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Recent Trends in Application of Mathematics in Sociology*

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Abstract: Mathematics has always been used in various disciplines to solve problems. Sociology is no exception to it[1]. Game Theory and little known Social Network Analysis which has grown rapidly in last 35 years are highly relevant examples of contribution of Mathematics towards Sociology. For the last 60 years, sociologists have been trying to highlight the significance and application of Mathematics in explaining Sociological theories.

With the advancement in society, individuals are related to each other in very complex manner. Thus task of a Sociologist to analyze a domain comprised of particular actors of the society is very difficult. A Social Scientist needs innovative sociological models and tools to explore and analyze situations in order to reach to a correct conclusion. Since inception of Modern Mathematical Sociology it has been repeatedly highlighted that mathematical concepts of Matrix, Mapping, Probability can be highly useful in mathematical and statistical modelling in a sociological theory. However no mere presentation or use of fanciful equations or mathematics can make a wrong theory right[1].

Keywords: Sociometry, Quantitative Sociology, Mathematical models, Social Network analysis.

1. Introduction

The origin of Mathematical Sociology can be traced back in 1950s (Fararo[2]). However, Mathematics is boon for all the quantitative empirical researches. Mathematics is of great significance and use for Natural Sciences. However, it has been useful in the branches of Social Sciences such as Economics, Psychology and for the past 60 years Sociologists take advantage of Mathematical tools to develop theories. Now, we also hear the terms such as Mathematical Biology and Mathematical Biophysics. Likewise, Mathematical Sociology is treated as application of mathematics in explaining complex social realities. This essay presents an

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overview of Mathematical Sociology and its main approaches.

There can be debate over the nature of Mathematical Sociology as to whether it is a Science or an Art. Its evolution as a Discipline or a Subject is debatable. However, this article will focus on the undeniable underlying truth of applying Mathematics in subject matters of Sociology e.g. concepts of set theory, mapping, matrix etc. can be used in representing social phenomena in human behaviour, migration, mobility etc. In this small article, we have illustrated few fundamentals of Maths which can be applied in quantitative social science research. The application may include from representing sociological phenomena or sentences in simple mathematical forms to develop models and equations.

2. History of Mathematical Sociology

The first contribution to Mathematical Sociology can be dated back to 1785 by Condorcet when he studied possibility of transitive outcomes of voting. Poison used probability for a jury to make verdict in 1837, Watson applied stochastic process model for family line extinction study in 1874. Voltera, Lotka carried out important work in Mathematical Demography and Diffusion modelling in 1930s. Dodd (1942) attempted to explain Society with mathematical approach by using Mathematical symbols and formulas, however it was later recognized in 1955 in diffusion model as a significant contribution in the field. In 1940s Rashevesky, a Biologist and his group in University of Chicago published many articles and in 1946 probably, Rashevesky first time used the term Mathematical sociology. Some significant publications from the group are Landau's work on hierarchy (Landau, 1951) and Rapoport's model of diffusion of networks (1953). Paul Lazars-field organized a group at Columbia and his early efforts led to the publication in 1954 of a collection of papers entitled 'Mathematical Thinking in the Social Sciences' which have an everlasting impact on the field. Simon, Homan and Festinger were some significant contributors of that period.

However, major contributions to the field came between late 1950s and middle 1960s. Coleman's 'Introduction to Mathematical Sociology' provides a significant way to conceptualize social processes by using Mathematical languages. White's study on kinship using Algebra in 1963 was another major contribution.

All these works belonged to US. Outside US a pioneering effort was done in by Richardson (1960) and Paris (1955) in Britain who developed differential equation model for armed races and stochastic model on social

mobility respectively. Similarly, Karlsson in 1958 and Boudon in 1960s were two other significant academicians from Scandinavia and France respectively. Later, many thinkers began to join e.g. Abell, Blalock, Dorein, Fararo, Zelditch, Anderson, Land, Spilerman, Krishnan, Heckathorn, Sorensen etc. Since last few decades, Social Network Analysis has gained popularity. This is also due to the fact of easily available and reliable data to study networks on World Wide Web. The concept can be traced to the seminal study conducted by Mark Granovetter (1974), who interviewed 282 professional and managerial men in Newton, Massachusetts but never concluded a causal relationship between network ties and actors (Lin[3]). Later in 1978, the small world study conducted in New York gives indirect linkage between ties and status attainment by actors. This has become a discipline with Social Network Analysis becoming one of the major focus area of it.

3. Applications

3.1. Developing theory:

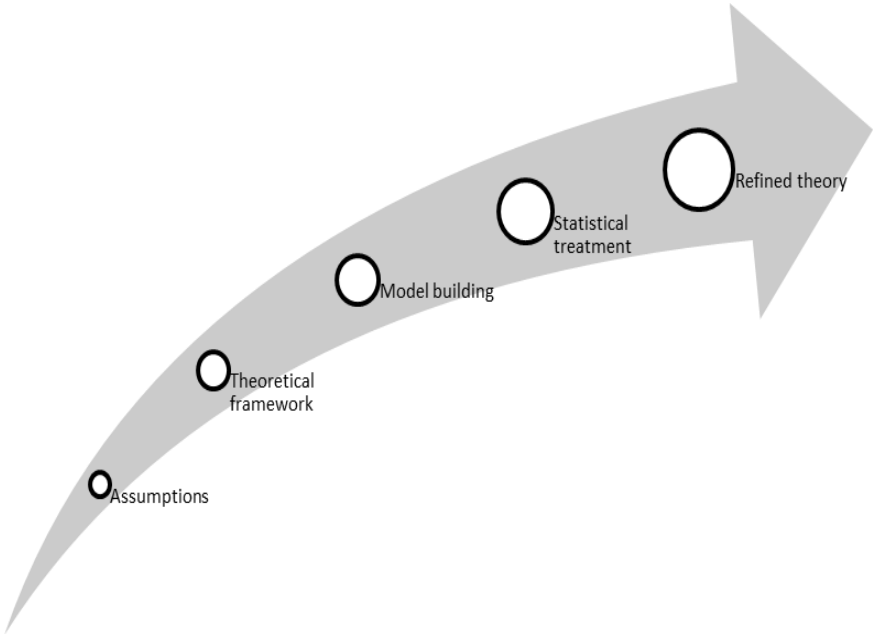

Figure1: Development of Social Theory Using Mathematical Modelling

In the evolution of Mathematical Sociology, Coleman's model on social process, Sorensen's vacancy competition model, Thomas Schelling's models of segregation, Duncan Watts' and Steven Strogatz's model of small

world dynamics, Markov process of social mobility, Harrison White's network analysis etc. have proven the usage of mathematical apparatus to analyze a social problem. There are many other examples which may be debated to be subject matter of other disciplines e.g. game theory by Von Neumann and Morgenstern and Nash which is of great importance in Economics however due to the very nature of individual choices involved in game theory, it is somehow touches the subject matter of psychology. For sociologists, game theories are of greater interest because it underlies the Weberian concept that action of ego depends upon action of alter (Swedberg2001).

Commonality of all these models or analysis is the logical flow of theoretical and model development resulting in a well improved theory which can be generalized in most of the cases. What has provided that logical simplicity? We may find the answer indeed the appropriate selection and usage of Mathematical and statistical tools. Yet, it is noteworthy that all the scholars remained focus on the social problem or theory they were in to and did not let the fanciful equations overshadow the subject. Fig.1 is an indicative process of theoretical development.

There are many concepts which can be applied to the field of Sociology. We are discussing few of them just to showcase the significance of the earlier in later without diminishing the 'boundaries' of the Sociology. Concepts along with suitable examples will be discussed in this section.

3.2. Mathematical languages and Social relations:

All the objects under the domain of one variable, making a sentential function or statement true or false when the object is put in place of variable, define a set

$$S = \{a : b\},$$

where, $S =$ set, $a =$ any variable, $b =$ function of a

Example: $S = \{x : x \text{ is scientist}\}$ if x is variable and domain is academicians.
So, S is set of all the scientists.

In case of more than one variable it may be represented in same way.

Example: Let *p, q* be two variables defined in domain of individuals) $R = \{p,\ q : p \text{ is related to } q\}$ relation may be family, friend etc. Supposing, P and Q are sets and f is relation which relates elements from P and Q in a

way that f has a unique pair say, (a, b) for each $a \in P$ then it will represent a mapping.

Mathematically, $f : P \to Q$, where P is domain and Q is co-domain or range. It is also written as $P \xrightarrow{f} Q$ which means P is mapped into Q by f. Technically, f is guiding elements of P in to Q. For, unique pair (a, b) the above relation will be represented as $\{f(a): a \in P\}$ Thus, variable $(a, b) \in f$, $a \in P$ and $b \in Q$. In sociology, choosing the variables and identifying the elements by defining the domain and range, we may represent the complex relationships using mapping as to develop an analytical model.

Example: Let P be set of individuals (elements) with a variable A (individuals under study) and Q be the set of human preferences (elements) with variable B. If the relation between the individuals and their choices is to be shown as f, we may represent it $f : P \to Q$.

Suppose the table below shows the data collected under the variables:

Variables	Values		
A	a_1	a_2	a_3
B	b_1	b_2	b_3

Assuming, everyone has at least a preference, we may find following relation defined by f in this case:

$$f = \{(a_1, b_1), (a_2, b_2), (a_3, b_3)\}.$$

It will be a tough task to curtail the vast application in to one essay however; we can discuss at least some basics to establish the importance of the same in sociology. Integration, Differential calculus, Probability and matrix are the other very useful tools in studying class migration in kinships or generation (Feraro[2]). Besides, Social mobility, imaging etc. has its wider application.

4. Conclusion

Sometimes, Mathematicians are faced with question of no standard solution or unrealistic modeling of existing social processes. However, efforts were continuously made in this regard. White (1970) applied Markov chain process to vacancies and not to individuals which proved out to be better fit of social mobility. In 1970s, the discipline focused on measurement of Mathematical theorizing. In 1990s, authors began to develop a common framework incorporating two or more theoretical basis in order to minimize the overlapping tendency of theories when developed

individually. It is evident in the work of Fararo and Skvoretz[4] when they assumed that actors are driven by payoffs or reinforcements and, second, that interactions and outcomes are generated by coupling two or more actors in situations where the choices made by one provide the payoffs or reinforcements experienced by the other and vice versa. Following these two principles, these two worked to develop a scheme for unifying various substantive theories of deviance and control (Fararo and Skvoretz[4]). With emergency of World Wide Web, physical boundaries in the social world have collapsed and we are able to connect with each other with the help of social networking sites. Presently, Social Networking Analysis has been a greater interest of the contributor to this discipline as it explains the influences, reasons attributed to individual decision making in any network. Thus we see how the discipline emerged from very simple mathematical language approach to a very complex study of networks. Despite subtle differences which may be due to uneasiness of sociologists to accept the social situations in to mathematical equations all the time, it is for sure that there is promising pact between the two disciplines on case to case basis. Beginning of 'Journal of Mathematical Sociology' in 1971 published by Taylor & Francis Group is significant milestone the discipline has achieved to keep the flame of Mathematical Sociology alive. Even the multi donor agencies like World Bank has acknowledged and adopted the SNA as a tool to study community structures as evident in the study by Hoff and Sen in 2004.

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