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Overstable Convection in Viscoelastic Nanofluid Layer Saturated By a Darcy-Brinkman Porous Medium Embedded by Dust Particles

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Abstract: The paper presents a linear stability analysis for the effect of dust particles on the onset of thermal convection in viscoelastic nanofluid layer saturated by a Darcy-Brinkman porous medium embedded with dust particles. The rheology of viscoelasticity is described by constitutive relations purposed by Oldroyd-B model. The nanofluid layer incorporates the effect of Brownian motion along with thermophoresis. The set of partial differential equations using the normal mode technique is reduced to non- dimensional ordinary differential equations in different non – dimensional parameters. Using the appropriate stress free boundary conditions, exact solutions are obtained. Both stationary and oscillatory convection are investigated using normal mode technique. It is found that instability sets in as oscillatory motions rather than stationary convection. For the stationary convection, the effects of the Lewis number (L_e), concentration Rayleigh number (R_n), modified diffusivity ratio (N_A) and Darcy number (D_a) the stability of the system has been investigated numerically. The oscillatory modes are introduced by the stress-relaxation time and strain- retardation parameter. The sufficient condition for the non-existence overstability is $\lambda_2 > \lambda_1$. The effect of various parameters for on thermal Rayleigh number the overstability has been presented graphically.

Keywords: Nanofluid Viscoelastic, Thermal convection, dust particles, Darcy-Brinkman porous medium.

1. Introduction

Heating or cooling of fluids is important for many industrial sectors, including energy supply and production, transportation and electronics. The thermal conductivity of these fluids plays a vital role in the development of energy coefficient heat transfer equipment. However, conventional heat transfer fluids have poor thermal transfer properties compared to most solids. In order to improve the thermal conductivity of these fluids numerous theoretical and experimental studies of the effective thermal conductivity of liquids containing suspended solid particles have been conducted.

Nanofluids are mixtures of a regular fluid, with a very small amount of suspended metallic or metallic oxide nanoparticles or nanotubes, which is first utilized by Choi [1]. Nanoparticles materials may be taken as oxide ceramics (Al2O3, CuO), metal carbides (SiC), nitrides (AlN, SiN) or metals (Al, Cu) etc. Base fluids are water, ethylene or tri-ethylene-glycols and other coolants, oil and other lubricants, bio-fluids, polymer solutions, other common fluids. Typical dimension of the nanoparticles is in the range of a few to about 100 nm. Characteristic features of nanofluids are the formation of very stable colloidal systems with very little settling and anomalous enhancement of the thermal conductivity compared to the base fluid[2, 3].

Buongiorno [4] noted that the nanoparticles absolute velocity can be viewed as the sum of the base fluid velocity and a relative (slip) velocity. Tzou [5] also studied thermal instability of nanofluids in natural convection whereas thermal instability in a porous medium layer saturated by a nanofluid was studied by Nield and Kuznetsov [6]. In all the above studies the nanofluid have been assumed to be Newtonian.

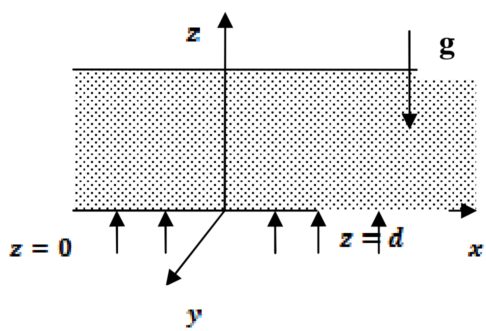
Non-Newtonian rheological behavior of nanofluids was indicated by many investigators (Chen et al. [7, 8] ; Schmidt et al. [9] . Convection of non-Newtonian fluids in a porous medium is of considerable importance in several applied fields such as oil recovery, food processing, and the spread of contaminants in the environment, and in various processes the chemical and materials industries. The onset of thermal convection of a viscoelastic fluid-saturated porous medium was studied by many authors (Rudraiah et al. [10]; Kim et al. [11]; Yoon et al. [12] ; Malashetty et al. [13]). Since elastic behavior is inherent in non-Newtonian fluids, oscillatory instabilities can set in before stationary modes. Sheu et al. [14] investigated the chaotic convection in an Oldroydian fluid-saturated porous medium using a weak non-linear analysis. The instability of a viscoelastic fluid-saturated porous layer was theoretically studied by Bertola and Cafaro [15] using a dynamic system approach. Sheu et al. [16] investigated chaotic thermal convection in an Oldroydian fluid-saturated porous medium using a thermal non-equilibrium model. Recently, Nield and Kuznetsov [17] studied thermal instability in a porous medium layer saturated by a nanofluid: Brinkman model while linear stability of convection in a in a viscoelastic nanofluid layer was studied by Sheu [18].

In geophysical situations, the fluid is often not pure but contains several suspended particles. Motivations for the study of certain effect of particles immersed in the fluid such as particle heat capacity, particle mass fraction and thermal force is due to the fact that the knowledge concerning fluid particles mixture is not commensurate with their industrial and scientific importance. Scanlon and Segel[19], Chand [20], Rana et al. [21, 22] studied the effect of suspended particles on Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of pure gas was supplemented by that of particles and it was found that suspended particles destabilize the fluid layer.

The investigation in porous media has been started with the simple Darcy law which states that the usual viscous term in the equations of motion is replaced by the resistance term $-\frac{\mu}{k_1}q$, where μ is the viscosity of the incompressible fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid and gradually was extended to Darcy-Brinkman model. Lapwood [23] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [24]. Nanofluids in porous media constitute an emerging topic; the review of recent literature points out to at least two possible applications. Porous foam and microchannel heat sinks (used for electronic cooling) are usually modeled and optimized utilizing the porous medium approach. The utilization of nanofluids for cooling such microchannel heat sinks has been suggested. Modelling of such heat sinks requires understanding of fundamentals of nanofluid convection in porous media. Another area relevant to nanofluid convection in porous media is the utilization of nanoparticles hyperthermia for cancer treatment. The objective is to induce the maximum damage on the tumor (this requires elevating the temperature of at least 90% of the tumor above 43^0 C) with the minimum damage to the normal tissue. Our objective in the present work is to study how the onset criterion for oscillatory convection is affected by interactions among Brownian diffusion, thermophoretic diffusion, suspended particles number density and viscoelasticity, and how is it related to the oscillatory instability of a binary viscoelastic base fluid with dust particles. The Oldroyd-B fluid model was employed to describe the rheological behavior of the nanofluid. The modified Darcy-Brinkman model was used to simulate conservation of momentum in the porous medium.

2. Theoretical Model

An infinite horizontal layer of an incompressible viscoelastic nanofluid which is heated from below, confined between two parallel planes $z = 0$ and $z = d$ where temperature and volumetric fraction of nanoparticles are kept constants: $T = T_0$ and $\phi = \phi_0$ at $z = 0$ and $T = T_1$ and $\phi = \phi_1$ at $z = d$. The both boundary surfaces are assumed to be free. The thermo physical properties of nanofluids (viscosity, density, thermal conductivity and specific heat) are as constants for the analytical formulation but these quantities are not



Geometrical configuration

constant and strongly depend on the volume fraction of nanoparticles. The fluid is acted on by the acceleration due to gravity $g(0,0,-g)$.

The mathematical equations describing the physical model are based upon the following assumptions:

i) Thermo physical properties expect for density in the buoyancy force (Boussinesq Hypothesis) are constant;
ii) The fluid phase and nanoparticles are in thermal equilibrium state;
iii) Nanoparticles are spherical;
iv) Nanofluid is incompressible, non-Newtonian and laminar;
v) Radiation heat transfer between the sides of wall is negligible when compared with other.

The continuity and momentum equations, using the Boussinesq approximation, relevant to the problem are:

(1) $$\nabla . q = 0$$

(2) $$\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)\left[\frac{\rho}{\varepsilon}\left(\frac{\partial q}{\partial t}+q\ .\nabla q\right)+\nabla p-\rho g-\frac{KN(q_d-q)}{\varepsilon}\right]=\left(1+\bar{\lambda}_2\frac{\partial}{\partial t}\right)\left(\mu\nabla^2 q-\frac{\mu}{k_1}q\right),$$

where $\bar{\lambda}_1$ is the stress relaxation time, $\bar{\lambda}_2$ is the strain retardation time, ε is the homogenous medium of porosity, q is the velocity vector, q_d is the velocity of suspended particles, μ is the viscosity of fluid, K is Stokes drag formula, N is number density of dust particles and p is the hydrostatic pressure, respectively. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion (2) for the particles.

The nanofluid density ρ in eq. (2) is

(3) $$\rho=\varphi\ \rho_p+\varphi\rho_f,$$

where φ is the nanoparticle volume fraction, ρ_p density of nanoparticles and ρ_f density of base fluid.

Taking the density of the nanofluid as that of the base fluid, as adopted by Tzou [5] the specific weight $(\rho\, g)$ in equation (2) thus becomes

(4) $$\rho g \cong \left\{\varphi\ \rho_p+(1-\varphi)\rho_{fo}\left[1-\beta(T-T_0)\right]\right\},$$

where β is the coefficient of thermal expansion.

Using equation (4), motion for nanofluid with suspended particles is given as

(5) $$\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)\left[\frac{\rho}{\varepsilon}\left(\frac{\partial \mathrm{q}}{\partial t}+\mathrm{q}\ .\nabla\mathrm{q}\ \right)+\nabla p-\left\{\varphi\,\rho_p+(1-\varphi)\,\rho_{fo}\left[1-\beta(T-T_0)\right]\right\}\right.$$
$$\left.-\frac{KN(\mathrm{q}_d-\mathrm{q})}{\varepsilon}\right]=\left(1+\bar{\lambda}_2\frac{\partial}{\partial t}\right)\left(\mu\nabla^2\mathrm{q}-\frac{\mu}{k_1}\mathrm{q}\right),$$

The buoyancy forces on the particles are neglected. Interparticle reaction are also not considered as we assume that distances between particles are quite large as compared with their diameter and if mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions, are

(6) $$\frac{\partial \mathrm{q}_d}{\partial t}=\frac{K}{m}(\mathrm{q}-\mathrm{q}_d)\ ,$$

(7) $$\frac{\partial N}{\partial t}+\nabla.(N\mathrm{q}_d)=0\ ,$$

The equation of continuity for the nanoparticles is

(8) $$\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}\mathrm{q}.\nabla\varphi=D_B\nabla^2\varphi+\frac{D_T}{T_b}\nabla^2 T.$$

where D_B is the Brownian diffusion coefficient, given by Einstein-Stokes equation and D_T is the thermophoretic diffusion coefficient of the nanoparticles.

The energy equation in nanofluid is

(9) $$(\rho c)_f\left(\frac{\partial T}{\partial t}+\mathrm{q}.\nabla T\right)+mNC_{pt}\left(\frac{\partial T}{\partial t}+\mathrm{q}_d.\nabla T\right)$$
$$=k_m\nabla^2 T+\varepsilon(\rho c)_p\left[D_B\nabla\varphi.\nabla T+(D_T/T_1)\nabla T.\nabla T\right].$$

where $(\rho c)_f$ is the heat capacity of fluid, C_{pt} is heat capacity of suspended particles, $(\rho c)_p$ is heat capacity of nanoparticles and k_m is thermal conductivity of the fluid.

Since the fluid under consideration is confined between two horizontal planes $z=0$ and $z=d$, on these two planes certain boundary conditions must be satisfied. We take case of free-free surface and assume the temperature and volumetric fraction of the nanoparticles are constant thus the boundary conditions are

(10) $$w=0, T=T_0, \varphi=\varphi_0 \ at \ z=0,$$

(11) $$w=0, T=T_1, \varphi=\varphi_1 \ at \ z=d.$$

We introduce non-dimensional variables as

$$(x^{'}, y^{'}, z^{'})=(x, y, z/d), \ t^{'}=t\kappa/d^2,$$

$$\mathrm{q}'(u^{'}, v^{'}, w^{'})=\mathrm{q}(u, v, w/\kappa)d,$$

$$\mathrm{q}'_d(l^{'}, r^{'}, s^{'})=\mathrm{q}_d(l, r, s/\kappa)d,$$

$$p^{'}=pd^2/\mu\kappa, \ \varphi^{'}=(\varphi-\varphi_0)/(\varphi_1-\varphi_0),$$

$$T^{'}=(T-T_1)/(T_0-T_1)$$

The equations (1), (5)-(9), in non-dimensional form can be written as,

(12) $$\nabla.\mathrm{q}=0$$

(13) $$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{V_a}\frac{\partial \mathrm{q}'}{\partial t}+\nabla p'+R_m\hat{e}_z-R_aT^{'}\hat{e}_z+R_n\varphi'\hat{e}_z-G\left(\mathrm{q}'_d-\mathrm{q}'\right)\right)$$
$$=\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(D_a\nabla^2\mathrm{q}'-\mathrm{q}'\right),$$

(14) $$\tau\frac{\partial \mathrm{q}_d^{'}}{\partial t^{'}}=(\mathrm{q}^{'}-\mathrm{q}_d^{'}),$$

(15) $$\frac{\partial M}{\partial t^{'}}+\nabla.(\mathrm{q}_d^{'})=0,$$

(16) $$\frac{\partial T'}{\partial t^{'}}-q^{'}.\nabla T^{'}+b\left(\frac{\partial}{\partial t^{'}}+\mathrm{q}_d^{'}\nabla\right)T^{'}$$
$$=\nabla^2T'+\frac{N_B}{L_e}\nabla\varphi^{'}.\nabla T^{'}+\frac{N_AN_B}{L_e}\nabla T^{'}.\nabla T^{'},$$

and

(17) $$\frac{\partial\varphi'}{\partial t}+\frac{1}{\varepsilon}\mathrm{q}'.\nabla\varphi'=\frac{1}{L_e}\nabla^2\varphi'+\frac{N_A}{L_e}\nabla^2T',$$

where non-dimensional parameters are:

Darcy-Prandtl number $P_r = \frac{\mu}{\rho\kappa}$, Lewis number $L_e = \frac{\kappa}{D_B}; b = \frac{mNC_{pt}}{\rho c}$; $G = \frac{KN_0 d^2}{\mu}$; $\tau = \frac{m\kappa}{Kd^2}$;, Darcy-number $D_a = \frac{k_1}{d^2}$, Vadasz number $V_a = \frac{\varepsilon P_r}{D_a}$, thermal Darcy-Rayleigh number $R_a = \frac{\rho g \beta d^3 (T_0 - T_1)}{\mu\kappa}$, basic density Rayleigh number $R_m = \frac{\rho_p \varphi_0 + \rho_{f0}(1-\varphi_0) g d^3}{\mu\kappa}$, concentration Rayleigh number $R_n = \frac{(\rho_p - \rho_{f0})(\varphi_1 - \varphi_0) g d^3}{\mu\kappa}$, Deborah number $\lambda_1 = \frac{\bar{\lambda}_1 \kappa}{d^2}$, retardation parameter $\lambda_2 = \frac{\bar{\lambda}_2 \kappa}{d^2}$, modified diffusivity ratio $N_A = \frac{D_T (T_0 - T_1)}{D_B T_1 (\varphi_1 - \varphi_0)}$, modified particle-density increment $N_B = \frac{(\rho c)_p}{(\rho c)_f}(\varphi_1 - \varphi_0)$ and Taylor number; $T_A = \frac{4\Omega^2 d^2}{\nu^2}$ and dropping the asterisks for convenience.

The basic state is assumed to be quiescent (no settling of dust particles and nanoparticles) and is given by

(18) $$u = v = w = 0, T = T_b(z), \varphi = \varphi_b(z), p = p(z), N = N_0 \text{ (a constant).}$$

To study the stability of the system, we superimpose infinitesimal perturbations onto the basic state, which are of the forms

(19) $$q' = 0 + q'', T' = T_b + T'', \varphi' = \varphi_b + \varphi'', p' = p_b + p''$$

Using the equation (19) in the equations (12)-(17), we obtain the linearized perturbation equations

(20) $$\nabla . \mathrm{q}'' = 0,$$

(21) $$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \frac{1}{V_a} \frac{\partial \mathrm{q}''}{\partial t} = \left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\left(-\nabla p'' + R_a T'' \hat{e}_z - R_n \varphi'' \hat{e}_z + G(\mathrm{q}_d'' - \mathrm{q}'')\right)$$

$$+\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(D_a\nabla^2 q''-q''\right),$$

(22) $$\tau\frac{\partial q_d''}{\partial t}=(q''-q_d''),$$

(23) $$\frac{\partial M}{\partial t}+\nabla.(q_d'')=0,$$

(24) $$\frac{\partial \varphi''}{\partial t}+w''=\frac{1}{L_e}\nabla^2\varphi''+\frac{N_A}{L_e}\nabla^2 T''.$$

and

(25) $$(1+b)\frac{\partial T''}{\partial t}-w''-bs''=\nabla^2 T''+\frac{N_B}{L_e}\left(\frac{\partial T''}{\partial z}-\frac{\partial \varphi''}{\partial z}\right)-\frac{2N_A N_B}{L_e}\frac{\partial T''}{\partial z},$$

Now eliminating p'', q_d'' from equations (21) making use of equations (20) and (22), we get

(26) $$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{V_a}\frac{\partial}{\partial t}\left(1+\tau\frac{\partial}{\partial t}\right)+G\tau\frac{\partial}{\partial t}\right]\nabla^2 w''$$
$$=\left(1+\tau\frac{\partial}{\partial t}\right)\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(R_a\nabla_H^2 T''-R_n\nabla_H^2\varphi''\right)$$
$$+\left(1+\tau\frac{\partial}{\partial t}\right)\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(D_a\nabla^4 w-\nabla^2 w\right)$$

,

Eliminating s (z component of q_d) from equation (25) by making use of equation (22), we have

(27) $$(1+b)\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial T''}{\partial t}=\left(1+\tau\frac{\partial}{\partial t}+b\right)w''+\left(1+\tau\frac{\partial}{\partial t}\right)\nabla^2 T''$$
$$+\frac{N_B}{L_e}\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial T''}{\partial z}-\frac{\partial \varphi''}{\partial z}\right)$$
$$-\frac{2N_A N_B}{L_e}\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial T''}{\partial z}.$$

In terms of the non- dimensional form, boundary conditions become

$$\begin{cases} w''=0, \ \dfrac{\partial^2 w''}{\partial z^2}=0, \ T''=1, \ \varphi''=0 \quad at \ z=0\,, \\ w''=0, \ \dfrac{\partial^2 w''}{\partial z^2}=0, \ T''=0, \ \varphi''=1 \quad at \ z=1\ . \end{cases} \tag{28}$$

Normal Mode Analysis Method

The partial differential equations (20) - (24) and (27) with boundary conditions (28) constitute a linear boundary-value problem that can be solved using the method of normal modes in the form in which the disturbances are analyzed into normal modes and the perturbation quantities of the form:

$$\left(w'',T'',\varphi''\right)=\left(W(z),\Theta(z),\Phi(z)\right)\exp\left[i(k_x x+k_y y)+nt\right], \tag{29}$$

where k_x and k_y are the wavenumber in the *x*- and *y*- directions and n is the growth rate and $k^2 = k_x^2 + k_y^2$. Using the expression (29), the equations (20) - (24) and (27) take the form

$$\left[\begin{array}{l}\left\{\dfrac{n}{Va}(D^2-a^2)+\dfrac{G\tau n}{1+\tau n}(D^2-a^2)\right\}(1+\lambda_1 n) \\ -(D^2-a^2)\left\{D_a(D^2-a^2)-1(1+\lambda_2 n)\right\}\end{array}\right]W$$
$$+R_a a^2\varphi''(1+\lambda_1 n)+R_n a^2\varphi''(1+\lambda_1 n)=0, \tag{30}$$

$$(B+\tau n)W+(1+\tau n)\left(\left(D^2-a^2\right)-nB+\frac{N_B}{L_e}D-\frac{2N_A N_B}{L_e}D\right)\Theta$$
$$-(1+\tau n)\frac{N_B}{L_e}D\Phi=0, \tag{31}$$

$$W-\frac{N_A}{L_e}(D^2-a^2)\Theta-\left(\frac{1}{L_e}(D^2-a^2)-n\right)\Phi=0, \tag{32}$$

where $D=\dfrac{d}{dz}$, $B=b+1$ and $a^2=k_x^2+k_y^2$ is dimensionless the resultant wave number.

Boundary conditions are

(32a) $W=0,\ D^2W=0,\ \Theta=0, \Phi=0$ at $z=0$ and $z=1$.

We assume the solution to W, Θ and Φ are of the form

(33) $$W = W_0 \sin\pi z,\quad \Theta=\Theta_0 \sin\pi z,\quad \Phi=\Phi_0 \sin\pi z\ ,$$

which satisfies the (32a) boundary conditions.

Substituting the equation (33) into equations (30) - (32) and integrating each equation from $z=0$ to $z=1$ and performing some integrations by parts, the following matrix equation is obtained:

(34) $$\begin{bmatrix} J\left\{\begin{matrix}(D_a J+1)(1+\lambda_2 n) \\ +\left\{\dfrac{n}{V_a}+\dfrac{G\tau n}{1+\tau n}\right\}(1+\lambda_1 n)\end{matrix}\right\} & -R_a a^2 & R_n a^2 \\ (B+\tau n) & -(1+\tau n)(J+nB) & 0 \\ \dfrac{1}{\varepsilon} & N_A L_\varepsilon^{-1} & L_\varepsilon^{-1} J+n \end{bmatrix}\begin{bmatrix} W_0 \\ \theta_0 \\ \phi_0 \end{bmatrix}=0$$

where $J=\pi^2+a^2$.

The non-trivial solution of the equations above matrix requires that

(35) $$R_a=\frac{(1+\tau n)(J+nB)J}{a^2(1+\lambda_1 n)(1+\tau n)}\left[(D_a J+1)(1+\lambda_2 n)+\left(\frac{n}{Va}+\frac{G\tau n}{1+\tau n}\right)(1+\lambda_1 n)\right]$$
$$-\frac{R_n\left[(B+\tau n)\dfrac{N_A}{L_e}J+\dfrac{(1+\tau n)(J+nB)}{\varepsilon}\right]}{(B+\tau n)\left(\dfrac{L}{L_e}+n\right)}$$

4. Natural Convection

a) Stationary Convection

For stationary convection $n=0\ (n_i=0)$, equation (35) reduces to

(36) $$R_a^{\,s}=\frac{\left[D_a(\pi^2+a^2)^3+(\pi^2+a^2)^2\right]}{a^2}-\left(N_A B+\frac{L_e}{\varepsilon}\right)R_N\,.$$

b) Oscillatory Convection

For oscillatory convection, putting $n = in_i$ in eq. (35) where n_i is real; we get

$$\text{(37)}\quad R_a = \frac{J}{a^2\left(1+\lambda_1^2 n^2\right)}\begin{bmatrix}(D_a J+1)\left\{J+\lambda_1 Bn^2-(\tau+\lambda_2)(B-J\lambda_1)n^2-\tau\lambda_2 Jn^2-\tau\lambda_1\lambda_2 Bn^4\right\} \\ -\frac{1}{V_a}(\tau+\lambda_2)n^2-G\lambda_1\tau n^2\end{bmatrix}$$

$$-\frac{1}{\left(\frac{J^2}{L_e^2}+n^2\right)}\left[\frac{BN_A J^2}{L_e^2}+\frac{N_A J\tau n^2}{L_e}+\frac{J^2}{L_e\varepsilon}+\frac{(J\tau+B)}{\varepsilon}n^2-\frac{B\tau J}{L_e}n^2\right]R_n$$

and

$$\text{(38)}\quad R_a = \frac{J}{\tau a^2(1+\lambda_1^2 n^2)}\left[(D_a J+1)\begin{Bmatrix}B+J(\lambda_2-\lambda_1)+J\tau+\tau\lambda_1 Bn^2+ \\ \lambda_1\lambda_2 Bn^2-\tau\lambda_2 Bn^2+\tau^2\lambda_1\lambda_2 n^2\end{Bmatrix}+\frac{1}{V_a}(1-\tau\lambda_1 n^2)+\tau\lambda\right]$$

$$-\frac{1}{\tau\left(\frac{J^2}{L_e^2}+n^2\right)}\left[\frac{N_A J}{L_e}\left(\frac{J\tau}{L_e}-B\right)+\frac{\tau J^2}{L_e\varepsilon}+\frac{BJ}{L_e\varepsilon}-\frac{J}{\varepsilon}+\frac{B\tau}{\varepsilon}n^2\right]R_n$$

From equations (37) and (38), we get

$$\text{(39)}\quad a_1(n^2)^3+a_2(n^2)^2+a_3(n^2)+a_4=0.$$

where
$$a_1 = J\,(D_a J+1)\,\tau^2\lambda_1\lambda_2 B,$$

$$a_4 = J(D_a J+1)B\frac{J^2}{L_e^2}\left(B+J(\lambda_2-\lambda_1)+J\tau\right)-J^2(D_a J+1)\frac{J^2}{L_e^2}$$

$$+\frac{BJ}{V_a}\frac{J^2}{L_e^2}+\tau GBJ\frac{J^2}{L_e^2}+\tau a^2\left(\frac{BN_A J^2}{L_e^2}+\frac{J^2}{L_e\varepsilon}\right)$$

$$-Ba^2\left(\frac{N_A J}{L_e}\left(\frac{J\tau}{L_e}-B\right)+\frac{\tau J^2}{L_e\varepsilon}+\frac{BJ}{L_e\varepsilon}-\frac{J}{\varepsilon}\right)R_n.$$

The constant a_2 and a_3 involving large number of terms has been not written here.

Results and Discussions

Expressions of the critical thermal Rayleigh number for stationary and oscillatory are given by Eqs. (36) and (37) respectively. According to Buongiorno [4] and Nield and Kuznetsov [6, 16], for most nanofluids investigated, Lewis number, Le, is large of the order $10^2 - 10^3$, while the modified diffusivity ratio, N_A, is no greater than about 10. In the following, we consider instability by taking values of L_e and N_A within these ranges.

The effect of the Deborah number, λ_1, is shown in Fig.1. The Deborah number is used in rheology to characterize how fluid a material is. It physically represents the ratio of the relaxation time to the thermal diffusion time. Fig. 1 shows that the oscillatory thermal Rayleigh number decreases with an increase in the Deborah number which indicates that the effect of the Deborah number is to advance the onset of convection in a viscoelastic nanofluid layer.

Fig. 2, shows the effect of the retardation parameter, λ_2, on the neutral curves. It is found that an increase in the value of the retardation parameter increases the minimum oscillatory Rayleigh number, indicating that it delays the onset of convection in a viscoelastic rotating nanofluid layer.

In Fig. 3, the effect of the Lewis number on neutral curves is displayed. It is noted that the effect of the Lewis number on the oscillatory thermal Rayleigh number is very slight, while its effect on the stationary mode is substantial.

The effect of the concentration Rayleigh number is shown in Fig. 4. It is clear from the Fig. 4 that the oscillatory thermal Rayleigh number decreases with an increase in R_N, which means that R_n, enhances oscillatory convection.

Fig. 5 depict the effect of dust particles parameter, B, on the neutral curves. It can be seen from figure 5 that the critical oscillatory thermal Rayleigh number increases with an increase in B, indicating that B, postpones the oscillatory onset and similarly for critical thermal Rayleigh number for the stationary convection.

In Fig. 6 the effect of Darcy number on the neutral curves is shown, which depicts that for oscillatory modes as well as stationary convection thermal Rayleigh number increases with the increase in Darcy number indicating that D_a delays for both type of convections.

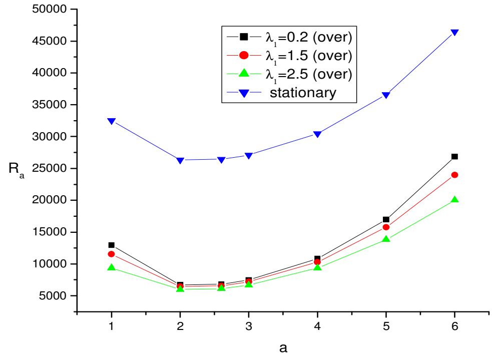


Figure 1: Neutral curves for different values of the Deborah number (λ_1).

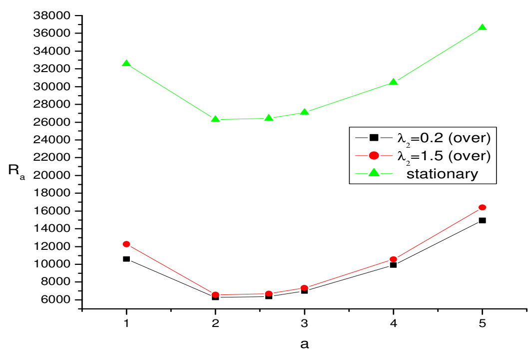


Figure 2: Neutral curves for different values of the retardation parameters (λ_2).

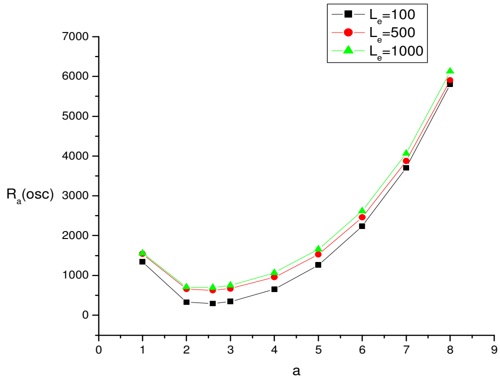


Figure 3: Neutral curves for different values of the Lewis number (L_ε).

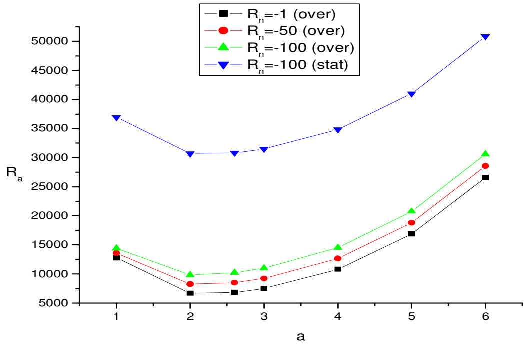


Figure 4: Neutral curves for different values of the concentration Rayleigh number (R_n).

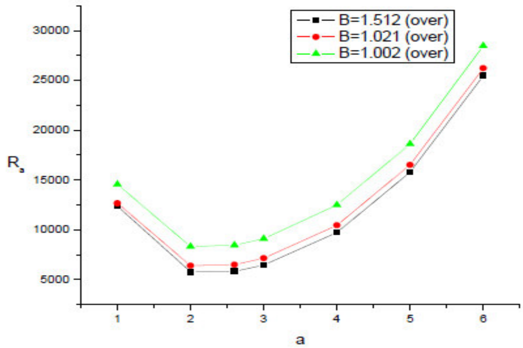


Figure 5: Neutral curves for different values of the dust particles parameter (B).

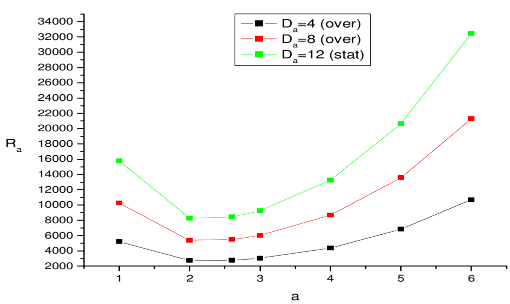


Figure 6. Neutral curves for different values of the Darcy number (D_a).

Conclusions

(i) The Lewis number L_e and modified diffusivity ratio N_A stabilize the system.

(ii) The effect of concentration Rayleigh number R_n is to destabilizes stationary convection and similarly R_n for overstability.

(iii) The Darcy number has stabilizing effect for the cases of stationary convection and overstability.

(iv) The dust particles parameters destabilize the thermal Rayleigh number for overstability.

(v) The sufficient condition for the non-existence of overstability is $\lambda_2 > \lambda_1$.

References

1. S. U. S. Choi, Enhancing thermal conductivity of fluids with nanoparticles in Developments and Applications of Non-Newtonian Flows, D. A. Siginer, H. P . Wang, (eds.), *ASME FED*, **231(66)** (1995) 99–105.

2. Masuda et al., Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles (dispersion of γ-Al_2O_3, SiO_2, and TiO2 ultra-fine particles), *Netsu Bussei,* **4(4)** (1993) 227–233.

3. J. A. Eastman et al., Anomalously increased effective thermal conductivity of ethylene glycol-based nanofluid containing copper nanoparticles, *Appl. Phys. Lett.,* **78(6)** (2001) 718–720.

4. J. Buongiorno, Convective transport in nanofluid, *ASME J. Heat Transf.*, **128(3)** (2006) 240–250.

5. D. Y. Tzou, Instability of nanofluids in natural convection, *ASME Journal of Heat Transfer*, **130** (2008) 372-401.

6. D. A. Nield, A. V. Kuznetsov, Thermal instability in a porous medium layer saturated by a nanofluid, *Int. J. Heat Mass Transf.,* **52(25-26)** (2009) 5796–5801.

7. H. S. Chen. et al., Y. L. Ding, Y. R. He., C. Q. Tan, Rheological behaviour of ethylene glycol based titania nanofluids, *Chem. Phys. Lett.*, **444** (2007b) 333–337.

8. Chen et al., Rheological behaviour of nanofluids containing tube/rod-like nanoparticles, *Power Technol*, **194** (2009) 132–141.

9. A. J. Schmidt et al., Experimental investigation of nanofluid shear and longitudinal viscosities, *Appl. Phys. Lett.*, **92** (2008) 244107.

10. N. Rudraiah. et al., Oscillatory convection in a viscoelastic fluid through a porous layer heated from below, *Rheol. Acta,* **28** (1989) 48–52 .

11. M. C. Kim et al., Thermal instability of viscoelastic fluids in porous media, *Int. J. Heat Mass Transf.,* **46** (2003) 5065–5072.

12. D. Y. Yoon et al., The onset of oscillatory convection in a horizontal porous layer saturated with viscoelastic liquid. *Transp, Porous Media*, **55** (2004) 275-284.

13. M. S. Malashetty et at., Convective instability of Oldroyd B fluid saturated porous layer heated from below using a thermal nonequilibrium model, *Transp. Porous Media* **64** (2006) 123–139.

14. L. J. Sheu et al., Chaotic convection of viscoelastic fluid in porous medium, *Chaos Solitons Fractals*, **37** (2008) 113–124.

15. B. Bertola, E. Cafaro, Thermal instability of viscoelastic fluids in horizontal porous layers as initial problem, *Int. J. Heat Mass Transf.*, **49** (2006) 4003–4012.

16. L. J. Sheu et al., A unified system describing dynamics of chaotic convection, *Chaos Solitons Fractals*, **41** (2009) 123–130.

17. A. V. Kuznetsov, D. A. Nield, Thermal instability in a porous medium layer saturated by a nanofluid: Brinkman Model, *Transp. Porous Medium*, **81** (2010) 409-422

18. Sheu, Linear stability of convection in a viscoelastic nanofluid layer, *World Academy of Sciences and Technology*, **58** (2011) 289-295.

19. J. W. Scanlon, and L. A. Segel, Effect of suspended particles on onset of Bénard convection, *Physics Fluids*, **16** (1973) 1573–78.

20. R. Chand, Effect of suspended particles on thermal instability of Maxwell viscoelastic fluid with variable gravity in porous medium, *Antarctica J. Math.*, **8(6)** (2011) 487-497.

21. G. C. Rana, and R. C. Thakur, Effect of suspended particles on thermal convection in Rivlin-Ericksen elastic-viscous fluid in a Brinkman porous medium, *Journal of Mechanical Engineering & Sciences*, **2** (2012) 162-171.

22. G. C. Rana, R. C. Thakur, and K. Kumar, Thermosolutal convection in compressible Walters' (model B)fluid permeated with suspended particles in a Brinkman porous medium, *Journal of computational Multiphase flows*, **4(2)** (2012) 211-224.

23. E. R. Lapwood, Convection of a fluid porous medium, *Proc. Camb. Phil. Soc.*, **44** (1948) 508-519.

24. R. A. Wooding, Rayleigh instability of a thermal boundary layer in flow through a porous medium, *J. Fluid Mech.*, **9** (1960)183-192.

25. S. Chand, G. C. Rana, onset of thermal convection in rotating nanofluid layer saturated by a Darcy-Brinkman porous medium, *International Journal of Engineering Research & Technology*, **1** (2012).

26. D. Laroze et al., Realistic rotating convection in a DNA suspension, *Physica A.,* **385** (2007a) 433-438.

27. D. Laroze et al. Thermal convection in a rotating binary viscoelastic liquid mixture, *Eur. Phys. J. Special Topics,* **146** (2007b) 291-300.