ISSN 0974 - 9373

Vol. 24 No. 1 (2020) **Journal of International Academy of Physical Sciences** pp. 13-24

*Ricci Solitons on Para-Sasakian Manifold

Sushil Shukla

Department of Mathematics
Uma Nath singh Institute of engineering and technology,
Veer Bahadur Singh Purvanchal University, Jaunpur 222001, India
Email: sushilcws@gmail.com

(Received December 26, 2019)

Abstract: The object of present paper is to study a special type of metric called *Ricci solitons on Para-Sasakian manifold.
Keywords: Ricci solitons, Para-Sasakian manifold, Einstein manifold.
2010 AMS Classification Number: 53C15, 53C25.

1. Introduction

Ricci soliton is a natural generalization of an Einstein metric and is defined on a Riemannian manifold (M, g). A Ricci soliton is a triple (g,V,λ) with g a Riemannian metric, V a vector field, and λ a real scalar such that

(1.1) $$L_V g + 2S + 2\lambda g = 0,$$

where S is a Ricci tensor of M and L_V denotes the Lie derivative operator along the vector field V.The Ricci soliton is said to be shrinking, steady, and expanding accordingly as λ is negative, zero, and positive, respectively[1]. In 1967, D. E. Blair[2] introduced the notion of quasi-Sasakian manifold to unify Sasakian and cosympletic manifolds and in 1977. The authors in[3–7] have studied Ricci solitons in contact and Lorentzian manifolds. G. Kaimakamis and K. Panagiotidou[8] initiated the notion of $*$-Ricci soliton where they essentially modified the definition of Ricci soliton by replacing the Ricci tensor *Ric* in (1.1) with the $*$-Ricci tensor Ric^*. A pseudo-Riemannian metric g on a smooth manifold M is called a $*$-Ricci soliton if there exists a smooth vector field V, such that

(1.2) $$\frac{1}{2}\left(\pounds_V g)(X,Y\right)+Ric^*\left(X,Y\right)=\lambda g(X,Y),$$

where

(1.3) $$Ric^*(X,Y)=\frac{1}{2}(trace\{\phi,R(X,\phi Y)\}),$$

for all vector fields X,Y on M

The notion of $*$-Ricci tensor was first introduced by S. Tachibana[9] on almost Hermitian manifolds and further studied by T. Hamada[10] on real hypersurfaces of non-flat complex space forms.

In the present paper, we have studied $*$-Ricci soliton on Para-Sasakian manifold and prove the following result:

Theorem 1.1: *Let $M(\varphi, \xi, \eta, g)$ be a $(2n+1)$-dimensional Para-Sasakian manifold. If g is a $*$-Ricci soliton on M, then either M is* D-*homothetic to an Einstein manifold, or the Ricci tensor of M with respect to canonical paracontact connection vanishes. In the first case, the soliton vector field is Killing and in the second case, the soliton vector field leaves φ invariant.*

2. Preliminaries

Let M be an almost contact manifold equipped with an almost contact metric structure (ϕ,ξ,η,g) consisting of a (1,1) tensor field ϕ, a vector field ξ, a 1-form η and a compatible Riemannian metric g satisfying

(2.1) $$\phi=-I+\eta\otimes\xi,\ \eta(\xi)=1,\ \phi(\xi)=0,\ \eta\circ\phi=0,$$

(2.2) $$g(X,Y)=g(\phi X,\phi Y)+\eta(X)\eta(Y),$$
$$g(X,\phi Y)=-g(\phi X,Y), g(X,\xi)=\eta(X),$$

for all $X,Y\in\chi(M)$.

An almost contact metric manifold M is a Para-Sasakian manifold if and only if

(2.3) $$(\nabla_X\phi)(Y) = -g(X, Y)\xi - \eta(Y)\phi(X) + 2\eta(X)\eta(Y), \ X, Y \in TM,$$

where ∇ is Levi-Civita connection of the Riemannian metric g.

From the above equation it follows that

(2.4) $$\nabla_X\xi = \phi(X), \quad X \in T(M),$$

(2.5) $$(\nabla_X\eta)Y = g(X, \phi Y) = (\nabla_Y\eta)X.$$

Moreover, the curvature tensor R and Ricci tensor S satisfy

(2.6) $$R(X, Y)\xi = \eta(X)Y - \eta(Y)X.$$

Let M be a three-dimensional Para-Sasakian manifold. The Ricci tensor S of M is given by

(2.7) $$S(\phi X, \phi Y) = S(X, Y) + (n-1)\eta(X)\eta(Y),$$

where R is the Riemannian curvature tensor and S is the Ricci tensor of type (0, 2) such that

(2.8) $$g(QX, Y) = S(X, Y),$$

where Q is the Ricci operator.

Lemma 2.1: *Let $M(\phi, \xi, \eta, g)$ be a Para-Sasakian manifold. Then (i) $\nabla_\xi Q = 0$, and (ii) $(\nabla_X Q)\xi = Q\phi X + \lambda\phi X$.*

Proof: Since ξ is Killing, we have $£_V Ric = 0$. This implies $(£_\xi Q)X = 0$ for any vector field X on M. From which it follows that

$$\begin{aligned} 0 &= £_\xi(QX) - Q(£_\xi X) \\ &= \nabla_\xi QX + \nabla_{QX}\xi - Q(\nabla_\xi X) + Q(\nabla_X\xi) \\ &= (\nabla_\xi Q)X + \nabla_{QX}\xi + Q(\nabla_X\xi). \end{aligned}$$

Using (2.4) in the above equation gives $\nabla_{\xi} Q = Q\phi - \phi Q$. Since the Ricci operator Q commutes with ϕ on Para-Sasakian manifold, we have (*i*). Next, taking covariant differentiation of (2.8) along an arbitrary vector field X on M and using (2.4), we obtain (*ii*). This completes the proof.

If the Ricci tensor of a Para-Sasakian manifold M is of the form

$$Ric(X, Y) = Ag(X, Y) + B\eta(X)\eta(Y),$$

for any vector fields X, Y on M, where A and B being constants, then M is called an $\eta-$Einstein manifold.

The 1-form η is determined up to a horizontal distribution and hence $D = Ker\,\eta$ connected by $\tilde{\eta} = \sigma\eta$ *for* a positive smooth function σ on a paracontact manifold M. This paracontact form $\bar{\eta}$ defines the structure tensor $(\bar{\phi}, \bar{\xi}, \bar{g})$ corresponding to η using the condition given in the paper[11]. We call the transformation of the structure tensors given by Lemma 4.1 of[11] a gauge (conformal) transformation of paracontact pseudo-Riemannian structure. When σ is constant this is a D-homothetic transformation. Let $M(\phi, \xi, \eta, g)$ be a paracontact manifold and

$$\bar{\phi} = \phi, \bar{\xi} = \frac{1}{\alpha}\xi, \bar{\eta} = \alpha\eta, \bar{g} = \alpha g + (\alpha^2 - \alpha)\eta \otimes \eta \;\; \alpha = \text{const.} \neq 0$$

to be D-homothetic transformation. Then $(\bar{\phi}, \bar{\xi}, \bar{\eta}, \bar{g})$ is also a para contact structure. Using the formula appeared in11 for D-homothetic deformation, one can easily verify that if $M(\phi, \xi, \eta, g)$ is a $(2n+1)-$dimensional $(n > 1)$ $\eta-$Einstein Para-Sasakian structure with scalar curvature $r \neq 2n$, then there exists a constant α such that $M(\bar{\phi}, \bar{\xi}, \bar{\eta}, \bar{g})$ is an Einstein Para-Sasakian structure. So we have following result.

Lemma 2.2: *Any* $(2n+1)-$*dimensional* $\eta-$*Einstein Para-Sasakian manifold with scalar curvature not equal to* $2n$ *is* D-*homothetic to an Einstein manifold.*

3. Proof of Theorem

First, we state and prove some lemmas which will be used to prove Theorem.

Lemma 3.1: *The* $*$*-Ricci tensor on a* $(2n+1)-$*dimensional Para-Sasakian manifold* $M(\phi, \xi, \eta, g)$ *is given by*

(3.1) $$Ric^{*}(X,Y) = -Ric(X,Y) - (2n-1)g(X,Y) - \eta(X)\eta(Y),$$

for any vector fields X, Y on M .

Proof: The Ricci tensor *Ric* of a $(2n+1)-$dimensional Para-Sasakian manifold $M(\phi, \xi, \eta, g)$ satisfies the relation (c.f. Lemma 3.15 in[11]:

(3.2) $$Ric(X,Y) = \sum_{i=1}^{2n+1} R'(X,\phi Y, e_i, \phi e_i) - (2n-1)g(X,Y) - \eta(X)\eta(Y),$$

for any vector fields X, Y on M. By the skew-symmetric property of ϕ, we have

$$\sum_{i=1}^{2n+1} R'(X,\phi Y, e_i, \phi e_i) = \sum_{i=1}^{2n+1} R(X,\phi Y, e_i, \phi e_i) = \sum_{i=1}^{2n+1} g(\phi R(X,\phi Y), e_i, e_i)$$

By this, (3.2) becomes

(3.3) $$\sum_{i=1}^{2n+1} g(\phi R(X,\phi Y), e_i, e_i) = -2Ric(X,Y) - 2(2n-1)g(X,Y) - 2\eta(X)\eta(Y).$$

By (1.3) and (3.3), we have (3.1).

Lemma 3.2: *For a Para-Sasakian manifold, we have the following relation*

(3.4) $$(\pounds_V \eta)(\xi) = -\eta(\pounds_V \xi) = \lambda.$$

Proof: By virtue of Lemma 3.1, the $*$-Ricci soliton equation (1.2) can be expressed as

(3.5) $$(\pounds_V g)(X,Y) = 2Ric(X,Y) + 2(2n-1+\lambda)g(X,Y) + 2\eta(X)\eta(Y).$$

Taking $Y=\xi$ in (3.5) and using (2.7) we have $(\pounds_V g)(X,\xi)=2\lambda\eta(X)$. Lie-differentiating the equation $\eta(X)=g(X,\xi)$ along V and by (3.5), we have

(3.6) $$(\pounds_V \eta)(X)-g(\pounds_V \xi, X)-2\lambda\eta(X)=0.$$

Now, Lie-derivative of $g(\xi,\xi)=1$ along V and equation (3.6) completes proof.

Lemma 3.3: *Let $M(\phi,\xi,\eta,g)$ be a $(2n+1)-$dimensional Para-Sasakian manifold. If g is a $*$-Ricci soliton, then M is an $\eta-$Einstein manifold and the Ricci tensor can be written as*

(3.7) $$Ric(X,Y)=-\left[2n-1+\frac{\lambda}{2}\right]g(X,Y)+\left[\frac{\lambda}{2}-1\right]\eta(X)\eta(Y),$$

for any vector fields X, Y on M.

Proof: Taking covariant differentiation of (3.5) along an arbitrary vector field Z, we get

(3.8) $$(\nabla_Z \pounds_V g)(X, Y)=2\left\{(\nabla_Z Ric)(X,Y)-g(X,\phi Z)\eta(Y)-g(Y,\phi Z)\eta(X)\right\}.$$

According to Yano[12], we have

$$(\pounds_V \nabla_Z g-\nabla_Z \pounds_V g-\nabla_{[V,Z]g})(X, Y)$$

$$=-g((\pounds_V \nabla)(Z, X),Y)-g((\pounds_V \nabla)(Z, Y), X),$$

for any vector fields X, Y, Z on M.
In view of the parallelism of the pseudo-Riemannian metric g, we have from above relation

(3.9) $$(\nabla_Z \pounds_V g)(X,Y)=g((\pounds_V \nabla)(Z,X),Y)+g((\pounds_V \nabla)(Z,Y),X).$$

From (3.8) and (3.9), we have

(3.10) $$g((\pounds_V \nabla)(Z, X),Y)+g((\pounds_V \nabla)(Z, Y), X)$$

$$= 2\left\{(\nabla_Z Ric)(X,\ Y) - g(X,\ \phi Z)\eta(Y) - g(Y,\ \phi Z)\eta(X)\right\}.$$

Which gives

(3.11) $$g((\pounds_V \nabla)(X,\ Y),\ Z) = -(\nabla_Z Ric)(X,Y) + (\nabla_X Ric)(Y,Z)$$
$$+(\nabla_Y Ric)(Z,X) + 2g(X,\phi Z)\eta(Y) + 2g(Y,\phi Z)\eta(X).$$

Taking ξ in place of Y in (3.11) and Lemma 2.1, we get

(3.12) $$(\pounds_V \nabla)(X,Y) = 2(2n-1)\phi X + 2Q\phi X.$$

Differentiating (3.12) covariantly along an arbitrary vector field Y on M and using the relations (2.3) and (2.8), we have

(3.13) $$(\nabla_Y \pounds_V \nabla)(X,\xi) + (\pounds_V \nabla)(X,\varphi Y)$$
$$= 2\left\{(\nabla_Y Q)\varphi X + \eta(X)QY + (2n-1)\eta(X)Y + g(X,\ Y)\xi\right\}.$$

According to Yano [12] we have

(3.14) $$(\pounds_V R)(X,Y)Z = (\nabla_X \pounds_V \nabla)(Y,Z) - (\nabla_Y \pounds_V \nabla)(X,Z).$$

Taking ξ in place of Z in (3.14) and by (3.13), we have

(3.15) $$(\pounds_V R)(X,Y)\xi + (\pounds_V \nabla)(Y,\varphi X) - (\pounds_V \nabla)(X,\varphi Y)$$
$$= 2\{(\nabla_X Q)\varphi Y - (\nabla_Y Q)\varphi X + \eta(Y)QX - \eta(X)QY$$
$$+(2n-1)(\eta(Y)X - \eta(X)Y)\}.$$

Taking ξ for Y in (3.15), then using (2.8), (3.12) and Lemma 2.1, we have (3.16)

$$(\pounds_V R)(X,\xi)\xi = 4\left\{QX + (2n-1)X + \eta(X)\xi\right\}.$$

Taking Lie-derivative of (2.6) along V and by (2.5) and (3.4) we have

(3.17) $$(\pounds_V R)(X,\xi)\xi = (\pounds_V \eta)(X)\xi - g(\pounds_V X,\xi) - 2\lambda X.$$

Comparing (3.16) with (3.17), and use of (3.6), gives the required result.

Proof of Theorem: By (3.7), the soliton equation (3.5) can be written as

(3.18) $$(\pounds_V g)(X,Y)=\lambda\left\{g(X,Y)+\eta\left(X\right)\eta(Y)\right\}.$$

Taking Lie-differentiation of (3.7) along the vector field V and using (3.5) we have

(3.19) $$(\pounds_V Ric)(X,Y)=\left(\frac{\lambda}{2}-1\right)\left\{\eta(Y)(\pounds_V\eta)(X)+\eta(X)(\pounds_V\eta)(Y)\right\}$$
$$-\left[\frac{\lambda}{2}+2n-1\right]\lambda\left\{g(X,Y)+\eta(X)\eta(Y)\right\}.$$

Differentiating (3.7) covariantly along an arbitrary vector field Z on M and using (2.4) we have

(3.20) $$(\nabla_Z Ric)(X,Y)=\left(1-\frac{\lambda}{2}\right)\left\{g(X,\phi Z)\eta(Y)+g(Y,\phi Z)\eta(X)\right\}.$$

By (3.20), equation (3.11) becomes

(3.21) $$(\pounds_V\nabla)(X,Y)=-\lambda\{\eta(Y)\ \varphi X+\eta\left(X\right)\varphi Y\}.$$

Differentiating (3.21) covariantly along an arbitrary vector field Z on M and by (2.3) and (2.4), we have

(3.22) $$(\nabla_Z\pounds_V\nabla)\left(X,Y\right)=\lambda\{g\left(Y,\ \phi Z\right)\phi X+g\left(X,\ \phi Z\right)\varphi Y$$
$$+g\left(X,\ Z\right)\eta\left(Y\right)\xi+\ g(Y,Z)\eta\left(X\right)\xi-2\eta\left(X\right)\eta(Y)\ Z\}.$$

Using (3.22) in (3.14) and using (2.4) we have

(3.23) $$\left(\pounds_V R\right)\left(X,Y\right)Z=\lambda\{g(\phi X,Z)\phi Y-g(\phi Y,Z)\phi X+2g(\phi X,Y\left)\phi Z\right.$$
$$+g(X,Z)\eta(Y)\xi-g(Y,Z)\eta\left(X\right)\xi-2\eta(Y)\eta\left(Z\right)X+2\eta\left(X\right)\eta\left(Z\right)Y\}.$$

Contracting (3.23) over Z, we get

(3.24) $$\left(\pounds_V Ric\right)(Y,Z)=2\lambda\left\{g(Y,Z)-\left(2n+1\right)\eta(Y)\eta\left(Z\right)\right\}.$$

By (3.19) and (3.24), we have

$$(3.25)\qquad \left(\frac{\lambda}{2}-1\right)\{\eta(Y)(£_V\eta)(Z)+\eta(Z)(£_V\eta)(Y)\}$$
$$-\left[\frac{\lambda}{2}+2n-1\right]\lambda\{g(Y,Z)+\eta(Y)\eta(Z)\}$$
$$=2\lambda\{g(Y,Z)-(2n+1)\eta(Y)\eta(Z)\}.$$

Replacing Y by $\phi^2 Y$ in (3.25) and then using (2.1) and (3.4) we get

$$(3.26)\qquad \left(\frac{\lambda}{2}-1\right)\left\{(£_V\eta)(Y)\eta(Z)=\lambda\left[1+2n+\frac{\lambda}{2}\right]g(Y,Z)-2n\lambda\eta(Y)\eta(Z)\right\}.$$

By (3.26) and (3.25) and then replacing Z by ϕZ, we have

$$(3.27)\qquad \lambda\left[1+2n+\frac{\lambda}{2}\right]g(Y,\phi Z)=0.$$

As $\phi(Y,Z)=g(Y,\phi Z)$ is non-vanishing everywhere on M, so either $\lambda=0$ or $\lambda=-2(2n+1)$.

Case I: If $\lambda=0$, from (3.18) we have $£_V g=0$, therefore, V is Killing. From (3.7) we have

$$(3.28)\qquad Ric(X,Y)=-(2n-1)g(X,Y)-\eta(X)\eta(Y).$$

Contracting the equation (3.28) we have $r=-4n^2$, where r is the scalar curvature of the manifold M. This shows that M is a $\eta-$Einstein manifold with scalar curvature $r\neq 2n$. So, M is $D-$homothetic to an Einstein manifold.

Case II: If $\lambda=-2(2n+1)$, then taking ξ in place of Z in (3.26) and then replace Y by ϕY the resulting equation gives

$$\left(\frac{\lambda}{2}-1\right)(£_V\eta)(\phi Y)=0.$$

Since $\lambda=-2(2n+1)$, we have $\lambda\neq 2$. Thus we have $(£_V\eta)(\varphi Y)=0$.

Replacing Y by ϕY and using (2.1), we have

(3.29) $$(\pounds_V \eta)(Y) = -2\ (2n+1)\eta\left(X\right).$$

Taking exterior differentiation d on (3.29) we have

(3.30) $$\left(\pounds_V d\eta)(X,Y\right) = -2\left(2n+1)g(X,\phi Y\right),$$

as d commutes with $\pounds_V$.

Taking the Lie-derivative of $d\eta(X,Y) = g(X,\phi Y)$ along the soliton vector field V provides

(3.31) $$\left(\pounds_V d\eta)(X,Y) = \left(\pounds_V g)(X,\phi Y\right) + g(X,\left(\pounds_V \phi\right)Y\right).$$

From (3.18) we have

(3.32) $$\left(\pounds_V g)(X,\phi Y\right) = -2\left(2n+1)g(X,\phi\right).$$

Using (3.30) and (3.32) in (3.31) we have $\pounds_V \phi = 0$. Therefore, soliton vector field V leaves ϕ invariant. Putting $\lambda = -2\left(2n+1\right)$ in (3.7) we have

(3.33) $$Ric\left(X,Y) = 2g(X,Y\right) - \left(2n+2)\eta\left(X\right)\eta(Y\right).$$

Contracting (3.33) we obtain $r = 2n$ (i.e., the manifold M cannot be $D-$ homothetic to an Einstein manifold. Ricci tensor $Ri\tilde{c}$ of a $\left(2n+1\right)$ dimensional Para-Sasakian manifold with respect to canonical paracontact connection $\tilde{\nabla}$ is defined as[11]

(3.34) $$Ri\tilde{c}\left(X,Y\right) = Ric\left(X,Y\right) - 2g\left(X,Y\right) + \left(2n+2\right)\eta\left(X\right)\eta\left(Y\right).$$

Using (3.33) in (3.34) we have $Ri\tilde{c}\left(\tilde{X},Y\right) = 0$. Therefore, the Ricci tensor with respect to the connection $\tilde{\nabla}$ vanishes. This completes the proof of theorem.

References

1. B. Chow, P. Lu, and L. Ni, Hamilton's *Ricci Flow*, vol. 77 of Graduate Studies in Mathematics, American Mathematical Society, Providence, RI, USA, 2006.

2. D. E. Blair, Theory of Quasi-Sasakian Structure, *J. Differential Geom.*, **1** (1967), 331-345.

3. A. M. Blaga, M. C. Crasmareanu, Torse-Forming $\eta-$Ricci Solitons in Almost Para-Contact $\eta-$Einstein Geometry, *Filomat*, **31(2)** (2017), 499–504.

4. M. Brozos-Vazquez, G. Calvaruso, E. Garcia-Rio and S. Gavino-Fernandez, Three-Dimensional Lorentzian Homogeneous Ricci Solitons, *Israel J. Math*., **188** (2012), 385–403.

5. G. Calvaruso and A. Fino, Four-Dimensional Pseudo-Riemannian Homogeneous Ricci Solitons, *Int. J. Geom. Methods Mod. Phys*., **12** (2015), 1550056.

6. G. Calvaruso and D. Perrone, Geometry of H-Paracontact Metric Manifolds, *Publ. Math. Debrecen*, **86** (2015), 325–346.

7. G. Calvaruso and A. Zaeim, A Complete Classification of Ricci and Yamabe Solitons of Non-Reductive Homogeneous, *J. Geom. Phys*., **80** (2014), 15–25.

8. G. Kaimakamis and K. Panagiotidou, $*$-Ricci Solitons of Real Hypersurfaces in Non-Flat Complex Space Forms, *J. Geom. Phys.*, **86** (2014), 408–413.

9. S. Tachibana, On Almost-Analytic Vectors in Almost Kahlerian Manifolds, *Tohoku Math. J*., **11** (1959), 247–265.

10. T. Hamada, Real Hypersurfaces of Complex Space Forms in Terms of Ricci $*$- Tensor, *Tokyo J. Math*., **25** (2002), 473-483.

11. S. Zamkovoy, Canonical Connections on Paracontact Manifolds, *Ann. Glob. Anal. Geom.*, **36(1)** (2009), 37–60.

12. K. Yano, *Integral Formulas in Riemannian Geometry*, Marcel Dekker, New York, 1970.

13. Sushil Shukla, On Relativistic Fluid Space Time Admitting Heat Flux of a Generalized Recurrent and Ricci Recurrent Kenmotsu Manifold, *Journal of International Academy Of Physical Sciences*, **15** (2011), 143-146.

14. Sushil Shukla, On Kenmotsu Manifold, *Journal of Ultra Scientist of Physical Sciences*, **21** (2009), 485-490 .

15. Uday Chand De, Ahmet Yildiz, Mine Turan and Bilal E. Acet, 3-Dimensional Quasi-Sasakian Manifolds with Semi-Symmetric Non-Metric Connection , *Hacettepe Journal of Mathematics and Statistics*, **41(1)** (2012), 127–137.

16. D. E. Blair, Riemannian Geometry of Contact and Sympletic Manifolds, *Progress in Mathematics*, **203** (2002).

17. U. C. De and A. K. Mondal, Three Dimensional Quasi-Sasakian Manifolds and Ricci Solitons, *SUT J. Math*., **48(1)** (2012), 71–81.

18. K. T. Pradeep, Venkatesh, and C. S. Bagawadi, On ϕ-Recurrent Para-Sasakian Manifold Admitting Quarter Symmetric Metric Connection, *ISRN Geometry*, (2012), Article ID 317253.

19. Z. Olszak, Normal Almost Contact Metric Manifolds of Dimension Three, *Ann. Polon.*

Math., **47** (1986), 41-50.

20. S. K. Hui , On Pseudo Symmetric Para-Sasakian Manifold, *Acta Universities Apulensis,* **39** (2014), 161-178.

21. M. D. Siddiqi, Conformal η-Ricci Solitons in δ-Lorentzian Trans Sasakian Manifolds, *Int. J. Maps Math. (IJMM.),* **1** (2018), 15–34.

22. M. M. Tripathi, Ricci Solitons in Contact Metric Manifolds, *arXiv:0801.4222*.

23. M. D. Siddiqi, Generalized Ricci Soliton on Trans Sasakian Manifolds, *Khayyam J. Math.,* **4** (2018), 178–186.

24. M. Turan, C Yetima and S. K. Chaubey, On Quasi-Sasakian 3-Manifolds Admitting η-Ricci Solitons, *Filomat*, **33** (2019), 4923–4930.

25. A. Sarkar, A Sil and A. K. Paul, Ricci Almost Solitons on Three-Dimensional Quasi-Sasakian Manifolds, *Proc. Nat. Inst. Sci. India*, **89** (2019), 705-710.