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A Related Fixed Point Theorem for Two Pairs of Mappings on Two Complete Fuzzy Metric Spaces

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Abstract: A related fixed point theorem for two pairs of mappings on two complete fuzzy metric spaces is obtained. Our result generalizes some results in the literature.

Keywords: Fuzzy Metric space, fixed point, related fixed point.

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1. Introduction and Preliminaries

The concept of fuzzy sets was introduced by L. Zadeh[1] in 1965. The concept is used in topology and analysis. It is used and developed by many mathematicians. Fuzzy set is applied in the field of decision making, population dynamics, computer science, artificial intelligence, operational research, industrial engineering, pattern recognition, medicine, group health underwriting, management science[2, 3, 4, 5, 6, 7, 8, 9, 10, 11] and many others. There was a remarkable and progressive development in the field of fuzzy topology in which one of the most important problems is to obtain a clear concept of fuzzy metric space which has been investigated by many authors in different ways. Particularly, Geroge and Veeramani[12] have introduced and studied an interesting notion of fuzzy metric space. Fuzzy metric space was introduced by Kramosil and Michalek[13] in 1975. Then, it was modified by George and Veeramani[12] in 1994. Related fixed point theorems were studied in[14, 15, 16, 17, 18, 19] and many others.

We list some definitions as follows.

1.1. Definition. [20] A binary operation $*:[0,1]\to[0,1]$ is continuous t-norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative,

(ii) $*$ is continuous,

(iii) $a*1=a \ \forall\, a\in[0,1]$,

(iv) $a*b\le c*d$ whenever $a\le c$, $b\le d$ and $a,b,c,d\in[0,1]$.

1.2. Definition. [12] The 3-tuple $(X,\mu,*)$ is called a fuzzy metric space if X is an arbitrary non-empty set, $*$ is a continuous t-norm and μ is a fuzzy set in $X^2\times(0,\infty)$ satisfying the following conditions :

(i) $\mu(x, y, t) > 0$,

(ii) $\mu(x, y, t) = 1$ if and only if $x = y$,

(iii) $\mu(x, y, t) = \mu(y, x, t)$,

(iv) $\mu(x, y, s) * \mu(y, z, t) \le \mu(x, z, t+s)$,

(v) $\mu(x; y; \cdot):(0,\infty)\to(0,1]$ is continuous
for all $x, y, z \in X$ and $t, s > 0$.

1.3. Definition. [21] Let $(X,\mu,*)$ be a fuzzy metric space. A sequence $\{x_n\}_n$ in X is said to converge to $x\in X$ if and only if $\lim\limits_{n\to\infty}\mu(x_n,x,t)=1$, for each $t>0$.

A sequence $\{x_n\}_n$ in X is called Cauchy sequence if and only if $\lim\limits_{n\to\infty}\mu(x_n,x_{n+p},t)=1$, for each $t>0$ and $p = 1, 2, 3,$.

A fuzzy metric space $(X,\mu,*)$ is said to be complete if and only if every Cauchy sequence in X is convergent in X.

The following theorem was proved in[22].

1.4. Theorem: *Let (X, d) and (Y,σ) be complete metric spaces. Let A, B be mappings of X into Y and S, T be mappings of Y into X satisfying the inequalities*

$$d(Sy,Ty')\,d(SAx,TBx')\le c\max\left\{\begin{array}{l} d(Sy,Ty')\sigma(Ax,Bx'),\ d(x',Sy)\sigma(y',Ax),\\ d(x,x')d(Sy,Ty'),\ d(Sy,SAx)d(Ty',TBx')\end{array}\right\}$$

$$\sigma(Ax,Bx')\sigma(BSy,ATy')\le c\max\left\{\begin{array}{l} d(Sy,Ty')\sigma(Ax,Bx'),\ d(x',Sy)\sigma(y',Ax),\\ \sigma(y,y')\sigma(Ax,Bx'),\ \sigma(Ax,BSy)\sigma(Bx',ATy')\end{array}\right\}$$

for all $x,x'\in X$ and $y,y'\in Y$, where $0\le c<1$. If one of the mappings A, B, S and T is continuous, then SA and TB have a unique common fixed point z in X and BS and AT have a unique common fixed point w in Y. Further, Az= Bz= w and Sw= Tw = z.

The following theorem was proved in[23].

1.5. Theorem: *Let (X, d_1) and (Y, d_2) be two complete metric spaces. Let A, B be mappings of X into B(Y) and S, T be mappings of Y into B(X) satisfying the inequalities*

$$\delta_1(Sy, Ty')\delta_1(SAx, TBx') \le c\max\left\{\begin{array}{l}\delta_1(Sy, Ty')\delta_2(Ax, Bx'),\ \delta_1(x', Sy)\delta_2(y', Ax),\\ d_1(x,x')\delta_1(Sy, Ty'),\ \delta_1(Sy, SAx)\delta_1(Ty', TBx')\end{array}\right\}$$

$$\delta_2(Ax, Bx')\delta_2(BSy, ATy') \le c\max\left\{\begin{array}{l}\delta_1(Sy, Ty')\delta_2(Ax, Bx'),\ \delta_1(x', Sy)\delta_2(y', Ax),\\ d_2(y,y')\delta_2(Ax, Bx'),\ \delta_2(Ax, BSy)\delta_2(Bx', Ty')\end{array}\right\}$$

for all $x, x' \in X$ *and* $y, y' \in Y$, *where* $0 \le c < 1$. *If one of the mappings A, B, S and T is continuous, then SA and TB have a unique common fixed point z in X and BS and AT have a unique common fixed point w in Y. Further,*

Az= Bz= w and Sw= Tw = z.

Now, we extend Theorem 1.4 and Theorem 1.5 for complete fuzzy metric spaces.

2. Main Result

2.1. Theorem: *Let $(X, \mu, *)$ and $(Y, \upsilon, *)$ be two complete fuzzy metric spaces. Let A, B be mappings of X into Y and S, T be mappings of Y into X satisfying the inequalities*

(1) $$k\mu(Sy, Ty', t)\mu(SAx, TBx', t) \ge \min\left\{\begin{array}{l}\mu(Sy, Ty', t)\nu(Ax, Bx', t),\\ \mu(x', Sy, t)\nu(y', Ax, t),\\ \mu(x, x', t)\mu(Sy, Ty', t),\\ \mu(Sy, SAx, t)\mu(Ty', TBx', t)\end{array}\right\}$$

(2) $$k\nu(Ax, Bx', t)\nu(BSy, ATy', t) \ge \min\left\{\begin{array}{l}\mu(Sy, Ty', t)\nu(Ax, Bx', t),\\ \mu(x', Sy, t)\nu(y', Ax, t),\\ \nu(y, y', t)\nu(Ax, Bx', t),\\ \nu(Ax, BSy, t)\nu(Bx', ATy', t)\end{array}\right\}$$

for all $x, x' \in X$ *and* $y, y' \in Y$, *where 0<k<1. If A and S or B and T are continuous, then SA and TB have a unique common fixed point z in X and BS and AT have a unique common fixed point w in Y. Further, Az= Bz= w and Sw= Tw = z.*

Proof: Let x be any arbitrary point in X. For n = 1, 2, 3,..., let $Sy_{2n-1} = x_{2n-1}$, $Bx_{2n-1} = y_{2n}$, $T y_{2n} = x_{2n}$, $Ax_{2n} = y_{2n-1}$.

Applying inequality (1), we get

$$k\mu(Sy_{2n-1},Ty_{2n},t)\mu(SAx_{2n}, TBx_{2n-1}, t) = k\mu(x_{2n-1}, x_{2n}, t)\mu(x_{2n-1}, x_{2n}, t)$$

$$\geq min\begin{Bmatrix}\mu(Sy_{2n-1}, T y_{2n}, t)\nu(Ax_{2n}, Bx_{2n-1}, t), \\ \mu(x_{2n-1}, Sy_{2n-1}, t)\nu(y_{2n}, Ax_{2n}, t), \\ \mu(x_{2n}, x_{2n-1}, t)\mu(Sy_{2n-1}, Ty_{2n}, t), \\ \mu(Sy_{2n-1}\ SAx_{2n}, t)\mu(Ty_{2n}, TBx_{2n-1}, t)\end{Bmatrix}$$

$$= min\begin{Bmatrix}\mu(x_{2n-1}, x_{2n}, t)\nu(y_{2n-1},y_{2n}, t), \\ \mu(x_{2n-1}, x_{2n-1}, t)\nu(y_{2n}, y_{2n-1}, t), \\ \mu(x_{2n},x_{2n-1}, t)\mu(x_{2n-1}, x_{2n},t), \\ \mu(x_{2n-1}, x_{2n-1}, t)\mu(x_{2n}, x_{2n}, t)\end{Bmatrix}$$

from which it follows that

(3) $$k\mu(x_{2n-1}, x_{2n}, t) \geq min\{\nu(y_{2n-1}, y_{2n}, t), \mu(x_{2n-1}, x_{2n}, t)\}$$

Applying inequality (2), we get

$$k\nu(Ax_{2n}, Bx_{2n-1}, t)\nu(BSy_{2n-1}, ATy_{2n},t) = k\nu(y_{2n-1}, y_{2n}, t)\nu(y_{2n-1}, y_{2n}, t)$$

$$\geq min\begin{Bmatrix}\mu(Sy_{2n-1}, Ty_{2n}, t)\nu(Ax_{2n}, Bx_{2n-1}, t), \\ \mu(x_{2n-1}, Sy_{2n-1}, t)\nu(y_{2n}, Ax_{2n}, t), \\ \nu(y_{2n-1}, y_{2n}, t)\nu(Ax_{2n}, Bx_{2n-1},t), \\ \nu(Ax_{2n}, BSy_{2n-1}, t)\nu(Bx_{2n-1}, ATy_{2n}, t)\end{Bmatrix}$$

$$= min\begin{Bmatrix}\mu(x_{2n-1}, x_{2n}, t)\nu(y_{2n-1}, y_{2n}, t), \\ \mu(x_{2n-1}, x_{2n-1}, t)\nu(y_{2n},y_{2n-1}, t), \\ \nu(y_{2n-1}, y_{2n}, t)\nu(y_{2n-1}, y_{2n}, t), \\ \nu(y_{2n-1}, y_{2n}, t)\nu(y_{2n}, y_{2n-1}, t)\end{Bmatrix}$$

from which it follows that

(4) $$k\nu(y_{2n-1}, y_{2n}, t) \geq min\{\mu(x_{2n-1}, x_{2n}, t), \nu(y_{2n-1}, y_{2n}, t)\}$$

From (3) and (4) can be written as

$$k\mu(x_{n-1}, x_n, t) \geq min\{\nu(y_{n-1}, y_n, t), \mu(x_{n-1}, x_n, t)\}$$

$$k\nu(y_{n-1}, y_n, t) \geq min\{\mu(x_{n-1}, x_n, t), \nu(y_{n-1}, y_n, t)\}$$

which can be again written as

(5) $$k\mu(x_{n+1}, x_n, t) \geq min\{\nu(y_n, y_{n+1}, t), \mu(x_n, x_{n+1}, t)\}$$

(6) $$k\nu(y_{n+1}, y_n, t) \geq min\{\mu(x_n, x_{n+1}, t), \nu(y_n, y_{n+1}, t)\}$$

From (5) and (6), by induction, we get

$$\mu(x_{n+1}, x_n, t) \geq \frac{1}{k^n} min\{\nu(y_1, y_2, t), \mu(x_1, x_2, t)\}$$

$$\nu(y_{n+1}, y_n, t) \geq \frac{1}{k^n} min\{\mu(x_1, x_2, t), \nu(y_1, y_2, t)\}$$

Let $t_1 = \frac{t}{p}$. Now,

$$\mu(x_n, x_{n+p}, t) = \mu(x_n, x_{n+p}, t_1 + t_1 + \ldots p \text{ times})$$

$$\geq \mu(x_n, x_{n+1}, t_1) * \mu(x_{n+1}, x_{n+2}, t_1) * .. * \mu(x_{n+p-1}, x_{n+p}, t_1)$$

$$\geq \frac{1}{k^n} min\{\nu(y_1, y_2, t), \mu(x_1, x_2, t)\} * \ldots$$

$$\ldots\ldots * \frac{1}{k^{n+p-1}}\{min\ \nu(y_1, y_2, t), \mu(x_1, x_2, t)\}$$

which implies that

$$lim\ \mu(x_n, x_{n+p}, t) \geq 1*1*\ldots*1 = 1$$

$\Rightarrow \{x_n\}$ is a Cauchy Sequence with a limit z in X.

Similarly, $\{y_n\}$ is a Cauchy sequence with a limit w in Y.

Now, on using the continuity of A and S respectively, we get

$$w = \lim y_{2n-1} = \lim Ax_{2n} = Az$$

and $$z = \lim x_{2n} = \lim Sy_{2n} = Sw$$

So that we get

(7) $Az = w$

(8) $Sw = z$

From (7) and (8), we get

(9) $SAz = z$.

Again applying inequality (1), we get

$$k\mu(Sy_{2n-1},Ty_{2n},t)\mu(SAx_{2n},TBx_{2n-1},t) \geq min\begin{Bmatrix}\mu(Sy_{2n-1},Ty_{2n},t)\nu(Ax_{2n},Bx_{2n-1},t),\\ \mu(x_{2n-1},Sy_{2n-1})\nu(y_{2n},Ax_{2n},t),\\ \mu(x_{2n},x_{2n-1},t)\mu(Sy_{2n-1},Ty_{2n},t),\\ \mu(Sy_{2n-1},SAx_{2n},t)\mu(Ty_{2n},TBx_{2n-1},t)\end{Bmatrix}$$

which implies that

(10) $$k\mu(SAx_{2n},TBx_{2n-1},t) \geq min\{\nu(Ax_{2n},Bx_{2n-1},t),\mu(x_{2n},x_{2n-1},t)\}$$

On letting $n \to \infty$, we get

$$k\mu(Sw, TBz, t) \geq min\{\nu(Az, w, t), 1\}$$

$$\Rightarrow \quad \mu(Sw, TBz, t) \geq \frac{1}{k}$$

$$\Rightarrow \quad \mu(Sw, TBz, t) \geq 1, \text{as } 0 < k < 1$$

$$\Rightarrow \quad \mu(Sw, TBz, t) = 1$$

which implies that

$$Sw = TBz$$

and from (8), we get

(11) $$z = TBz$$

From (9) and (11), we get

(12) $$SAz = z = TBz$$

Now, (10) gives

$$k\mu(x_{2n-1},Ty_{2n},t) \geq min\{\nu(Ax_{2n},Bx_{2n-1},t),\ \nu(y_{2n},Ax_{2n},t),\ \mu(x_{2n},x_{2n-1},t)\}$$

On letting $n \to \infty$, we get

$$\mu(z, Tw, t) = 1$$

which implies that

(13) $$z = Tw$$

Again applying inequality (2), we get

$$k\nu(Ax_{2n},Bx_{2n-1},t)\nu(BSy_{2n-1},ATy_{2n},t)$$

$$\geq \min \begin{Bmatrix} \mu(Sy_{2n-1}, Ty_{2n}, t)\nu(Ax_{2n}, Bx_{2n-1}, t), \\ \mu(x_{2n-1}, Sy_{2n-1}, t)\nu(y_{2n}, Ax_{2n}, t), \\ \nu(y_{2n-1}, y_{2n}, t)\nu(Ax_{2n}, Bx_{2n-1}, t), \\ \nu(Ax_{2n}, BSy_{2n-1}, t)\nu(Bx_{2n-1}, ATy_{2n}, t) \end{Bmatrix}$$

which implies that

(14) $$k\nu(BSy_{2n-1}, ATy_{2n}, t) \geq \min \begin{Bmatrix} \mu(Sy_{2n-1}, Ty_{2n}, t), \mu(x_{2n-1}, Sy_{2n-1}, t), \\ \nu(y_{2n-1}, y_{2n}, t), \nu(Ax_{2n}, Bx_{2n-1}, t) \end{Bmatrix}$$

On letting $n \to \infty$, we get

$$\nu(BSw, ATw, t) = 1$$

which implies that

(15) $BSw = ATw$

Now, (14) gives

$$k\nu(y_{2n}, ATy_{2n}, t) \geq \min \begin{Bmatrix} \mu(Sy_{2n-1}, T\,y_{2n}, t), \mu(x_{2n-1}, Sy_{2n-1}, t), \\ \nu(y_{2n-1}, y_{2n}, t), \nu(Ax_{2n}, BSy_{2n-1}, t) \end{Bmatrix}$$

On letting $n \to \infty$, we get

$$\nu(w, ATw, t) = 1$$

which implies that

(16) $w = ATw$.

From (15) and (16), we get

(17) $BSw = w = ATw$

From (8) and (17), we get

(18) $Bz = w$

From (7) and (18), we get

(19) $Az = Bz = w$

From (8) and (13), we get

(20) $Sw = Tw = z$

Similarly, on using the continuity of B and T, the above results hold.

To prove the uniqueness, let *SA* and *TB* have a second distinct common fixed point z' in *X* and *BS* and *AT* have a second distinct common fixed point w' in *Y*.

Applying inequality (1), we get

$$k\mu(Sy, Ty', t)\,\mu(SAz, TBz', t) \geq \min\begin{Bmatrix} \mu(Sy, T\,y', t)\nu(Az, Bz'\ t), \\ \mu(z'\ Sy, t)\nu(y', Az, t), \\ \mu(z, z', t)\mu(Sy, T\,y', t), \\ \mu(Sy, SAz, t)\mu(T\,y', T\,Bz', t) \end{Bmatrix}$$

$$\Rightarrow \qquad k[\mu(z, z', t)]^2 \geq \min\begin{Bmatrix} \mu(z, z', t)\nu(Az, Bz', t), \\ \mu(z', z', t)\nu(Bz', Az, t), \\ \mu(z, z', t)\mu(z, z', t), \\ \mu(z', z', t)\mu(z, z, t) \end{Bmatrix}$$

$$\Rightarrow \qquad k\mu(z, z', t) \geq \min\{\nu(Az, Bz', t), \mu(z, z', t)\}$$

which implies that

(21) $\qquad k\mu(z, z', t) \geq \nu(Az, Bz', t)$

Applying inequality (2), we get

$$k\nu\,(Az, Bz', t)\nu(BSy, ATy', t) \geq \min\begin{Bmatrix} \mu(Sy, Ty', t)\nu(Az, Bz', t), \\ \mu(z', Sy, t)\nu(y', Az, t), \\ \nu(y, y', t)\nu(Az, Bz', t), \\ \nu(Az, BSy, t)\nu(Bz', ATy', t) \end{Bmatrix}$$

$$\Rightarrow \qquad k[\nu(Az, Bz', t)]^2 \geq \min\begin{Bmatrix} \mu(z, z', t)\nu(Az, Bz', t), \\ \mu(z', z', t)\nu(Bz', Az, t), \\ \nu(Az, Bz', t)\nu(Az, Bz', t), \\ \nu(Az, Bz', t)\nu(Bz', Az, t) \end{Bmatrix}$$

$$\Rightarrow \qquad k\nu(Az, Bz', t) \geq \min\{\nu(Az, Bz', t), \mu(z, z', t)\}$$

which implies that

(22) $\qquad k\nu(Az, Bz', t) \geq \mu(z, z', t)$

From (21) and (22), we get

$$k^2\mu(z, z', t) \geq k\nu(Az, Bz', t) \geq \mu(z, z', t)$$

$$\Rightarrow \quad \mu(z, z', t) \geq \frac{1}{k^2}\mu(z, z', t)$$

$$\Rightarrow \quad \mu(z, z', t) \geq \frac{1}{k^2}\mu(z, z', t) \geq \ldots \geq \frac{1}{k^n}\mu(z, z', t)$$

$$\Rightarrow \quad 1 \geq \mu(z, z', t) \geq \lim \frac{1}{k^n}\mu(z, z', t) > 1$$

$$\Rightarrow \quad \mu(z, z', t) = 1$$

which implies that, $z = z'$.

This proves the uniqueness of *z*. Similarly, the uniqueness of *w* can be proved. The following corollary considering one pair of mappings can be discussed.

2.2. Corollary: *Let (X, μ, ∗) and (Y, υ, ∗) be two complete fuzzy metric spaces. Let S be mapping of X into Y and T be mapping of Y into X satisfying the inequalities*

$$k\nu(Sx, Sx', t)\, \nu(STy, STy', t) \geq \min \begin{Bmatrix} \mu(Ty, Ty', t)\nu(Sx, Sx', t), \\ \mu(x', Ty, t)\nu(y', Sx, t), \\ \nu(y, y', t)\nu(Sx, Sx', t), \\ \nu(Sx, STy, t)\nu(Sx', STy', t) \end{Bmatrix}$$

$$k\mu(Ty, Ty', t)\mu(TSx, TSx', t) \geq \min \begin{Bmatrix} \mu(Ty, Ty', t)\nu(Sx,Sx', t), \\ \mu(x', Ty, t)\nu(y', Sx, t), \\ \mu(x, x', t)\mu(Ty, Ty', t), \\ \mu(Ty, TSx, t)\mu(Ty', TSx', t) \end{Bmatrix}$$

for all $x, x' \in X$ *and* $y, y' \in Y$ *,where 0<k<1. If S or T is continuous, then TS has a unique fixed point z in X and ST has a unique fixed point w in Y.Further, Sz= w and Tw = z.*

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