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Efficient Estimation Procedure of Finite Population Mean Using Supplementary Information in Stratified Random Sampling

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Abstract: This paper addresses the problem of estimating the population mean $\bar{Y}$ of the study variable y using known population mean $\bar{X}$ of the auxiliary variable x in stratified random sampling. We adopted the procedure of Kadilar and Cingi[1] is proposing some chain ratio-type, combined chain ratio-type exponential, combined chain ratio-ratio-type exponential estimators and their generalized versions. Suggested estimators' properties are studied up to the first order of approximation. We obtained the regions of preferences under which the proposed estimators are better than some existing estimators. In support of the present study, numerical illustration is being provided.
Keywords: Study variate, Auxiliary variate, Bias, Mean squared error, Efficiency.

1. Introduction

It is a well-established fact that the use of auxiliary information improves the precision of estimates. A large amount of work has been carried out in estimating the population mean $\bar{Y}$ of the auxiliary variable y when the population mean $\bar{X}$ of the auxiliary variable x is known under simple random sampling, for instance, see Cochran[2], Sukhatme et al.[3], Singh, H. P.[4], Singh S.[5] and the references cited therein. However, some works have been carried out in situations where information on the auxiliary variable x is available for all the units in the population. Thus, the information on several parameters such as population mean $\bar{X}$, (or total $X = N\bar{X}$) Coefficient of variation C_x, Coefficients of skewness $\beta_1(x)$, and kurtosis $\beta_2(x)$ of the auxiliary variable x can be made known. From the previous studies or experience gathered in due course of time the correlation coefficient ρ between y and x is also known. The above studies have been carried out under simple random sampling. However, due to several reasons the well known stratified random sampling scheme has its wide applications in practical situations. This led few authors to pay attention to the estimation of the population mean $\bar{Y}$ of the study variable y in stratified random sampling its wide application in practical situations.

Consider a finite population $\Omega = (\Omega_1, \Omega_2, ..., \Omega_N)$ of size N and let y and x, respectively, be the study and auxiliary variables associated with each unit Ω_j (j = 1,2,…, N) of the population Ω. Let the population of size N is stratified into L strata with hth stratum containing N_h units, where h = 1,2,…,L such that $\sum_{h=1}^{L} N_h = N$.

Let y be the study variate and x be the simple random sample of size n_h is drawn from each stratum which constitutes a sample of size $n = \sum_{h=1}^{n} n_h$ and we define:

$\bar{Y}_h = \dfrac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$: h^{th} Stratum mean for the study variate y,

$\bar{X}_h = \dfrac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: h^{th} Stratum mean for the auxiliary variate x,

$\bar{Y} = \dfrac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \dfrac{1}{N}\sum_{h=1}^{L} N_h\bar{Y}_h = \sum_{h=1}^{L} W_h\bar{Y}_h$: Population mean of the study variate y ,

$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \frac{1}{N}\sum_{h=1}^{L} W_h \bar{X}_h$: Population mean of the auxiliary variate x,

$\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$: Sample mean of the study variate y for h^{th} stratum,

$\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$: Sample mean of the auxiliary variate x for h^{th} stratum,

$W_h = \frac{N_h}{N}$: Stratum weight of h^{th} stratum.

1.1 Some Traditional Estimators: Usual unbiased estimators of population means $\bar{Y}$, $\bar{X}$ in stratified random sampling are defined respectively as

$$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h \ , \ \bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h \ .$$

When population mean $\bar{X}$ of auxiliary variable x is known, Hansen et al.[6] defined the combined ratio estimator for population mean $\bar{Y}$ as

(1.1) $$\bar{y}_{RC} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right).$$

The conventional product estimator for population mean $\bar{Y}$ in stratified random sampling is defined by

(1.2) $$\bar{y}_{PC} = \bar{y}_{st}\left(\frac{\bar{x}_{st}}{\bar{X}}\right).$$

The biases and mean squared errors of $\bar{y}_{RC}$ and $\bar{y}_{PC}$, up to order n^{-1} , are obtained as

(1.3) $$B(\bar{y}_{RC}) = \left(\frac{1}{\bar{X}}\right)(RV_{20} - V_{11}) = \frac{\bar{Y}V_{20}}{\bar{X}^2}(1-K),$$

(1.4) $$B(\bar{y}_{PC}) = \left(\frac{1}{\bar{X}}\right)V_{11} = \frac{\bar{Y}}{\bar{X}^2} K V_{20},$$

(1.5) $$MSE(\bar{y}_{RC}) = \left(V_{02} - 2R\, V_{11} + R^2 V_{20}\right),$$

(1.6) $$MSE(\bar{y}_{PC}) = \left(V_{02} + 2RV_{11} + R^2 V_{20}\right),$$

where $V_{02} = \sum_{h=1}^{L} \gamma_h W_h^2 S_{yh}^2$, $V_{20} = \sum_{h=1}^{L} \gamma_h W_h^2 S_{xh}^2$, $V_{11} = \sum_{h=1}^{L} \gamma_h W_h^2 S_{yxh}^2$, $R = \frac{\bar{Y}}{\bar{X}}$,

$$\gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right), S_{yh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}\right)^2, S_{xh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(x_{hi} - \bar{X}\right)^2,$$

and

$$S_{yxh} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}\right)\left(x_{hi} - \bar{X}\right).$$

1.2 Some Exponential Type Estimators: Singh et al.[7] suggested ratio-type and product-type exponential estimators respectively for population mean $\bar{Y}$ as

(1.7) $$\bar{y}_{RCe} = \bar{y}_{st} \exp\left\{\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right\},$$

(1.8) $$\bar{y}_{PCe} = \bar{y}_{st} \exp\left\{\frac{\bar{x}_{st} - \bar{X}}{\bar{x}_{st} + \bar{X}}\right\}.$$

To the first degree of approximation (*fda*) the bias and *MSE* of the estimator $\bar{y}_{RC}$ and $\bar{y}_{PCe}$ are respectively given by

(1.9) $$B(\bar{y}_{RCe}) = \frac{\left[3RV_{20} - 4V_{11}\right]}{8\bar{X}} = \frac{\bar{Y}V_{20}}{8\bar{X}^2}\left[3 - 4K\right],$$

(1.10) $$B(\bar{y}_{PCe}) = -\frac{1}{8\bar{X}}\left[RV_{20} - 4V_{11}\right]$$

$$= -\frac{\bar{Y}V_{20}}{8\bar{X}^2}[1-4K],$$

(1.11) $$MSE(\bar{y}_{RCe}) = \left[V_{02} + \frac{R^2V_{20}}{4} - RV_{11}\right]$$

$$= \left[V_{02} + \frac{R^2V_{20}}{4}(1-4K)\right],$$

(1.12) $$MSE(\bar{y}_{PCe}) = \left[V_{02} + \frac{R^2V_{20}}{4} + RV_{11}\right]$$

$$= \left[V_{02} + \frac{R^2V_{20}}{4}(1+4K)\right].$$

The variance/*MSE* of the usual stratified sample mean $\bar{y}_{st}$ is given by

(1.13) $$MSE(\bar{y}_{st}) = V_{02}.$$

From (1.5) and (1.13), we have

$$MSE(\bar{y}_{RC}) - MSE(\bar{y}_{st}) = (R^2V_{20} - 2RV_{11}),$$

which is negative if

(1.14) $$K > \frac{1}{2},$$

where $K = \dfrac{V_{11}}{RV_{20}}$.

From (1.6) and (1.13) we have

$$MSE(\bar{y}_{PC}) - MSE(\bar{y}_{st}) = R(R\ V_{20} + 2RV_{11}),$$

which is less than zero if

(1.15) $$K < -\frac{1}{2}.$$

From (1.11) and (1.13) we have

$$MSE\left(\bar{y}_{RCe}\right) - MSE\left(\bar{y}_{st}\right) = \left(\frac{R^2 V_{20}}{4} - RV_{11}\right),$$

which is negative if

(1.16) $$K > \frac{1}{4}.$$

From (1.12) and (1.13) we have

$$MSE\left(\bar{y}_{PCe}\right) - MSE\left(\bar{y}_{st}\right) = \left(\frac{R^2 V_{20}}{4} + V_{11}\right)$$

which is less than zero if

(1.17) $$K < -\frac{1}{4}.$$

From (1.15) and (1.17) we have made the following conclusions:

(i) the combined ratio estimator $\bar{y}_{RC}$ is more efficient than $\bar{y}_{st}$ if, $K > \frac{1}{2}$.

(ii) the combined product estimator $\bar{y}_{PC}$ is more efficient than $\bar{y}_{st}$ if, $K < -\frac{1}{2}$.

(iii) the ratio-type exponential estimator $\bar{y}_{RCe}$ is more efficient than $\bar{y}_{st}$ if, $K > \frac{1}{4}$.

(iv) the product-type exponential estimator $\bar{y}_{PCe}$ is better than $\bar{y}_{st}$ if, $K < -\frac{1}{4}$.

Further from (1.5) and (1.11) we have

$$MSE(\bar{y}_{RC}) - MSE(\bar{y}_{RCR}) = R^2 V_{20}\left(\frac{3}{4} - K\right),$$

which is positive if

(1.18) $$K < \frac{3}{4}.$$

Expression (1.18) clearly indicates that the ratio-type exponential estimator $\bar{y}_{RCe}$ is more efficient than ratio estimator $\bar{y}_{RC}$ as long as the condition (1.18) is satisfied,

From (1.6) and (1.12) we have

$$MSE(\bar{y}_{PC}) - MSE(\bar{y}_{PCe}) = R^2 V_{20}\left(K + \frac{3}{4}\right)$$

which is positive if

(1.19) $$K > -\frac{3}{4}.$$

Thus under the condition (1.19), the product-type exponential estimator $\bar{y}_{PCe}$ is more efficient than the combined product estimator $\bar{y}_{PC}$.

Combining the inequalities (1.14) and (1.19) we get that the ratio-type exponential estimator $\bar{y}_{RCe}$ is more efficient than $\bar{y}_{st}$ and ratio estimator $\bar{y}_{RC}$ if

(1.20) $$\frac{1}{2} < K < \frac{3}{4}.$$

Further combining the inequalities (1.15) and (1.20) we note that the product type exponential estimator $\bar{y}_{PCe}$ will dominate over $\bar{y}_{st}$ and $\bar{y}_{PC}$ if

(1.21) $$-\frac{3}{4} < K < -\frac{1}{4}.$$

In this paper we recommend a chain-ratio-ratio-type exponential estimator in stratified random sampling. The bias and *MSE* of the suggested estimator

have been obtained under large sample approximation. Conditions are obtained under which the suggested estimator is more efficient than the usual unbiased estimator, usual ratio combined estimator and ratio-type exponential estimator due to Singh et al.[8].
We use the following notations and expected values for deriving the biases and *MSEs* of the estimators suggested in Section 2.
We write

$$\bar{y}_{st} = \bar{Y}(1+\zeta_0), \quad \bar{x}_{st} = \bar{X}(1+\zeta_1)$$

such that

$$E(\zeta_0) = E(\zeta_1) = 0 \text{ and } E(\zeta_0^2) = \frac{V_{02}}{\bar{Y}^2}, E(\zeta_1^2) = \frac{V_{20}}{\bar{X}^2}, \ E(\zeta_0\zeta_1) = \frac{V_{11}}{\bar{Y}\bar{X}}.$$

2. Study of Some Chain-Type Estimators in Stratified Random Sampling

2.1 Chain Estimator in Stratified Random Sampling: Considering the procedure as given by Kadilar and Cingi[1], if stratified sample mean $\bar{y}_{st}$ in (1.1) is replaced by with $\bar{y}_{RC}$, the chain ratio-type estimator for $\bar{Y}$ is obtained as

(2.1) $$\bar{y}_{CRC} = \bar{y}_{RC}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)$$

$$= \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\left(\frac{\bar{X}}{\bar{x}_{st}}\right) = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)^2.$$

Further, if stratified random sample mean $\bar{y}_{st}$ in (1.7) is replaced with $\bar{y}_{RCe}$ the chain ratio-type exponential estimator for $\bar{Y}$ is given by

(2.2) $$\bar{y}_{CRCe} = \bar{y}_{RCe}\exp\left(\frac{\bar{X}-\bar{x}_{st}}{\bar{X}+\bar{x}_{st}}\right)$$

$$= \bar{y}_{st}\exp\left\{\frac{2(\bar{X}-\bar{x}_{st})}{\bar{X}+\bar{x}_{st}}\right\}.$$

Such an estimator for $\bar{Y}$ in simple random sampling without replacement is suggested by Singh and Pal[9] and also defined in systematic and cluster sampling by Pal et al.[11-12].
If we replace $\bar{y}_{st}$ in (1.7) by usual combined ratio estimator

$$\bar{y}_{RC} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right),$$

then we get a chain combined ratio-ratio-type exponential estimator for mean $\bar{Y}$ as

$$\text{(2.3)} \qquad \bar{y}_{CRC\,\mathrm{Re}} = \bar{y}_{RC}\,\exp\left(\frac{\bar{X}-\bar{x}_{st}}{\bar{X}+\bar{x}_{st}}\right)$$

$$= \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\exp\left(\frac{\bar{X}-\bar{x}_{st}}{\bar{X}+\bar{x}_{st}}\right).$$

2.2 Bias and MSE of $\bar{y}_{CRC}$: Expressing (2.1) we have

$$\text{(2.4)} \qquad \bar{y}_{CRC} = \bar{Y}(1+\zeta_0)(1+\zeta_1)^{-2}$$

$$= \bar{Y}(1+\zeta_0 - 2\zeta_1 - 2\zeta_0\zeta_1 + 3\zeta_1{}^2 \ldots)$$

which is approximated as

$$\bar{y}_{CRC} \cong \bar{Y}(1+\zeta_0 - 2\zeta_1 - 2\zeta_0\zeta_1 + 3\zeta_1{}^2)$$

or

$$\text{(2.5)} \qquad \left(\bar{y}_{CRC} - \bar{Y}\right) \cong \bar{Y}(\zeta_0 - 2\zeta_1 - 2\zeta_0\zeta_1 + 3\zeta_1{}^2).$$

So the bias of $\bar{y}_{CRC}$ to the *fda* is given by

$$\text{(2.6)} \qquad B\left(\bar{y}_{CRC}\right) = \frac{\left(3RV_{20} - 2V_{11}\right)}{\bar{X}},$$

$$= \frac{RV_{20}}{\bar{X}}(3-2K).$$

The approximate value of $\left(\bar{y}_{CRC}-\bar{Y}\right)^2$ is given by

(2.7) $$\left(\bar{y}_{CRC}-\bar{Y}\right)^2=\bar{Y}(\zeta_0{}^2-4\zeta_0\zeta_1+4\zeta_1{}^2)\,.$$

The expectation of both sides of (2.7) gives the *MSE* of $\bar{y}_{CRC}$ to the *fda* as

(2.8) $$\begin{aligned}MSE\left(\bar{y}_{CRC}\right)&=\left(V_{02}-4RV_{11}+4R^2V_{20}\right)\\&=\left\{V_{02}+4R^2V_{20}(1-K)\right\}.\end{aligned}$$

2.3 Bias and MSE of $\bar{y}_{CRCe}$: Putting $\bar{y}_{st}=\bar{Y}(1+\zeta_0)$ and $\bar{x}_{st}=\bar{X}(1+e_1)$ in (2.2), expanding, multiplying out and neglecting terms of ζ 's having power greater two, we have

$$\bar{y}_{CRCe}\cong\bar{Y}(1+\zeta_0-\zeta_1-\zeta_0\zeta_1+\zeta_1{}^2)$$

or

(2.9) $$\left(\bar{y}_{CRCe}-\bar{Y}\right)\cong\bar{Y}(\zeta_0-\zeta_1-\zeta_0\zeta_1+\zeta_1{}^2)$$

So the bias of $\bar{y}_{CRCe}$ to the *fda* is given by

(2.10) $$\begin{aligned}B\left(\bar{y}_{CRCe}\right)&=\frac{RV_{20}}{\bar{X}}(1-K)\\&=B\left(\bar{y}_{RC}\right)\,,\end{aligned}$$

which is same as the bias of the combined ratio estimator $\bar{y}_{RC}$.
Retaining upto second power of ζ 's in $\left(\bar{y}_{CRCe}-\bar{Y}\right)^2$ we have

(2.11) $$\left(\bar{y}_{CRCe}-\bar{Y}\right)^2=\bar{Y}(\zeta_0{}^2-2\zeta_0\zeta_1+\zeta_1{}^2)\,.$$

So the *MSE* of $\bar{y}_{CRCe}$ to the *fda* is given by

$$\begin{aligned}MSE\left(\bar{y}_{CRCe}\right)&=\left\{V_{02}+R^2V_{20}(1-2K)\right\}\\&=MSE\left(\bar{y}_{RC}\right)\,,\end{aligned}$$

which is same as *MSE* of the combined ratio estimator $\bar{y}_{RC}$ [see, equation (1.6)].

2.4 Bias and *MSE* of $\bar{y}_{CCR\mathrm{Re}}$: Inserting $\bar{y}_{st} = \bar{Y}(1+\zeta_0)$ and $\bar{x}_{st} = \bar{X}(1+\zeta_1)$ in (2.3) we have

$$\bar{y}_{CCR\mathrm{Re}} = \bar{Y}(1+\zeta_0)(1+\zeta_1)^{-1}\exp\left\{\frac{\bar{X}-\bar{X}(1+\zeta_1)}{\bar{X}+\bar{X}(1+\zeta_1)}\right\}$$

$$= \bar{Y}(1+\zeta_0)(1+\zeta_1)^{-1}\exp\left\{\frac{-\zeta_1}{2}\left(1+\frac{\zeta_1}{2}\right)^{-1}\right\}$$

$$= \bar{Y}(1+\zeta_0)(1+\zeta_1)^{-1}\left\{1-\frac{\zeta_1}{2}+\frac{\zeta_1^{\,2}}{4}+\frac{\zeta_1^{\,2}}{8}-\ldots\right\}$$

$$= \bar{Y}(1+\zeta_0)(1+\zeta_1+\zeta_1^{\,2}-\ldots)\left\{1-\frac{\zeta_1}{2}+\frac{3\zeta_1^{\,2}}{8}-\ldots\right\}$$

(2.12) $$\bar{y}_{CCR\mathrm{Re}} = \bar{Y}\left(1+\zeta_0-\frac{3\zeta_1}{2}-\frac{3\zeta_0\zeta_1}{2}+\frac{15\zeta_1^2}{8}+\ldots\right).$$

Retaining terms of ζ 's upto second power, we have

(2.13) $$\left(\bar{y}_{CCR\mathrm{Re}}-\bar{Y}\right) \cong \bar{Y}\left(\zeta_0-\frac{3\zeta_1}{2}-\frac{3\zeta_0\zeta_1}{2}+\frac{15\zeta_1^2}{8}\right).$$

The expected value of (2.13) yields the bias of $\bar{y}_{CCR\mathrm{Re}}$ as

(2.14) $$B\left(\bar{y}_{CCR\mathrm{Re}}\right) = \left(\frac{15}{8}RV_{20}-\frac{3}{2}V_{11}\right)\frac{1}{\bar{X}}$$

$$= \frac{3RV_{20}}{8\bar{X}}(5-4K).$$

Retaining terms of ζ 's in $\left(\bar{y}_{CCR\mathrm{Re}}-\bar{Y}\right)^2$ up to second power, we have

(2.15) $$\left(\bar{y}_{CCR\mathrm{Re}} - \bar{Y}\right)^2 \cong \bar{Y}^2\left(e_0^{\ 2} - 3e_0e_1 + \frac{9e_1^2}{4}\right)$$

So the MSE of $\bar{y}_{CCR\mathrm{Re}}$ to the *fda* is given by

(2.16) $$MSE\left(\bar{y}_{CCR\mathrm{Re}}\right) = \left(V_{02} + \frac{9}{4}R^2V_{20} - 3RV_{11}\right)$$
$$= \left(V_{02} + \frac{3R^2}{4}V_{20}(3-4K)\right).$$

2.5 Efficiency Comparison: From (1.13) and (2.8) we have

(2.17) $$MSE\left(\bar{y}_{st}\right) - MSE\left(\bar{y}_{CRC}\right) = 4R^2V_{20}\left(K-1\right) > 0 \text{ if } K > 1.$$

From (1.5) and (2.17) we have

(2.18) $$MSE\left(\bar{y}_{RC}\right) - MSE\left(\bar{y}_{CRC}\right) = \bar{Y}^2V_{20}\left(2K-3\right) > 0 \text{ if } K > \frac{3}{2} = 1.50.$$

From (1.11) and (2.8) we have

$$MSE\left(\bar{y}_{RCe}\right) - MSE\left(\bar{y}_{CRC}\right) = 3R^2V_{20}\left(K - \frac{5}{4}\right)$$

which is non-negative if

(2.19) $$K > \frac{5}{4} = 1.25.$$

Thus the proposed chain combined ratio estimator $\bar{y}_{CRC}$ is better than:

(i) $\bar{y}_{st}$ if $K > 1$,

(ii) $\bar{y}_{RC}$ if $K > 1.50$,

(iii) $\bar{y}_{RCe}$ if $K > 1.25$.

The condition $K > 1.50$ is sufficient for $\bar{y}_{CRC}$ to be more accurate than $\bar{y}_{st}$, $\bar{y}_{RC}$ and $\bar{y}_{RCe}$.

Motivated by Swain[12], using square root transformation, we define a combined ratio- type estimator in stratified random sampling for $\bar{Y}$ as

$$\bar{y}_{SQRC} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)^{1/2}. \tag{2.20}$$

To the *fda*, the bias and MSE of $\bar{y}_{SQRC}$ are respectively given by

$$\begin{aligned} B(\bar{y}_{SQRC}) &= \frac{V_{20}R}{8\bar{X}}(3-4K) \\ &= B(\bar{y}_{RCe}), \end{aligned} \tag{2.21}$$

$$\begin{aligned} MSE(\bar{y}_{SQRC}) &= \left[V_{02} + \frac{R^2V_{20}}{4}(1-4K)\right], \\ &= MSE(\bar{y}_{RCe}). \end{aligned} \tag{2.22}$$

We note that the bias and *MSE* of $\bar{y}_{SQRC}$ are same as that of combined ratio-type exponential estimator $\bar{y}_{RCe}$. Also the bias and *MSE* of $\bar{y}_{CRCe}$ are same as that of the combined ratio estimator $\bar{y}_{RC}$. So the comparison of the proposed chain combined ratio estimator $\bar{y}_{CRC}$ and the chain combined ratio-ratio-type exponential estimator $\bar{y}_{CRC\mathrm{Re}}$ made with the estimators $\bar{y}_{RC}$ and $\bar{y}_{RCe}$ will hold also for $\bar{y}_{SQRC}$ and $\bar{y}_{CRCe}$.

From (1.13) and (2.16) we have

$$MSE(\bar{y}_{st}) - MSE(\bar{y}_{CCR\mathrm{Re}}) = \frac{3R^2}{4}V_{20}(4K-3) > 0 \text{ if } K > \frac{3}{4} = 0.75. \tag{2.23}$$

Subtracting (2.16) from (1.5) we have

$$MSE(\bar{y}_{RC}) - MSE(\bar{y}_{CRC\mathrm{Re}}) = R^2V_{20}\left(K - \frac{5}{4}\right) > 0 \text{ if } K > \frac{5}{4} = 1.25. \tag{2.24}$$

We note from (2.11) and (2.16) that

$$MSE(\bar{y}_{RCe}) - MSE(\bar{y}_{CRC\mathrm{Re}}) = 2R^2V_{20}(K-1)$$

which is larger than zero if

(2.25) $K > 1$.

Further the difference of (2.8) and (2.16) we have

$$MSE(\bar{y}_{CRC}) - MSE(\bar{y}_{CRC\mathrm{Re}}) = R^2 V_{20}\left(\frac{7}{4} - K\right)$$

which is positive if

(2.26) $K < \frac{7}{4} = 1.75$.

We note that from (2.23), (2.24), (2.25) and (2.26) that the $\bar{y}_{CCR\mathrm{Re}}$ is more accurate than

(i) $\bar{y}_{st}$ if $K > \frac{3}{4} = 0.75$,

(ii) $\bar{y}_{RC}$ if $K > \frac{5}{4} = 1.25$,

(iii) $\bar{y}_{RCe}$ if $K > 1$,

(iv) $\bar{y}_{CRC}$ if $K < \frac{7}{4} = 1.75$,

Thus we see that the suggested chain combined ratio-ratio-type exponential estimator $\bar{y}_{CCR\mathrm{Re}}$ is better than $\bar{y}_{st}$, $\bar{y}_{RC}$, $\bar{y}_{RCe}$ and $\bar{y}_{CRC}$ as long as the condition :

(2.27) $1.25 = \frac{5}{4} < K < \frac{7}{4} = 1.75$

is satisfied.

Remark 2.1: If the correlation between y and x is negative, the chain combined product-type estimator $\bar{y}_{CPCe}$ (say), chain combined product-type exponential estimator $\bar{y}_{CPCe}$ (say) and chain combined product-product-type exponential estimator $\bar{y}_{CPCPe}$ (say) are respectively defined by

(2.28) $$\bar{y}_{CPC} = \bar{y}_{st}\left(\frac{\bar{x}_{st}}{\bar{X}}\right)^2,$$

(2.29) $$\bar{y}_{CPCe} = \bar{y}_{st}\exp\left\{\frac{2\left(\bar{x}_{st}-\bar{X}\right)}{\bar{X}+\bar{x}_{st}}\right\},$$

(2.30) $$\bar{y}_{CPCPe} = \bar{y}_{st}\left(\frac{\bar{x}_{st}}{\bar{X}}\right)\exp\left\{\frac{\left(\bar{x}_{st}-\bar{X}\right)}{\bar{X}+\bar{x}_{st}}\right\}.$$

Proceeding as earlier the properties of the estimators $\bar{y}_{CPC}$, $\bar{y}_{CPCe}$ and $\bar{y}_{CPCPe}$ can be easily studied.

3. A General Class of Chain -Type Estimators

Adopting the procedure due to Srivastava[13], Singh and Pal[9] and Pal et al.[10-11], a class of chain type estimators for $\bar{Y}$ is defined by

(3.1) $$t = \bar{y}_{st}\left(\frac{\alpha\bar{X}+\delta}{\alpha\bar{x}_{st}+\delta}\right)^p \exp\left\{\frac{q\alpha\left(\bar{X}-\bar{x}_{st}\right)}{\alpha\left(\bar{X}+\bar{x}_{st}\right)+2\delta}\right\},$$

where $(\alpha \neq 0, \delta)$ are real constants or the functions of the known parameters of x and y such
as

$$\wp_1 = \sum_{h=1}^{L} W_h s_{xh}, \wp_2 = \sum_{h=1}^{L} W_h C_{xh}, \wp_3 = \sum_{h=1}^{L} W_h \beta_{1h}(x),$$

$$\wp_4 = \sum_{h=1}^{L} W_h \beta_{2h}(x)$$

$$\wp_5 = \sum_{h=1}^{L} W_h \rho_h, \wp_6 = \sum_{h=1}^{L} W_h \Delta_h(x), \wp_7 = \sum_{h=1}^{L} W_h C_{yh},$$

$$\wp_8 = \sum_{h=1}^{L} W_h \beta_{2h}(y) \text{ etc.,}$$

where

$$\Delta_h(x) = \{\beta_{2h}(x) - \beta_{1h}(x) - 1\} > 0, C_{2h} = S_{xh}/\bar{X}_h,\ C_{yh} = S_{yh}/\bar{Y}_h,$$

$$\beta_{1h}(x)=\left\{\frac{\left\{E\left(x_h-\bar{X}_h\right)^3\right\}^2}{\left\{E\left(x_h-\bar{X}_h\right)^2\right\}^3}\right\},\ \beta_{1h}(y)=\left\{\frac{\left\{E\left(y_h-\bar{Y}_h\right)^3\right\}^2}{\left\{E\left(y_h-\bar{Y}_h\right)^2\right\}^3}\right\},$$

$$\beta_{2h}(x)=\left\{\frac{E\left(x_h-\bar{X}_h\right)^4}{\left\{E\left(x_h-\bar{X}_h\right)^2\right\}^2}\right\},\ \beta_{2h}(y)=\left\{\frac{E\left(y_h-\bar{Y}_h\right)^4}{\left\{E\left(x_h-\bar{Y}_h\right)^2\right\}^2}\right\},$$

and (p, q) being suitably chosen scalars.

3.1 Bias and MSE of t: Substituting $\bar{y}_{st}=\bar{Y}(1+\zeta_0)$ and $\bar{x}_{st}=\bar{X}(1+\zeta_1)$ in (3.1) we have

$$t=\bar{Y}(1+\zeta_0)\left\{\frac{\alpha\bar{X}+\delta}{\alpha\bar{X}(1+\zeta_1)+\delta}\right\}^p \exp\left\{\frac{q\alpha\left(\bar{X}-\bar{X}(1+\zeta_1)\right)}{\alpha\left(\bar{X}+\bar{X}(1+\zeta_1)\right)+2\delta}\right\}$$

$$=\bar{Y}(1+\zeta_0)(1+\tau\zeta_1)^{-p}\exp\left\{\frac{-q\alpha\bar{X}\zeta_1}{2(\alpha\bar{X}+\delta)+\alpha\bar{X}\zeta_1}\right\}$$

$$=\bar{Y}(1+\zeta_0)(1+\tau\zeta_1)^{-p}\exp\left\{\frac{-q\tau\zeta_1}{2}\left(1+\frac{\tau\zeta_1}{2}\right)^{-1}\right\}$$

$$=\bar{Y}(1+\zeta_0)\left\{1-p\tau\zeta_1+\frac{p(p+1)}{2}\tau^2\zeta_1^2-...\right\}\left\{1-\frac{q\tau\zeta_1}{2}+\frac{q\tau^2\zeta_1^2}{4}+\frac{q^2\tau^2\zeta_1^2}{8}-....\right\}$$

$$=\bar{Y}\left[\begin{array}{l}1+\zeta_0-\left(p\tau+\frac{q\tau}{2}\right)\zeta_1-\left(p\tau+\frac{q\tau}{2}\right)\zeta_0\zeta_1+\frac{\tau^2}{8}\left\{4pq+4p^2+4p+q^2+2q\right\}\zeta_1^2\\ \qquad\qquad+\left\{\frac{p(p+1)}{2}+\frac{q(q+2)}{2}\right\}\zeta_0\zeta_1^2-\frac{p\tau q(q+2)\tau^2\zeta_1^3}{8}+...\end{array}\right],$$

where $\tau=\alpha\bar{X}/\left(\alpha\bar{X}+\delta\right)$.

The above expression is approximated as

$$t\cong\bar{Y}\left[1+\zeta_0-\frac{H\tau}{2}\zeta_1-\frac{H\tau}{2}\zeta_1\zeta_1+\frac{H(H+2)\tau^2\zeta_1^2}{8}\right] \tag{3.2}$$

So the bias of t to the *fda* is given by

$$B(t) \cong R\frac{HV_{20}}{8\bar{X}}\{(H+2)\tau-4K\}, \tag{3.3}$$

where $G=(p+q)$ and $H=(2p+q)$.

The approximate value of $(t-\bar{Y})^2$ is given by

$$(t-\bar{Y})^2 \cong \bar{Y}^2\left[\zeta_0^{\,2}-H\tau\zeta_0\zeta_1+\frac{H^2}{4}\tau^2\zeta_1^{\,2}\right]. \tag{3.4}$$

So the *MSE* of t to the *fda* is given by

$$MSE(t)=\left[V_{02}+\frac{R^2V_{20}H\tau}{2}\left\{\frac{H\tau}{2}-2K\right\}\right]. \tag{3.5}$$

For the purpose of comparisons we consider the two classes of estimators for $\bar{Y}$ as

$$t_1=\bar{y}_{st}\left(\frac{\alpha\bar{X}+\delta}{\alpha\bar{x}_{st}+\delta}\right)^p \tag{3.6}$$

and

$$t_2=\bar{y}_{st}\exp\left\{\frac{q\alpha(\bar{X}-\bar{x}_{st})}{\left(\alpha(\bar{X}+\bar{x}_{st})+2\delta\right)}\right\}, \tag{3.7}$$

which are obtained by putting (p, q)=(p, 0) and (0, q) in (3.1) respectively.

The biases and *MSE*s of t_1 and t_2 to the *fda* are respectively given by

$$B(t_1)=R\frac{p\tau V_{20}}{2\bar{X}}[(p+1)\tau-2K], \tag{3.8}$$

$$B(t_2)=R\frac{p\tau V_{20}}{8\bar{X}}[(q+2)\tau-4K], \tag{3.9}$$

(3.10) $$MSE(t_1) = [V_{02} + R^2 V_{20} p\tau(p\tau - 2K)],$$

(3.11) $$MSE(t_2) = \left[V_{02} + \frac{R^2 V_{20} q\tau}{2}\left\{\frac{q\tau}{2} - 2K\right\}\right].$$

3.2 **Efficiency Comparison:** Subtracting (3.5) from (1.13) we have

$$MSE(\bar{y}_{st}) - MSE(t) = \frac{R^2 H\tau}{2} V_{20}\left\{2K - \frac{H\tau}{2}\right\}$$

which is non-negative if

$$\frac{H\tau}{2}\left\{2K - \frac{H\tau}{2}\right\} > 0, \text{ if}$$

(3.12) $$\left.\begin{array}{ll} either & K > \dfrac{H\tau}{4}, \dfrac{H\tau}{2} > 0 \\ or & K < \dfrac{H\tau}{4}, \dfrac{H\tau}{2} < 0 \end{array}\right\},$$

Further subtracting (3.10) from (3.6) we have

$$MSE(t_1) - MSE(t) = R^2 V_{20}\left[p\tau(p\tau - 2K) - \frac{H\tau}{2}\left\{\frac{H\tau}{2} - 2K\right\}\right]$$

$$= -R^2 V_{20}\tau\frac{q}{2}\left[2K - \tau\frac{(2p+H)}{2}\right]$$

which is positive if

(3.13) $$\left.\begin{array}{ll} either & K > \dfrac{(2p+H)\tau}{4}, \tau q > 0 \\ or & K < \dfrac{(2p+H)\tau}{4}, \tau q < 0 \end{array}\right\}$$

Now the difference of (3.6) and (3.11) is given by

$$MSE(t_2) - MSE(t) = R^2 V_{20}\left[\frac{q^2\tau^2}{4} - Kq\tau - \frac{H^2\tau^2}{4} + KH\tau\right]$$
$$= R^2 V_{20}\, p\tau[2K - \tau G] > 0 \text{ if}$$

(3.14)
$$\left.\begin{array}{l} either\ K > \dfrac{G\tau}{2},\ p\tau > 0 \\ or \qquad K < \dfrac{G\tau}{2},\ p\tau < 0 \end{array}\right\}$$

Now from (3.12) to (3.14) we observed that t is more precise than:

(i) $\bar{y}_{st}$ if
$$\left.\begin{array}{l} either\ K > \dfrac{H\tau}{4}, \dfrac{H\tau}{2} > 0 \\ or \qquad K < \dfrac{H\tau}{4}, \dfrac{H\tau}{2} < 0 \end{array}\right\},$$

(ii) t_1 if
$$\left.\begin{array}{l} either K > \dfrac{(2p+H)\tau}{4},\ \tau q > 0 \\ or \qquad K < \dfrac{(2p+H)\tau}{4},\ \tau q < 0 \end{array}\right\},$$

(iii) t_2 if ,

$$\text{where} \quad \left.\begin{array}{l} either\ K > \dfrac{G\tau}{2},\ p\tau > 0 \\ or \qquad K < \dfrac{G\tau}{2},\ p\tau < 0 \end{array}\right\} G = (p+q) \text{ and } H = (2p+q).$$

Two other classes of estimators for $\bar{Y}$ are defined by

(3.15)
$$t_1^{\otimes} = \bar{y}_{st}\left(\frac{\alpha\bar{X} + \delta}{\alpha\bar{x}_{st} + \delta}\right)^{p^{\otimes}},$$

and

(3.16) $$t_2^{\otimes} = \bar{y}_{st} \exp\left\{ \frac{\alpha\left(\bar{X} - \bar{x}_{st}\right)q^{\otimes}}{\left(\alpha\left(\bar{X} + \bar{x}_{st}\right) + 2\delta\right)} \right\},$$

where $p^{\otimes}$ (different from p) and $q^{\otimes}$ (different from q) are suitable chosen scalars. To the *fda* biases and *MSE*s of $t_1^{\otimes}$ and $t_2^{\otimes}$ are respectively given by

(3.17) $$B\left(t_1^{\otimes}\right) = \frac{Rp^{\otimes}\tau V_{20}}{2\bar{X}}\left[(p^{\otimes}+1)\tau - 2K\right],$$

(3.18) $$B\left(t_2^{\otimes}\right) = \frac{Rq^{\otimes}\tau V_{20}}{8}\left[(q^{\otimes}+2)\tau - 4K\right],$$

(3.19) $$MSE\left(t_1^{\otimes}\right) = \left[V_{02} + R^2 p^{\otimes}\tau V_{20}\left(p^{\otimes}\tau - 2K\right)\right],$$

(3.20) $$MSE\left(t_2^{\otimes}\right) = \left[V_{02} + \frac{R^2 q^{\otimes}\tau V_{20}}{2}\left(\frac{q^{\otimes}\tau}{2} - 2K\right)\right].$$

From (3.10) and (3.19) we have

$$MSE\left(t_1^{\otimes}\right) - MSE(t) = \tau R^2 V_{20}\left[p^{\otimes}\tau - 2Kp^{\otimes} - \frac{H^2}{4}\tau + HK\right]$$

$$= \tau R^2 V_{20}\left\{p^{\otimes} - \frac{H}{2}\right\}\left[\tau\left\{p^{\otimes} + \frac{H}{2}\right\} - 2K\right]$$

which is positive if

(3.21) $$\left.\begin{array}{ll} either\ K < \frac{\tau}{2}\left\{p^{\otimes} + \frac{H}{2}\right\}, \tau\left\{p^{\otimes} - \frac{H}{2}\right\} > 0 \\ or \qquad K > \frac{\tau}{2}\left\{p^{\otimes} + \frac{H}{2}\right\}, \tau\left\{p^{\otimes} - \frac{H}{2}\right\} < 0 \end{array}\right\}.$$

Further, the difference between (3.11) and (3.19) is given by

$$MSE\left(t_2^{\otimes}\right)-MSE(t)=\tau R^2V_{20}\left[\frac{q^{\otimes}\tau^2}{4}-Kq^{\otimes}-\frac{H^2}{4}+HK\right]$$

$$=\tau R^2V_{20}\left(q^{\otimes}-H\right)\left\{\frac{\tau}{4}\left(q^{\otimes}+H\right)-K\right\}$$

which is non negative if

$$\left.\begin{array}{l} either K<\frac{\tau}{4}\left(q^{\otimes}+H\right),\left(q^{\otimes}-H\right)\tau>0 \\ or \quad K>\frac{\tau}{4}\left(q^{\otimes}+H\right),\left(q^{\otimes}-H\right)\tau<0 \end{array}\right\}. \tag{3.22}$$

So the envisaged class of estimators t is more efficient than the class of estimations t_1^* and t_2^* as long as the conditions (3.21) and (3.22) are respectively satisfied.

3.3 Optimal Choices of Scalars (p,q): We express the $MSE(t)$ at (3.5) as

$$MSE(t)=[V_{02}+R^2V_{20}\theta\tau(\theta\tau-2K)], \tag{3.23}$$

where $\theta=\frac{H}{2}$. Setting $\frac{\partial MSE(t)}{\partial\theta}=0$, we get the optimum value of θ as

$$\theta=\frac{K}{\tau}=\theta_{opt}(say)$$

or

$$\left\{\frac{H}{2}\right\}_{opt}=\frac{K}{\tau}. \tag{3.24}$$

Thus the resulting $MMSE$ of t is given by

$$MSE_{\min}(t)=(V_{02}-R^2V_{20}K^2). \tag{3.25}$$

Thus we state the following theorem.

Theorem 3.1: *Up to first order approximation,* $MSE(t) \geq (V_{02} - R^2 V_{20} K^2)$ *with equality holding if* $\left\{\frac{H}{2}\right\} = \frac{K}{\tau}$. *From equation (3.24) one can calculate the optimum value of* p *by fixing the value of* q *and vice-versa.*

4. An Improved Version of the Class of Estimators t

An improved version of the class of estimators t for $\bar{Y}$ is given by

$$t_M = Mt \tag{4.1}$$

$$= M\bar{y}_{st}\left(\frac{\alpha\bar{X} + \delta}{\alpha\bar{x}_{st} + \delta}\right)^p \exp\left\{\frac{q\alpha\left(\bar{X} - \bar{x}_{st}\right)}{\alpha\left(\bar{X} + \bar{x}_{st}\right) + 2\delta}\right\},$$

where M is a suitably chosen constant. Setting $\bar{y}_{st} = \bar{Y}(1+\zeta_0)$ and $\bar{x}_{st} = \bar{X}(1+\zeta_0)$ in (4.1), we have

$$t_M = M\bar{Y}(1+\zeta_0)(1+\tau\zeta_1)^{-p} \exp\left\{\frac{-q\tau\zeta_1}{2}\left(1+\frac{\tau\zeta_1}{2}\right)^{-1}\right\}, \tag{4.2}$$

which can be approximated as

$$t_M = M\bar{Y}\left[1+\zeta_0 - \frac{H\tau}{2}\zeta_1 - \frac{H\tau}{2}\zeta_0\zeta_1 + \frac{H(H+2)\tau^2\zeta_1^{\ 2}}{8}\right]$$

or

$$\left(t_M - \bar{Y}\right) \cong \bar{Y}\left[M\left\{1+\zeta_0 - \frac{H\tau}{2}\zeta_1 - \frac{H\tau}{2}\zeta_0\zeta_1 + \frac{H(H+2)\tau^2\zeta_1^{\ 2}}{8}\right\} - 1\right]. \tag{4.3}$$

So the bias of t_M to the *fda* is given by

(4.4) $$B(t_M) \cong \left[M\left\{ \bar{Y} + \frac{H\tau}{2} RV_{20}\left(\frac{(H+2)\tau}{8} - K \right) \right\} - \bar{Y} \right].$$

The value of $(t_M - \bar{Y})^2$ is approximately given by

(4.5) $$(t_M - \bar{Y})^2 \cong \bar{Y}^2 \left[\begin{array}{l} 1 + M^2\left\{ 1 + 2\zeta_0 - H\tau\zeta_1 - H\tau\zeta_0\zeta_1 + \zeta_0^{\,2} + \dfrac{H(H+2)\tau^2\zeta_1^{\,2}}{2} \right\} \\ -2M\left\{ 1 + \zeta_0 - \dfrac{H\tau}{2}\zeta_1 - \dfrac{H\tau}{2}\zeta_0\zeta_1 + \zeta_0^{\,2} + \dfrac{H(H+2)\tau^2\zeta_1^{\,2}}{8} \right\} \end{array} \right]$$

So the *MSE* of t_M to the *fda* is given by

(4.6) $$MSE(t_M) = \bar{Y}^2 \left[\begin{array}{l} 1 - 2M\left\{ 1 + \dfrac{H\tau}{2} V_{20}^{*} \left(\dfrac{(H+2)\tau}{4} - K \right) \right\} \\ + M^2\left\{ 1 + V_{20}^{*} + \dfrac{H\tau}{2} V_{02}^{*} \left((H+1)\tau - 4K \right) \right\} \end{array} \right]$$

where $V_{11}^{*} = \dfrac{V_{11}}{\bar{X}\bar{Y}}$, $V_{20}^{*} = \dfrac{V_{20}}{\bar{X}^2}$, $V_{02}^{*} = \dfrac{V_{02}}{\bar{Y}^2}$ and $K = \dfrac{V_{11}^{*}}{V_{20}^{*}} = \dfrac{V_{11}}{RV_{20}}$.

The $MSE(t_M)$ at (4.6) is minimized for

$$M = \frac{\left[1 + \dfrac{H\tau}{2} V_{20}^{*} \left[\dfrac{(H+2)\tau}{4} - K \right] \right]}{\left[1 + V_{02}^{*} + \dfrac{H\tau}{2} V_{20}^{*} \left[(H+1)\tau - 4K \right] \right]} = M_{opt} \text{ (say)}$$

This leads to the minimum *MSE* of t_M as

(4.7) $$MSE_{\min}(t_M) = \bar{Y}^2 \left[1 - \frac{\left\{ 1 + \dfrac{H\tau}{2} V_{20}^{*} \left\{ \dfrac{(H+2)\tau}{8} - K \right\} \right\}^2}{\left\{ 1 + V_{02}^{*} + \dfrac{H\tau}{2} V_{20}^{*} \left\{ (H+1)\tau - 4K \right\} \right\}} \right].$$

Subtracting (4.7) from (3.5) we have

(4.8) $$MSE(t) - MSE_{\min}(t_M) = \bar{Y}^2 \frac{(A-B)^2}{B},$$

where

$$A = \left\{1 + \frac{H\tau}{2} V_{20}^{*} \left(\frac{(H+2)\tau}{8} - K\right)\right\},$$

$$B = \left\{1 + V_{02} + \frac{H\tau}{2} V_{20}^{*} \left((H+1)\tau - 4K\right)\right\}.$$

Expression (4.8) clearly indicates that the difference $[MSE(t) - MSE(t_M)]$is positive. Therefore the t_M family of estimators is more efficient than the t family of estimators.

5. Empirical Study

To examine the performance of the proposed estimators over other existing estimators, we consider a natural population whose description is given below:

Population [Source: Murthy[14]] y : Output; x : fixed capital,

$$N = 10, N_1 = 5, N_2 = 5, \ n_1 = 2, n_2 = 2, L = 2, \ \bar{Y}_1 = 1925.8, \bar{Y}_2 = 315.6,$$

$$\bar{X}_1 = 214.4, \ \bar{X}_2 = 333.8, S_{y1} = 615.92, \ S_{y2} = 340.38, \ S_{x1} = 74.87,$$

$$S_{x2} = 66.35, \ S_{yx1} = 39308.68, S_{yx2} = 22356.50.$$

We use the following formula to compute the percent relative efficiency (PRE) of various estimators of population mean $\bar{Y}$ with respect to stratified random sample mean $\bar{y}_{st}$:

(5.1) $$PRE(\bar{y}_{RC} or \ \bar{y}_{CRCe}, \bar{y}_{st}) = \frac{V_{02}}{[V_{02} + R^2 V_{20}(1-2K)]} \times 100,$$

(5.2) $$PRE(\bar{y}_{RCe} or\ \bar{y}_{SQRC}, \bar{y}_{st}) = \frac{V_{02}}{\left[V_{02} + \frac{R^2 V_{20}}{4}(1-4K)\right]} \times 100,$$

(5.3) $$PRE(\bar{y}_{CRC}, \bar{y}_{st}) = \frac{V_{02}}{[V_{02} + 4R^2 V_{20}(1-K)]} \times 100,$$

(5.4) $$PRE(\bar{y}_{CCR\mathrm{Re}}, \bar{y}_{st}) = \frac{V_{02}}{\left[V_{02} + \frac{3R^2 V_{20}}{4}(3-4K)\right]} \times 100,$$

(5.5) $$PRE(t_{opt}, \bar{y}_{st}) = \frac{V_{02}}{\left[V_{02} - R^2 V_{20} K^2)\right]} \times 100.$$

Further to illustrate, the improvement over optimum estimator $t_{opt}\,(say)$ in the class of estimators t_M , we consider the following estimators for $\bar{Y}$ as

(5.6) $$t_M^* = M\ \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\exp\left\{\frac{\left(\bar{X} - \bar{x}_{st}\right)}{\left(\bar{X} + \bar{x}_{st}\right)}\right\},$$

which is obtained by putting $(p, q, \alpha, \delta) = (1, 1, 1, 1)$ and $\tau = 1$ in (4.1).
We below give the optimum value of M from (4.7) and thus the resulting *MMSE* of t_M^* from (4.8) respectively as

(5.7) $$M_{opt} = \frac{[1 + (3/2)V_{20}^*\left((5/4) - K\right)]}{\left[1 + V_{02}^* + 6V_{20}^*(1-K)\right]}$$

This leads to the *MMSE* of t_M as

(5.8) $$MSE_{\min}\left(t_M^*\right) = \bar{Y}^2\left[1 - \frac{\left\{1 + (3/2)V_{20}^*\left\{(5/4) - K\right\}\right\}^2}{\left\{1 + V_{02}^* + 6V_{20}^*\left\{1 - 2K\right\}\right\}}\right].$$

To see the performance of the estimators t_M^* we have computed the percent relative efficiency (*PRE*) of t_M^* with respect to $\bar{y}_{st}$ by using the following formula:

(5.9)
$$MSE_{\min}\left(t_M^*, \bar{y}_{st}\right) = \frac{V_{02}^*}{\left[1 - \dfrac{\left\{1 + (3/2)V_{20}^*\left\{(5/4) - K\right\}\right\}^2}{\left\{1 + V_{02}^* + 6V_{20}^*\left\{1 - 2K\right\}\right\}}\right]} \times 100$$

Findings are given in Table 5.1.

Table 1. *PRE*s of the estimators with respect to $\bar{y}_{st}$

S. No.	Estimator	$PRE(\bullet, \bar{y}_{st})$
1.	$\bar{y}_{st}$	100.00
2.	$\bar{y}_{RC}$ and $\bar{y}_{CRCe}$	313.70
3.	$\bar{y}_{RCe}$ and $\bar{y}_{SQRC}$	173.93
4.	$\bar{y}_{CRC}$	319.26
5.	$\bar{y}_{CCR\mathrm{Re}}$	431.89
6.	t (at optimum value of θ)	431.94
7.	t_M^*	432.03

It is observed from Table 1. that the proposed estimators $\bar{y}_{CRC}$ and $\bar{y}_{CCR\mathrm{Re}}$ are more efficient than $\bar{y}_{st}, \bar{y}_{RC}, \bar{y}_{CRCe}$ and Singh et al.[7] estimator $\bar{y}_{RCe}$ and the proposed $\bar{y}_{SQRC}$ with substantial gain in efficiency. It is also observed that there is very marginal gain in efficiency by using the optimum estimators $\left(t_{opt}, t_{Mopt}^*\right)$ over the proposed estimator $\bar{y}_{CCR\mathrm{Re}}$ which is at par with optimum estimator t_{opt}. Thus the proposed estimator $\bar{y}_{CCR\mathrm{Re}}$ is recommended for its use in practice as compared to optimum estimators $\left(t_{opt}, t_{Mopt}^*\right)$ because the optimum estimators t_{opt} depend on the unknown parameters.

6. Conclusion

The present paper deals with estimation of $\bar{Y}$ of y using auxiliary information in stratified random sampling. Some chain type estimators based on Kadilar and Cingi[1] and Singh and Pal[9] have been developed along with their properties upto first order of approximation. Inequalities are obtained under which the suggested chain-type estimators are more precise than other existing competitors. Further a class of chain type estimators for $\bar{Y}$ has been suggested along with their properties under large sample approximation. A large number of estimators can be shown as the members of the suggested class of chain-type estimators. Regions of preferences have been obtained in which the suggested class of chain-type estimators better than existing estimators. The niceness of the study is that it unifies several results at one place. An improved version t_M of class of chain-type estimators t is also given along with its properties. We have also carried out an empirical study to see the performance of the suggested estimators over other exiting estimators.

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