

# An Introductory Review on Big Data and Distributed Delay Framework of Neuron Model

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**Abstract:** This article introduces the big data application in neuroscience and distributed Delay Framework (DDF) of neuron model. Understanding of DDF is helpful in information mechanism of neuron with its previous membrane potential values which can further be used to analyze in-vitro and in-vivo data. This understanding reveals the way of integration of past potential values with current values at artificial intelligence implementation time.

**Keywords:** Artificial Intelligence, Delay Kernel, IF Model, LIF Model, Power Law, Information Processing.

## 1. Introduction

Big data, the buzzword, deals with information extraction from a large amount of heterogeneous data, where systematic traditional data processing and information retrieval techniques are not successful<sup>1,2</sup>. It has four dimensions, namely, volume, variety, velocity and veracity. In 21<sup>st</sup> century, big data is widely applicable in all branches of engineering, science and

social science as it provides an in-depth and new understanding for the subject. Now-a-days, it is entering into a new branch of science, namely, *Neuroscience*<sup>3</sup>. Neuroscience deals with information processing mechanism into neurons<sup>4-6</sup>. Neuroscience has its own big data sets which includes fMRI images, *in-vitro* and *in-vivo* empirical data, cell recording etc<sup>2,3,7,8,9</sup>. Neuron processes information in truly parallel mode and in order to get insight into neuronal dynamics and information processing mechanism, horizontal and vertical, both, data-processing mechanism are applicable<sup>3,8</sup>. It requires a delicate balance among biological detail precision, inferences and generalized principles, thus, needs a systematic, standardized way to collection, synthesize and integrate data for heterogeneous level of analysis across the species<sup>2,7</sup>. Heterogeneity of neuronal data makes the analysis too complex for mathematical point of view. A neuron model has to take care of certain neuronal constraints for explaining specific neuronal properties.

Lapicque<sup>10</sup> has proposed the Integrate Fire model to explain the spiking nature of neuron in 1907. It is the first model which describes that how a biological neuron functions. In order to explain the neuronal functionality and information processing, a number of its variant like Leaky integrate and Fire model (LIF model)<sup>10-12</sup>, Stein's model<sup>13</sup>, Spike-Response model (SRM model)<sup>5,11</sup>, Hybrid Spiking model (HSM)<sup>5</sup> etc., and other neuron models viz. Hodgkin-Huxley model (HH model)<sup>14,15</sup>, Morris-Lecar model (ML model)<sup>10-16</sup> have been proposed in literatures in previous more than 100 years. Each proposed neuron model captures certain neuron properties and explains underlying information processing mechanism. Understanding of neuron functionality and information processing mechanism help researchers to develop similar algorithms at software level as well as devices at hardware level resulting into biological brain features, which is also known as artificial intelligence (AI). IF and LIF models are prime choice in AI and in other related fields due to its mathematical simplicity and implement ability. IF and LIF models have been investigated in different input and parameter scenarios. Burkitt's<sup>11,12</sup> review literatures contains few input choices in term of homogeneous synaptic input and heterogeneous synaptic input. McDonnell et. al<sup>17</sup> review literature contains basics of neuronal information processing mechanism. Karmeshu et. al.<sup>18</sup> has suggested the distributed delay framework around LIF model. DDF provides a way to integrate the past values of neuron membrane potential with current input; and, then to investigate neuron spiking activity and other related features. This article contains DDF and several neuron models investigated in this framework.

It is well assumed that a neuron uses rate encoding and temporal encoding techniques to encode information<sup>9,19</sup>. This encoding takes place in form of spikes, also known as membrane potential epochs. The spike transmits in form of sequences in nervous system. Temporal encoding technique uses time interval between to spikes, which is also known as inter-spike-interval (ISI), to encode information. The probability distribution of interval-spike-interval provides inter-spike-interval distribution of neuron.

The article is structured in 6 sections. After a brief introduction about big-data, neurons and its information processing in first section, second section explains LIF Model, DDF for the LIF model. Section 3 contains LIF model in DDF with gamma distributed delay and their findings. Section 4 contains LIF model with hypo-exponential distributed delay and its finding. Section 5 deals with Modified hybrid spiking neuron model and its findings. Conclusion is outlined in the last section 6.

## 2. LIF Neuron Model and DDF Framework

LIF model is an extension of IF model in term of membrane decay constant. Mathematically, it may be given by the following equation.

$$(2.1) \quad \frac{dV}{dt} = -\beta V + I(t).$$

Here,  $\beta$  is the membrane decay constant and  $I(t)$  is the applied input stimulus.

Karmeshu et. al.<sup>18</sup> has suggested a framework to incorporate the past values of membrane potential (also known as memory) of neuron for LIF model. The proposed framework includes a delay kernel function for incorporating entire membrane potential values developed on time-line. Incorporation of kernel function  $K(t)$  into Eq. (2.1) modifies the LIF model as

$$(2.2) \quad \frac{dV}{dt} = -\beta \int_0^t K(t-\tau)V(\tau)d\tau + I(t),$$

with initial condition  $V(t)=0$  at  $t=0$ .

Depending on the choice of parameter investigated,  $K(t)$  may acquire different kernel function viz. exponential function, gamma function, hypo-exponential function, hyper-exponential function etc. The membrane

potential evolution process represented by Eq. (2.2) is a non-Markovian process. By applying linear-chain-trick, this non-Markovian process can be transformed into a Markovian process in an extended space<sup>20</sup>. Karmeshu et. al.<sup>18</sup> and Sharma and Karmeshu<sup>21</sup> studied Eq. (2.2) with gamma kernel function. Karmeshu et. al.<sup>18</sup> further has extended his study with exponential kernel function whereas Sharma and Karmeshu<sup>21</sup> have investigated Eq. (2.2) with Gamma kernel function. Their studies are outlined in next section III. Choudhary et. al.<sup>22,23</sup> has investigated LIF model in DDF with hypo-exponential distribution which is covered in section IV. Distributed delay kernel function reflects zero effect on stationary state membrane potential distribution which is a Gaussian distribution in general<sup>24</sup>.

### 3. LIF Neuron Model with Gamma Distributed Delay Kernel

Gamma function may be written as<sup>25,26</sup>

$$(3.1) \quad K(t) = \frac{\eta^{m+1} t^m e^{-\eta t}}{m!}, \quad m=0,1,2,\dots$$

Here,  $m$  and  $\eta$  represents shape and delay parameters. Choice of parameter value  $m$  in  $K(t)$  transforms Eq. (3.1) in exponential function and gamma kernel function<sup>18, 21, 23, 24, 25, 27</sup>. Substitution of  $K(t)$  form Eq. (3.1) in Eq. (2.1) results the LIF model in DDF as given below

$$(3.2) \quad \frac{dV}{dt} = -\beta \int_0^t \frac{\eta^{m+1} (t-\tau)^m e^{-\eta(t-\tau)}}{m!} V(\tau) d\tau + I(t).$$

For  $m=0$ , kernel function shown in Eq. (3.1) transforms in exponential kernel. Karmeshu et. al.<sup>18</sup> has used this parameter values in Eq. (3.2) and enhanced it in an extended space as

$$(3.3) \quad \begin{aligned} \frac{dV}{dt} &= -\eta\beta U_0(t) + I(t), \\ \frac{dU_0(t)}{dt} &= -\eta\{U_0(t) - \eta V(t)\}, \end{aligned}$$

with initial condition  $[V(t)=0 \text{ and } U(t)=0 \text{ at } t=0]$ .

Detailed analysis of Eq. (3.3) with stochastic input leads the ISI distribution into a power law<sup>18</sup>. For  $m \geq 1$ , kernel function shown in Eq. (3.1) transforms

in gamma kernel. Following Sharma and Karmeshu<sup>21</sup>, choice of parameter  $m=1$  in Eq. (3.2) results LIF model in extended space as

$$(3.3) \quad \begin{aligned} \frac{dV}{dt} &= -\eta\beta U_1(t) + \mu + \xi(t), \\ \frac{dU_1(t)}{dt} &= -\eta\{U_1(t) - U_0(t)\}, \\ \frac{dU_0(t)}{dt} &= -\eta\{U_0(t) - \eta V(t)\}, \end{aligned}$$

with initial condition  $[V(t)=0 \text{ and } U_j(t)=0 \forall j \in \{0,1,2\} \text{ at } t=0]$ .

Detailed analysis of Eq. (3.3) with stochastic input leads the ISI distribution into a power law<sup>18,23</sup>.

In simulation based studied of Eq. (3.3) with stochastic input, Sharma and Karmeshu<sup>21</sup> have noticed that this extension of LIF model has transient bi-modality feature i.e. underlying parameter values leads ISI distribution to transform from uni-model to bi-modal.

#### 4. LIF Neuron Model with Hypo-exponential Distributed Delay Kernel

Hypo-exponential function with parameter  $n$  parameters  $(\lambda_1, \lambda_2, \dots, \lambda_n)$  may be written as Ross<sup>23, 26, 27</sup>

$$(4.1) \quad K_{X_1+X_2+\dots+X_n}(t) = \sum_{i=1}^n \binom{n}{i} \lambda_i e^{-\lambda_i(t)}.$$

Here  $\binom{n}{i} = \prod_{j \neq i} \frac{\lambda_j}{\lambda_j - \lambda_i}$ ,  $\lambda_i \neq \lambda_j \forall i \neq j$ .

Choudhary *et. al.*<sup>28</sup> has used two parameters  $\lambda_E$  and  $\lambda_I$  ( $\lambda_E \neq \lambda_I$ ) so that Eq. (2.1) takes the form as

$$(4.2) \quad \frac{dV}{dt} = -\frac{\beta\lambda_E\lambda_I}{\lambda_I - \lambda_E} \int_0^t (e^{-\lambda_E(t-\tau)} - e^{-\lambda_I(t-\tau)}) V(\tau) d\tau + I(t),$$

with initial condition  $V(t) = V_0$  at  $t = 0$ .

LIF model in DDF given by Eq. (4.2) may be transformed into a system of three coupled linear equation in a four dimensional extended space as shown below.

$$\begin{aligned} \frac{dV}{dt} &= -\frac{\beta\lambda_E\lambda_I}{\lambda_I - \lambda_E}(X - Y) + I(t), \\ (4.3) \quad \frac{dY}{dt} &= -\lambda_I Y + V. \end{aligned}$$

This extended space model has capacity to generate uni-model, bi-modal, multi-model, exponential ISI distribution pattern during simulation based study with stochastic input<sup>23</sup>. Out of these ISI distribution patterns, few patterns show power-law behavior<sup>23</sup>.

### 5. Hybrid Spiking Neuron Model in DDF

Hybrid spiking model is proposed by Izhikevich<sup>29</sup>. It has two parameters membrane potential ( $V$ ) and recovery variable ( $U$ ). This model may be given as

$$\begin{aligned} \frac{dV}{dt} &= f(V) - U(E - V) + I, \\ (5.1) \quad \frac{dU}{dt} &= a(bV - U). \end{aligned}$$

with a reset condition if  $V \geq V_T$  then  $V \leftarrow V_R$ , and  $U \leftarrow U + U_I$ ,  $f(V)$  is a membrane potential and current relationship.  $I$  is applied input and  $E$  is known as reversal potential.  $a, b, V_T$  and  $U_I$  are constants. Bharti et. al.<sup>30</sup> has generalized the inclusion of delay in DDF as

$$(5.2) \quad f(V) = \int_0^t K(t - \tau)V(\tau)d\tau.$$

Here,  $K(t)$  is a memory kernel. Bharti et. al.<sup>30</sup> used gamma function for  $K(t)$  shown in Eq. (5.2) so that Hybrid spiking neuron model takes form in DDF as

$$\frac{dV}{dt} = \int_0^t \eta e^{-\eta(t-\tau)} V(\tau) d\tau - U + I ,$$

(5.3)

$$\frac{dU}{dt} = a(bV - U) .$$

Choudhary et. al<sup>22</sup> , Choudhary and Solanki<sup>24</sup>, Choudhary and Singh<sup>31</sup> have investigated this model and noticed that distributed delay is effect less on stationary state probability distribution of membrane potential.

## 6. Conclusion

Underlying information processing mechanism of brain has made a revolution in this century in term of AI and deep learning. Fundamental neuron models like IF model, LIF model, HSN model etc., form the building block in this revolution. DDF provides an aspect to integrate past values of neuron membrane potential in term kernel function, which opens a gate to look into new horizon of neuron information processing so that the revolution in term of AI and deep learning can further be facilitated.

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