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# Magnetohydrodynamic Flow of Dusty Fluids between Two Inclined Co - axial Cylinders

**R. K. Khare and Jyoti Singh**
Department of Mathematics & Statistics
Sam Higginbottom Institute of Agriculture, Technology & Sciences, Allahabad
Email: drrajeevkhare@gmail.com

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**Abstract:** In this paper, we have studied the MHD flow of a dusty viscous non Newtonian fluid flowing between two inclined co-axial cylinders such that the flow is influenced by gravity. Within the frame work of some physically realistic approximation, derivations for velocity profile of both phases with suitable boundary conditions have been obtained by varying the magnetic field and inclination of the co-axial cylinders. The result shows that on increasing the magnetic field, the velocities of both phases increase in the beginning and taking a maximum value, they start decreasing for every inclination of the co-axial cylinders. The results have got remarkable applications in various fields involving nuclear reactor cooling, powder technology, sedimentation, flow of oil, gases and molten metals as these flows occur between two inclined co-axial cylinders.

**Keywords:** MHD flow, Newtonian fluid, magnetic field
**2010 Mathematics Subject Classification No**.: 76W05

## 1. Introduction

The study of flow of dusty fluids through annular pipes has got wide applications in many industries as this helps in designing the cooling system of reactors, purification of water and performance of power generators. Due to its growing applications, it has drawn attention of several authors. Micheal and Norey[1] studied the motion of a dusty gas contained between two co axial cylinders rotated impulsively from rest. Singh studied the flow of a dusty gas through the annular space between two concentric cylinders with uniform dust particles. Rukmagadachari[2] discussed dusty viscous flow between oscillating co axial cylinders. Guha[3] studied the flow of a dusty viscous conducting gas through the annular space between two co axial cylinders under the influence of a uniform magnetic field. Gupta and Gupta[4] investigated the unsteady flow of a dusty visco elastic fluid through a coaxial pipe. Rao and Murthy[5] gave a numerical study of microstructure fluid through concentric pipes. Khare and Avinash[6] examined the MHD flow of a dusty viscous fluid through a co axial cylinder. In this paper we have studied the motion of an incompressible dusty fluid through a co axial circular cylindrical pipe placed under a transverse magnetic field such that the flow is affected by gravity and the expressions for fluid and dust velocity have been obtained and corresponding effects have also been discussed with the help of graphs.

## 2. Formulation of the problem and basic equation

Consider the flow of a dusty viscous incompressible fluid bounded by two infinitely long inclined co axial circular cylinders in which $r_1$ and $r_2$ are the radii of the outer and inner cylinders respectively placed under transversely applied magnetic field. The flow is occurring along the axis of the channel. Considering the flow to be fully developed and symmetric and the velocity of fluid phase and particle phase are the function of radial distance r and time t only.

The equations of motion of conducting unsteady viscous incompressible fluid with uniform distribution of dust particles are given by:

$$(2.1)\ -\frac{1}{\rho}\frac{\partial p}{\partial r}=0,$$

$$(2.2)\ \frac{\partial u_z}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left[\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}\right]+\frac{kN}{\rho}\left(v_z-u_z\right)-\frac{\sigma B_0^2}{\rho\left(1+m^2\right)}u_z+g_z\sin\theta,$$

$$(2.3)\ \frac{\partial v_z}{\partial t}=\frac{k}{m}\left(u_z-v_z\right)+g_z\sin\theta,$$

where

$u \rightarrow$ velocity of the fluid phase,

$\vec{v} \rightarrow$ velocity density of the dust phase,

$\rho \rightarrow$ density of the gas,

$p \rightarrow$ pressure of the fluid,

$N \rightarrow$ number of density of dust particles,

$\nu \rightarrow$ kinematics viscosity,

$k = 6\pi a\mu \rightarrow$ Stoke's resistance (drag coefficient),

$a \rightarrow$ spherical radius of dust particle,

$m \rightarrow$ mass of the dust particle,

$\mu \rightarrow$ the co-efficient of viscosity of fluid particles,

$t \rightarrow$ time,

$\theta \rightarrow$ inclination of the channel with the horizontal.

$\sigma \rightarrow$ the electrical conductivity of the fluid

Boundary conditions are given by

$$(2.4) \qquad \begin{cases} u_r = 0; u_\theta = 0; u_z = u_z(r,t); \\ v_r = 0; v_\theta = 0; v_z = v_z(r,t); \end{cases}$$

where $(u_r, u_\theta, u_z)$ and $(v_r, v_\theta, v_z)$ are the velocity components of fluid and dust particles respectively.

Let us introduce the following non – dimensional quantities;

$$(2.5) \begin{cases} R = \dfrac{r}{(r_1 - r_2)}, \bar{z} = \dfrac{z}{(r_1 - r_2)}, \bar{p} = \dfrac{p(r_1 - r_2)^2}{\rho \nu^2}, T = \dfrac{t\nu}{(r_1 - r_2)^2}, u = \dfrac{u_z(r_1 - r_2)}{\nu}, \\ v = \dfrac{v_z(r_1 - r_2)}{\nu}, \beta = \dfrac{l}{\gamma} = \dfrac{Nk(r_1 - r_2)^2}{\rho \nu}, l = \dfrac{Nm}{\rho}, \gamma = \dfrac{\nu m}{k(r_1 - r_2)^2}, g = \dfrac{g_z(r_1 - r_2)}{\nu} \end{cases}.$$

Now we transform the equations (2.1) to (2.3) to the non dimensional form which become

$$(2.6) \qquad -\frac{1}{\rho}\frac{\partial p}{\partial r} = -\frac{\nu^2}{(r_1 - r_2)^3}\frac{\partial \bar{p}}{\partial R} = 0,$$

$$(2.7) \; \frac{\partial u}{\partial T} = -\frac{\partial \bar{p}}{\partial \bar{z}} + \left(\frac{\partial^2 u}{\partial R^2} + \frac{1}{R}\frac{\partial u}{\partial R}\right) + \beta(v - u) - \frac{M^2}{(1 + m^2)}u + g\frac{(r_1 - r_2)^2}{\nu}\sin\theta,$$

$$(2.8) \qquad \gamma\frac{\partial v}{\partial T} = (u - v) + g\frac{m}{k}\sin\theta,$$

where $M = B_0(r_1 - r_2)\sqrt{\left(\sigma/\mu\right)}$ = the Hartmann number.

Since we have assumed that the time dependent pressure gradient to be impressed on the system for t >0, so we can write

$$-\left(\frac{\partial \bar{p}}{\partial \bar{z}}+\gamma\frac{\partial}{\partial T}\left(\frac{\partial \bar{p}}{\partial \bar{z}}\right)\right)=C+De^{\alpha T}$$

where $C$, $D$ and $\alpha$ are real.

Eliminating $v$ from (2.7) and (2.8) and then substituting the expression for pressure gradient, one can get

$$(2.9)\quad \gamma\frac{\partial^2 u}{\partial T^2}-\gamma\frac{\partial}{\partial T}\left(\frac{\partial^2 u}{\partial R^2}+\frac{1}{R}\frac{\partial u}{\partial R}\right)+\left(\gamma\frac{M^2}{(1+m^2)}+l+1\right)\frac{\partial u}{\partial T}=C+De^{\alpha T}+\left(\frac{\partial^2 u}{\partial R^2}+\frac{1}{R}\frac{\partial u}{\partial R}\right)-\frac{M^2}{(1+m^2)}u+g\frac{(r_1-r_2)^2}{\nu}(l+1)\sin\theta.$$

Let the solution of the equation (2.9) be

$$(2.10)\qquad u=U(R)+V(R,T),$$

where $U$ is steady part and $V$ is unsteady part of the fluid velocity.

Separating the equation (2.9)

$$(2.11)\qquad \left(\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}\right)-\frac{M^2}{(1+m^2)}U+g\frac{(r_1-r_2)^2}{\nu}(l+1)\sin\theta=-C.$$

$$(2.12)\qquad \gamma\frac{\partial^2 V}{\partial T^2}-\gamma\frac{\partial}{\partial T}\left(\frac{\partial^2 V}{\partial R^2}+\frac{1}{R}\frac{\partial V}{\partial R}\right)+\left(\gamma\frac{M^2}{(1+m^2)}+l+1\right)\frac{\partial V}{\partial T}$$
$$=De^{\alpha T}+\left(\frac{\partial^2 V}{\partial R^2}+\frac{1}{R}\frac{\partial V}{\partial R}\right)-\frac{M^2}{(1+m^2)}V+g\frac{(r_1-r_2)^2}{\nu}(l+1)\sin\theta.$$

By solving the equation (2.11) with initial conditions

$$U=0,\ at\ R=1 \text{ and } U=finite,\ at\ R=0,$$

which give the solution as

$$(2.13)\qquad U=\frac{(1+m^2)}{M^2}\left(C+g\frac{(r_1-r_2)^2}{\nu}(l+1)\sin\theta\right)\left[1-\frac{J_0\left(\frac{RM}{\sqrt{1+m^2}}\right)}{J_0\left(\frac{M}{\sqrt{1+m^2}}\right)}\right],$$

where $J_0$ is Bessel's function of zeroth order.

Assume the solution of the equation (2.12) in the form

(2.14) $$V = h(R)e^{\alpha T},$$

where $h(R)$ is an unknown function to be determined.

Now we obtained the solution of the equation (2.14) with the following boundary conditions:

(2.15) $$U = V \; at \, T = 0, V = 0 \; at \, R = 1, V = 0 \; at \, R = 0.$$

Using equation (2.15) in (2.12), we get

(2.16) $$\frac{\partial^2 h(R)}{\partial R^2} + \frac{1}{R}\frac{\partial h(R)}{\partial R} + \lambda_1 h(R) = \lambda_2,$$

where

$$\frac{\left[\gamma\alpha^2 + \frac{M^2}{(1+m^2)} + \left(\gamma\frac{M^2}{(1+m^2)} + l + 1\right)\alpha\right]}{(1+\gamma\alpha)} = -\lambda_1$$

and

$$\frac{D + g\frac{(r_1 - r_2)^2}{\nu e^{-\alpha T}}(l+1)\sin\theta}{(1+\gamma\alpha)} = -\lambda_2.$$

(2.17) $$h(R) = \frac{\lambda_2}{\lambda_1}\left[1 - \frac{J_0(\lambda_1 R)}{J_0(\lambda_1)}\right].$$

Using this in equation (2.14), we get V as

(2.18) $$V = e^{\alpha T}\frac{\lambda_2}{\lambda_1^2}\left[1 - \frac{J_0(\lambda_1 R)}{J_0(\lambda_1)}\right].$$

Now using equations (2.13) and (2.18) in (2.10), we obtain the fluid velocity u as,

$$(2.19) \qquad u = \frac{\left(1+m^2\right)}{M^2}\left(C + g\frac{\left(r_1 - r_2\right)^2}{\nu}\left(l+1\right)\sin\theta\right)\left[1 - \frac{J_0\left(\frac{RM}{\sqrt{1+m^2}}\right)}{J_0\left(\frac{M}{\sqrt{1+m^2}}\right)}\right]$$

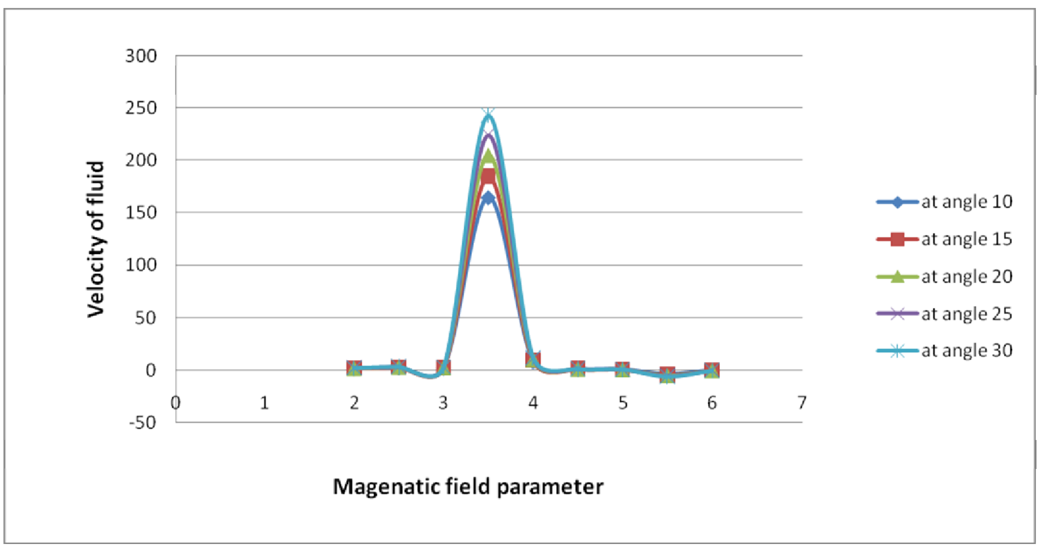
$$+ e^{\alpha T}\frac{\lambda_2}{\lambda_1}\left[1 - \frac{J_0\left(\lambda_1 R\right)}{J_0\left(\lambda_1\right)}\right].$$

Also the dust phase velocity is obtained from equation (2.13) as

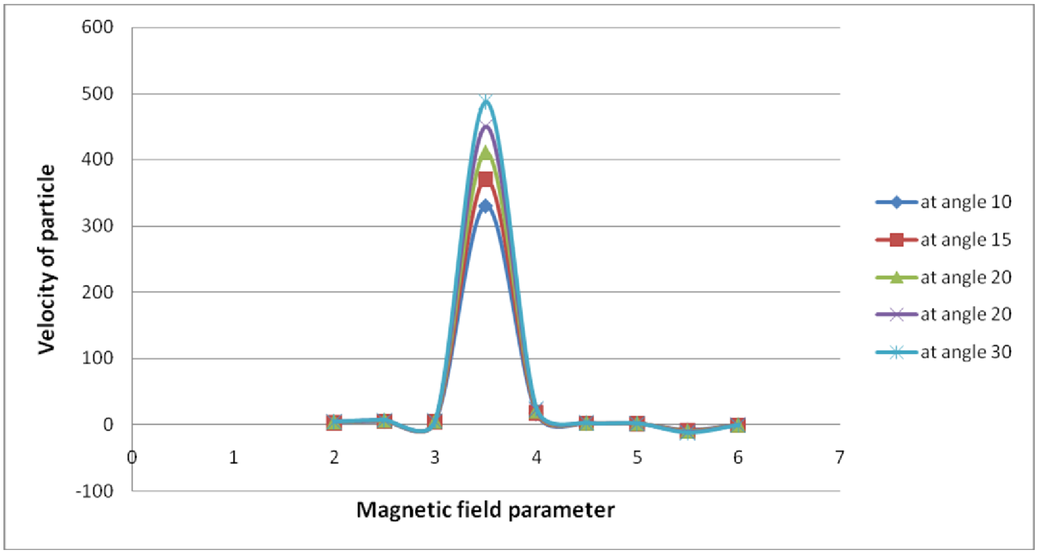
$$(2.20) \qquad v = \frac{\left(1+m^2\right)}{M^2}\left(C + g\frac{\left(r_1 - r_2\right)^2}{\nu}\left(l+1\right)\sin\theta\right)\left[1 - \frac{J_0\left(\frac{RM}{\sqrt{1+m^2}}\right)}{J_0\left(\frac{M}{\sqrt{1+m^2}}\right)}\right]\left(1 - e^{\frac{1}{\gamma}T}\right)$$

$$+ e^{\alpha T}\frac{\lambda_2}{\lambda_1}\left[1 - \frac{J_0\left(\lambda_1 R\right)}{J_0\left(\lambda_1\right)}\right]\left(e^{\alpha T} - e^{\frac{1}{\gamma}T}\right).$$

Now, we draw graphs between the velocity of both phases and magnetic field parameter by assuming suitable values of different terms present in their expressions.




**Graph between Velocity of Fluid and Magnetic Field Parameter**




**Graph between Velocity of Particle & Magnetic Field Parameter**

## Result & Discussion

(1) The study indicates that for every inclination of the cylinder, both velocities show a resonance character which occurs nearly magnetic field parameter ($B_0$ = 3.5) and that has been verified mathematically also.

(2) The value of the velocity in case of particle is almost double in comparison to that of fluid. The reason is clear that magnetic field is more effective on magnetic sensitive particle.

(3) Both graphs are appearing similar in nature but the magnitudes of the changes due to parameter are different.

(4) Also the maximum/minimum velocities for both phases are occurring at the same inclinations.

Thus, the results obtained may be applied in the aforesaid mentioned field having multiphase flow between two coaxial inclined cylinders placed in magnetic field.

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