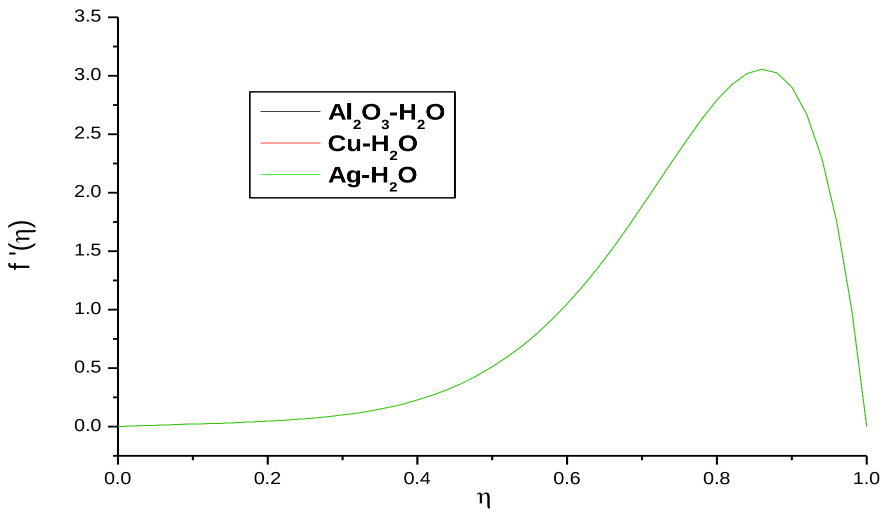


Fig.6:Re=M=1,Ec=Da=0.01,Pr=5.784,$\phi$=0.04($H_2O$-Cu)

**a. Temperature Profiles with Parameter Variation:** The effects of Da on temperature profile is shown in figure (7). It is noteworthy that temperature of $H_2O$-Ag nanofluid is generally higher than that of $H_2O$-Cu and $H_2O$-$Al_2O_3$ nanofluids is indicated in figure (8). Figure (9) depicts that increment in value of Re causes decrement in temperature profile $\theta(\eta)$due to increment of viscocity. Figure(10) displays that $\theta(\eta)$

is increasing with increment in M. It is observed by the figure(11) that temperature profile is increasing in increasing direction of Ec. The flow is same as Makinde et al.[27]. It is noticed that the nanofluid temperature increases with growing in nanoparticles volume fraction due to enhanced properties of nanofluid which is shown in figure (12).

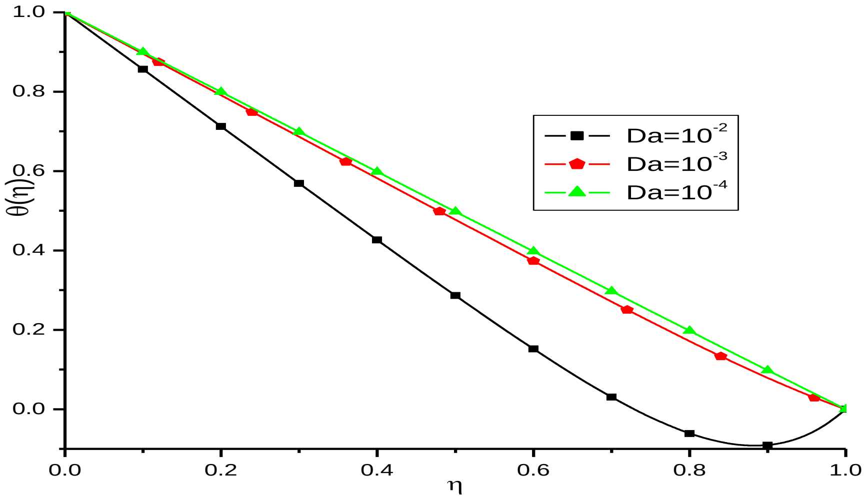


Fig.7: $\phi$=5.784,Phi=5.784,Re=M=1,Ec=0.01

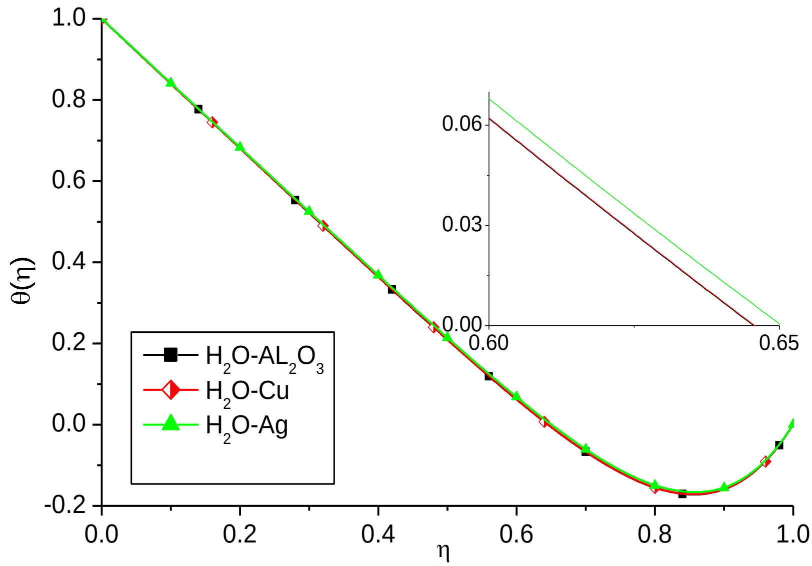


Fig.8: Re=M=1,Ec=Da=0.01,Pr=7,$\phi$=0.04

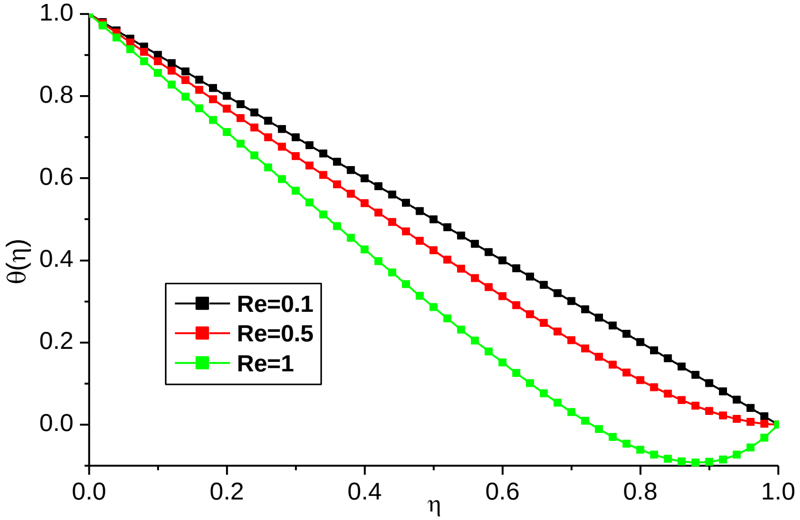


Fig.9: Pr=5.784,Ec=0.01=Da,M=1=Re,$\phi$=0.04($H_2O$-Cu)

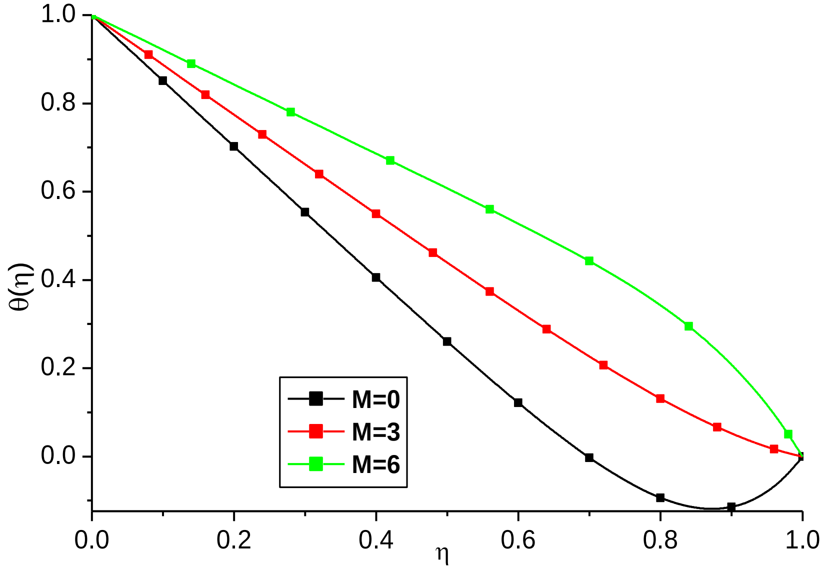


Fig.10:Pr=5.784,Re=1,Ec=Da=0.01,$\phi$=0.04($H_2O$-Cu)

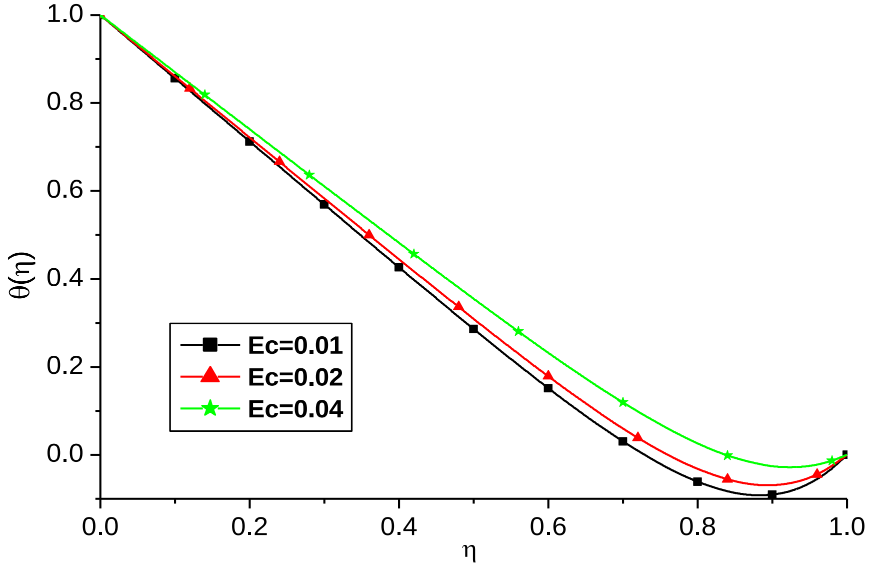


Fig.11:Pr=5.784,M=Re=1,Da=0.01,$\phi$=0.04($H_2O$-Cu)

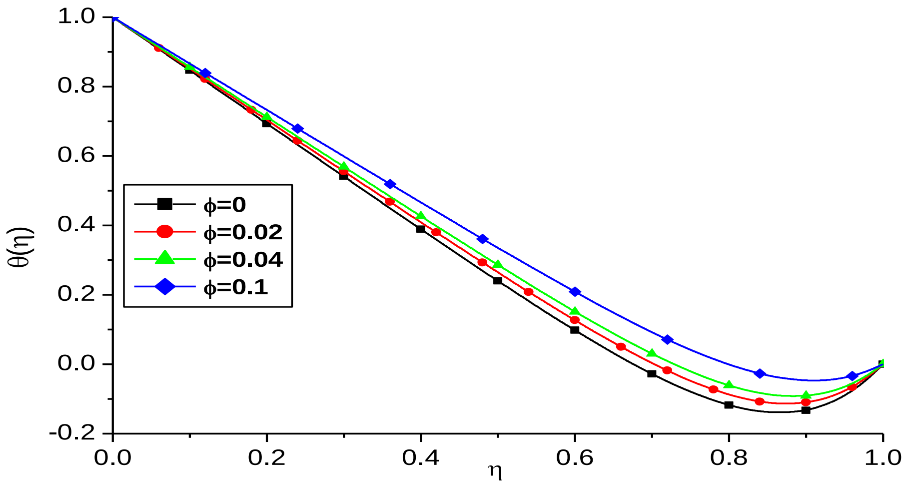


Fig.12:M=Re=1,Ec=Da=0.01,pr=5.784($H_2O$-Cu)

**b. Nusselt Number and Skin Friction:** The effect of Da on Nusselt Number is depicted in figure (13). Figure (14) elucidates the Nusselt number profile for different values of Ec. It is displayed by figure (15) that Nusselt number is highest in case of $H_2O$-Ag nanofluid with growing M. Increment in solid volume fraction increases Nusselt number and decreases Skin friction which is visible in figures 16-17. Skin friction increases with growing value of Da as indicated in figure (18). In figure (19), it is noticed that skin friction increases with increasing value of Reynold number. Figure (20) demonstrates that Nusselt number for is decreasing when Re is increasing.

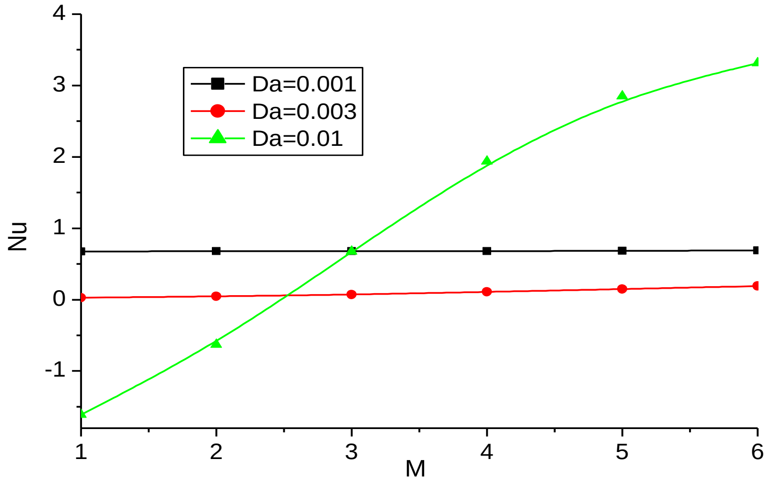


Fig.13:Re=1,Ec=0.01,Pr=5.784,$\phi$=0.04($H_2O$-Cu)

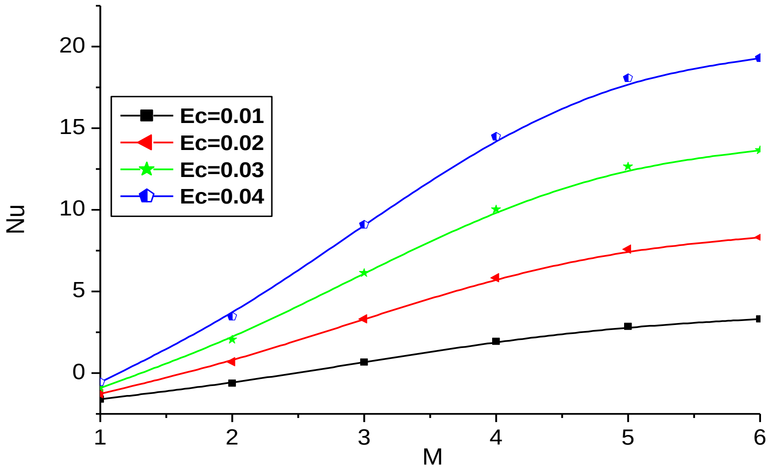


Fig.14:Re=1,Da=0.01,Pr=5.784,$\phi$=0.04($H_2O$-Cu)

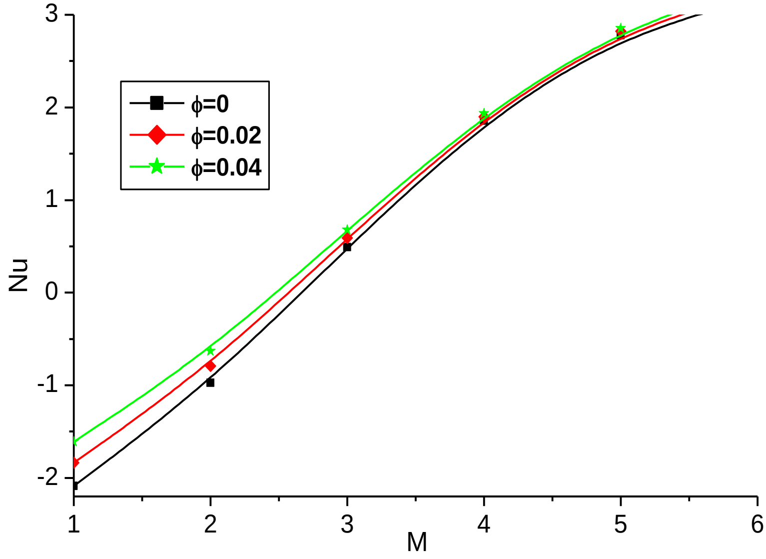


Fig.16:Re=1,Da=Ec=0.01,Pr=5.784($H_2O$-Cu)

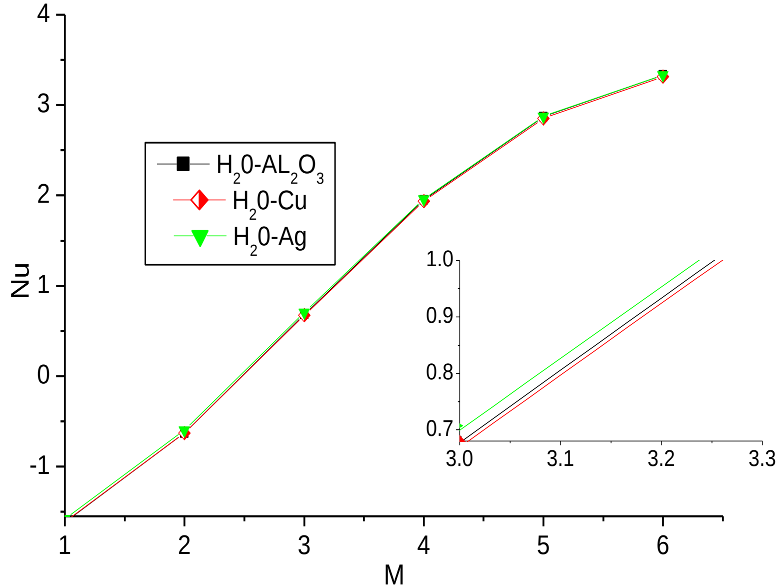


Fig.15:Re=1,Ec=Da=0.01,Pr=5.784,$\phi$=0.04

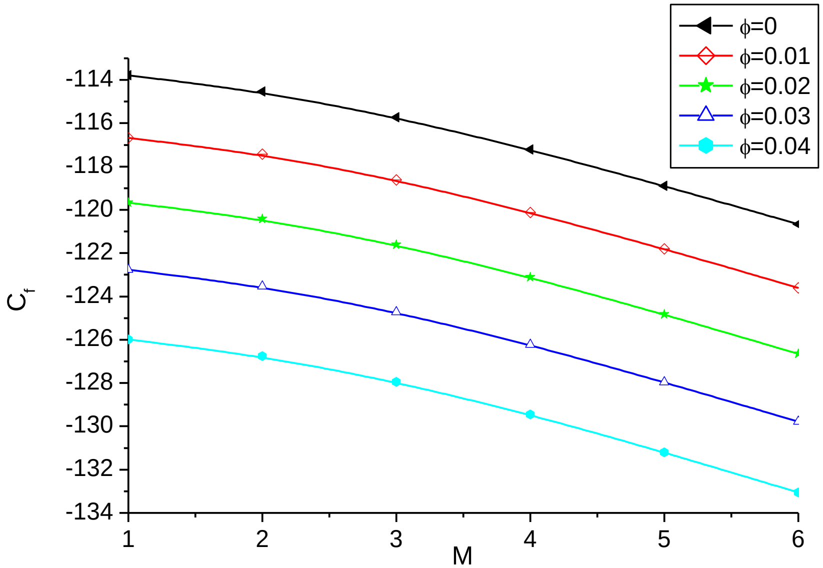


Fig.17:Re=1,Ec=Da=0.01,Pr=5.784,($H_2O$-Cu)

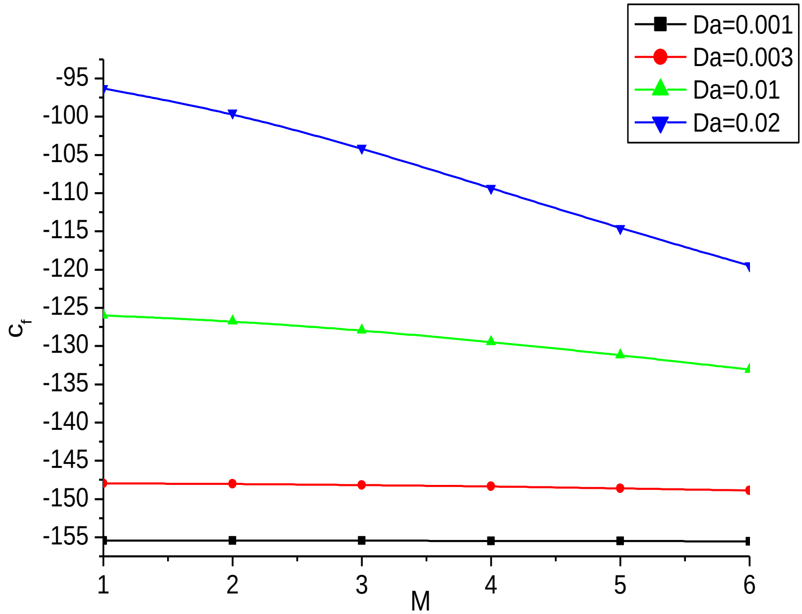


Fig.18:Re=1,Ec=0.01,Pr=5.784,$\phi$=0.04($H_2O$-Cu)

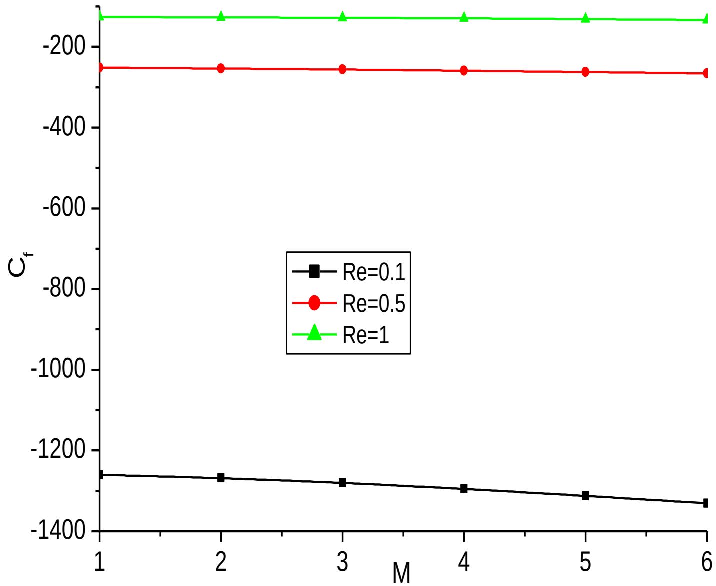


Fig.19:M=1,Ec=Da=0.01,Pr=5.784,$\phi$=0.04($H_2O$-Cu)

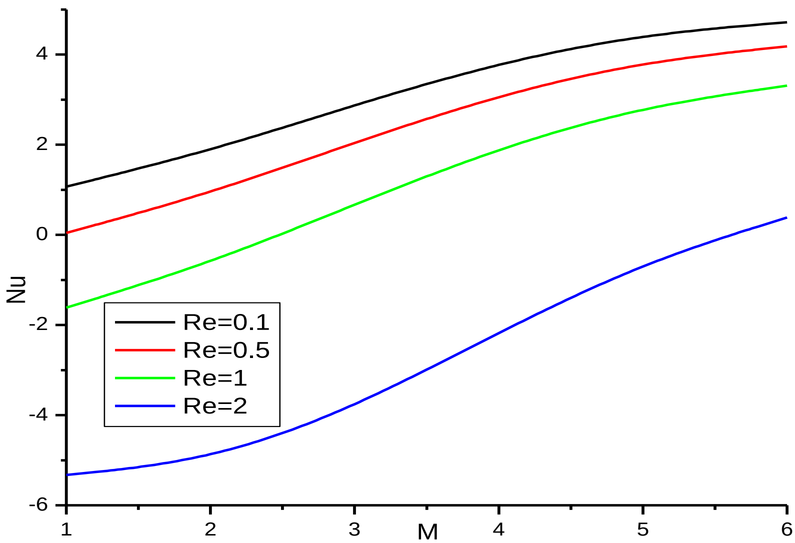


Fig.20: Ec=0.01=Da,Pr=5.784,$\phi$=0.04

## 5. Conclusion

In this study, differential transformation method (DTM) is applied to solve the problem of flow and heat convection of nanofluid in a porous channel. By solution of this problem the following points is concluded:

(1) In general, by M velocity in the channel is reduced and temperature attains maximum amount.

(2) Increment in amount of Re causes slightly increment in velocity and decrement in temperature.

(3) Growing Ec. causes increment in temperature profile.

(4) Temperature and Nu is highest in case of $H_2O$-Ag nanofluid but $C_f$ is lowest.

(5) Temperature as well as Nu increases with solid volume fraction of nanoparticles.

(6) Skin friction increases with increasing value of Re.

(7) Skin friction decreases with increasing value of M.

(8) Increment in amount of $\phi$ increases Nu and decreases $C_f$.

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