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Effects of Viscous Dissipation on MHD Free Convection Flow of Radiation Absorbing Fluid through a Porous Medium with Heat Source/Sink

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Abstract: Viscous dissipation effects on a steady two-dimensional flow of a viscous incompressible electrically conducting fluid past a vertical plate through porous medium with heat source\sink and radiation absorption is studied here. The coupled non-linear partial differential equations are solved by perturbation method. The influence of various material parameters on velocity, temperature and concentration are studied and discussed with the help of graphs. The expressions for Skin friction and Nusselt number are also derived and discussed numerically.

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Keywords: MHD, heat source, radiation absorption, viscous dissipation, free convection.

1. Introduction

In the flow of all fluids, mechanical energy is degraded into heat and this process is called viscous dissipation. The problem of viscous dissipation in the fluid flow have many practical application, an example is oil products transportation through ducts and polymer processing. The viscous dissipation effects in natural convection process have been studied by Gebhart[1] and Gebhart and Mollendorf[2]. Soundalgekar et. al[3] studied the viscous dissipation effect on unsteady free convection flow past an infinite, vertical porous plate with constant suction. Mamun et. al[4] presented a paper on MHD–conjugate heat transfer analysis for a vertical flat plate in presence of viscous dissipation and heat generation. Soundalgekar[5] studied viscous dissipative effects on unsteady free convective flow past a vertical porous plate with constant suction. The problem of Dissipation effects on MHD

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nonlinear flow and heat transfer past a porous surface with prescribed heat flux have been studied by Anjali Devi and Ganga[6]. Radiation and viscous dissipation effects on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with hall current in the presence of chemical reaction is explained by Reddy[7]. In view of several physical problems such as fluids undergoing exothermic or endothermic chemical reactions, heat source/sink has great significances. It has also numerous industrial applications viz., manufacturing of ceramics or glass ware, polymer production and food processing. Singh et. al[8] studied the effects of chemical reaction and radiation on MHD convective heat and mass transfer flow past a semi-infinite vertical moving plate with time dependent suction. Effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical moving in a porous medium with heat source and suction studied by Seethamahalakshmi et. al[9]. Raju[10] studied unsteady MHD free convection boundary layer flow of radiation absorbing kuvshinski fluid through porous medium. Sudarshan Reddy et. al[11] reported effects of radiation and chemical reaction on free convection MHD flow through a porous medium bounded by vertical surface.

In this study, effects of viscous dissipation on MHD free convection flow of a viscous incompressible and electrically conducting fluid past a semi-infinite vertical plate are investigated. We obtain velocity, temperature and concentration profile by perturbation method and the effects of various parameters discussed in detail.

1. Formulation of the Problem and Governing Equations

Let's us considered a viscous electrically conducting fluid which steadily flows through a porous medium subjects to a uniform transverse magnetic field B_0. The x axis is taken along the vertical infinite plate in the upward direction and y axis normal to the plate. The temperature and concentration at wall are maintained T_w and C_w and far away from fluid are assumed to be T_∞ and C_∞ respectively. It is also assumed that induced magnetic field is neglected in comparison with applied magnetic field as Reynolds number is much less than unity. Now under Boussinesq's approximation the flow is governed by the following equations:

(1.1) $$v_y = 0.$$

(1.2) $$vv_y = \upsilon u_{yy} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \left(\frac{\sigma B_0^2}{\rho}\right)u + \left(\frac{\upsilon}{k}\right)u,$$

(1.3) $$vT_y = \left(\frac{\lambda}{\rho C_p}\right)T_{yy} - \frac{Q_0}{\rho C_p}(T - T_\infty) + Q_1^*(C - C_\infty) + \frac{\upsilon}{\rho C_p}u_y^2,$$

(1.4) $$vC_y = DC_{yy} - Kr'(C - C_\infty),$$

where u and v are velocity component in x, y direction, g is gravitational acceleration, ρ is the fluid density, υ kinematic viscosity, β, β^* coefficient of thermal and volumetric expansion, σ electric conductivity of the fluid, k permeability of porous medium, T the temperature of fluid near the plate and C is the corresponding concentration, λ thermal conductivity, C_p specific heat at constant pressure, Q_0 heat source/sink, Q_1^* coefficient of proportionality of the radiation, D diffusion coefficient, Kr chemical reaction parameter.

The boundary conditions for the problem are:

(1.5) $$u = 0, T = T_w, C = C_w, y = 0$$

$$u \to 0, T \to T_\infty, C \to C_\infty, y \to \infty$$

It is clear from equation of continuity (1.1), that the suction velocity is a constant.

(1.6) $v = -v_0$ (constant); $v_0 > 0$

On the introducing non dimensional quantities

(1.7) $$f(\eta) = \frac{u}{v_0},\ \eta = \frac{v_0 y}{\upsilon},\ M = \frac{\sigma B_0^2 \upsilon}{v_0^2 \rho},\ \theta = \frac{(T - T_\infty)}{T_w - T_\infty},\ C = \frac{(C - C_\infty)}{C_w - C_\infty}$$

$$Sc = \frac{\upsilon}{D}, \Pr = \frac{\rho \upsilon C_p}{\lambda}, Gr = \frac{g\beta\upsilon(T_w - T_\infty)}{v_0^3}, Gm = \frac{g\beta^*\upsilon(C_w - C_\infty)}{v_0^3}$$

$$E = \frac{v_0^2}{(T_w - T_\infty)C_p}, Kr = \frac{\upsilon Kr'}{v_0^2}, \alpha = \frac{v_0^2 k}{\upsilon^2}, \phi = \frac{Q_0 \upsilon}{\rho C_p v_0^2}, Q_1 = \frac{Q_1^* \upsilon(C_w - C_\infty)}{v_0^2(T_w - T_\infty)}.$$

In the view of above non dimensional variables, the governing equations (1.2), (1.3) and (1.4) in dimensionless form are:

(1.8) $$f'' + f' - f(\alpha^{-1} + M) = -Gr\theta - GmC\,,$$

(1.9) $$-\Pr\theta' = \theta'' - \Pr\phi\theta - \Pr Q_1 C + E\Pr(f')^2\,,$$

(1.10) $$C'' + ScC' - KrScC = 0\,.$$

and corresponding boundary conditions are

$$\eta = 0 : f = 0, \theta = 1, C = 1$$

(1.11) $$\eta \to \infty : f \to 0, \theta \to 0, C \to 0\,,$$

where Gr is Grashof number, Gm is solutal Grashof number, $\Pr$ is Prandtl number, M is magnetic field parameter, α is permeability parameter, ϕ is the heat source parameter and Q_1 is the absorption of radiation parameter, Sc is Schmidt number and Kr is chemical reaction parameter.

1.1 Method of Solution:

The numerical solutions of the non-linear partial differential equations (1.8), (1.9) and (1.10) under the boundary condition (1.11) has been performed by applying perturbation method. First the governing equations are converted into ordinary differential equation and solved analytically. This can be done by representing the velocity, temperature and concentration as:

(1.1.1) $$f(\eta) = f_0(\eta) + Ef_1(\eta) + O(E^2)$$

$$\theta(\eta) = \theta_0(\eta) + E\theta_1(\eta) + O(E^2)$$

$$C(\eta) = C_0(\eta) + EC_1(\eta) + O(E^2)$$

Substituting (1.1.1) in Equations (1.8) – (1.10) and equating the harmonic and non – harmonic terms, and neglecting the higher order terms of $O(E^2)$, we obtain zeroth order terms

(1.1.2) $$f_0'' + f_0' - f_0(\alpha^{-1} + M) = -Gr\theta_0 - GmC_0,$$

(1.1.3) $$\theta_0'' + \Pr\theta_0' - \Pr\phi\theta_0 = -\Pr Q_1 C_0,$$

(1.1.4) $$C_0'' + ScC_0' - KrScC_0 = 0,$$

Subject to boundary conditions

$$\eta = 0,\ f_0 = 0, \theta_0 = 1, C_0 = 1,$$

(1.1.5) $$\eta \to \infty,\ f_0 \to 0, \theta_0 \to 0, C_0 \to 0.$$

First order terms

(1.1.6) $$f_1'' + f_1' - f_1(\alpha^{-1} + M) = -Gr\theta_1 - GmC_1,$$

(1.1.7) $$\theta_1'' + \Pr\theta_1' - \Pr\phi\theta_1 = -\Pr Q_1 C_1 - \Pr(f')^2,$$

(1.1.8) $$C_1'' + ScC_1' - KrScC_1 = 0.$$

Subject to boundary conditions

$$\eta = 0,\ f_1 = 0, \theta_1 = 0, C_1 = 0,$$

(1.1.9) $$\eta \to \infty,\ f_1 \to 0, \theta_1 \to 0, C_1 \to 0.$$

Solving equations (1.1.2)-(1.1.4) and (1.1.6)-(1.1.8) subjected to (1.1.5) and (1.1.9) we get

$$\begin{aligned} f &= A_7 \exp(-m_3\eta) + A_3 \exp(-m_2\eta) + A_6 \exp(-m_1\eta) + E\big[A_{22}\exp(-m_3\eta) \\ &\quad + A_{15}\exp(-m_2\eta) + A_{16}\exp(-2m_3\eta) + A_{17}\exp(-2m_2\eta) + A_{18}\exp(-2m_1\eta) \\ &\quad + A_{19}\exp(-m_4\eta) + A_{20}\exp(-m_5\eta) + A_{21}\exp(-m_6\eta)\big] \end{aligned}$$

$$\begin{aligned} \theta &= A_2 \exp(-m_2\eta) + A_1 \exp(-m_1\eta) + E\big[A_{14}\exp(-m_2\eta) \\ &\quad + A_8\exp(-2m_3\eta) + A_9\exp(-2m_2\eta) A_{10}\exp(-2m_1\eta) \\ &\quad + A_{11}\exp(-m_4\eta) + A_{12}\exp(-m_5\eta) + A_{13}\exp(-m_6\eta)\big], \end{aligned}$$

$$C = \exp(-m_1\eta).$$

The physical quantities of interest for the present problem are wall shear stress (τ_w) and rate of heat transfer (q_w) which are given by the expression

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \rho v_0^2 f^{'}(0)\,.$$

Therefore, local skin friction τ is defined by

$$\tau = \frac{\tau_w}{\rho v_0^2} = f^{'}\left(0\right), -m_3 A_7 - m_2 A_3 - m_1 A_6 + E\begin{bmatrix} -m_3 A_{22} - m_2 A_{15} - 2m_3 A_{16} - 2m_2 A_{17} \\ -2m_1 A_{18} - m_4 A_{19} - m_5 A_{20} - m_6 A_{21} \end{bmatrix}.$$

The rate of heat transfer at surface is defined as

$$q_w = -\lambda \frac{\partial T}{\partial y}\bigg|_{y=0} = -\lambda \frac{v_0}{\upsilon}(T_w - T_\infty)\theta^{'}(0).$$

By the definition of local Nusselt number

$$Nu_x = \frac{q_w}{(T_w - T_\infty)}\frac{x}{\lambda},$$

$$\frac{Nu_x}{\mathrm{Re}_x} = -\theta^{'}(0) = -m_2 A_2 - m_1 A_1 + E\left[-m_2 A_{14} - 2m_3 A_8 - 2m_2 A_9 - 2m_1 A_{10} - m_4 A_{11} - m_5 A_{12} - m_6 A_{13}\right],$$

where $\mathrm{Re}_x = \dfrac{v_0 x}{\upsilon}$ is Reynolds number.

2. Results and Discussions

The effects of various parameters such as magnetic field parameter, Schmidt number, chemical reaction parameter, radiation absorption parameter, heat source parameter, viscous dissipative heat, Prandtl number and permeability parameter on dimensional less velocity, temperature and concentration profile are reported graphically in Fig.1-15.

Fig.1 presents the effects of magnetic field parameter (M) on velocity profile it is obvious that velocity decreases with increases in magnetic field parameter. Because the magnetic force which is normal to the plate, retards the flow. Hence this type of retarding force reduces the fluid velocity.The variation of velocity, temperature and concentration profile with Schmidt number (Sc) shows that lesser molecular diffusivity higher the velocity and concentration also as Schmidt number increases temperature decreases. These results are clearly shown in Fig.2-4.

Fig.5-7 displays the effect of chemical reaction parameter (Kr) on velocity, temperature and concentration profile. We observe that velocity, temperature and concentration decreases with increasing chemical reaction parameter.This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity.

The effect of absorption of radiation parameter (Q_1) on the velocity and temperature profile is shown in Fig.8-9. We observe that large Q_1 values increases buoyancy force which accelerates the flow rate and in the boundary layer as the radiated heat is absorbed by the fluid which in turn increases the temperature of the fluid.

Fig.10-11 exhibits the influence of heat source parameter (ϕ) on velocity and temperature profile. It is noticed that velocity and temperature decrease as heat source parameter increases. This is because when heat is absorbed, the buoyancy force decreases the temperature.

From Fig.12-13, we noticed that an increase in viscous dissipative heat ((E)) leads to increase in both velocity and temperature .This is due to the heat energy stored in the liquid because of frictional heating.

Fig.14 depicts the effect of Prandtl number $\Pr$ on velocity profile. It is observed that the velocity increases with increasing of Prandtl number. The effect of permeability parameter α on velocity profile is shown in Fig.15.the velocity increases with an increase in permeability parameter.

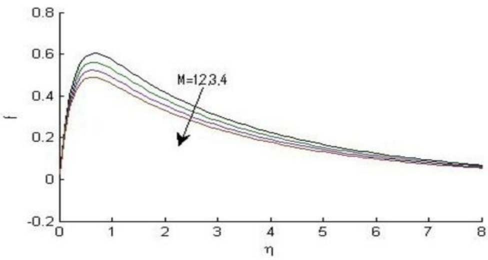

Figure 1: Effect of M on velocity profile with $Gr = 5, Gm = 5$
$Kr = 0.1, Q_1 = 1, \alpha = 0.1, \phi = 1, E = 0.01, \Pr = 0.71, Sc = 0.22$

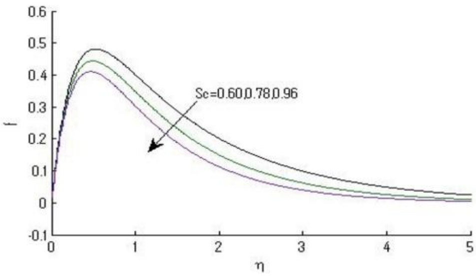

Figure 2: Effects of Sc on velocity profile with $Gr = 5, Gm = 5$
$Kr = 0.1, Q_1 = 1, M = 1, \alpha = 0.1, \phi = 1, E = 0.01, \Pr = 0.71$

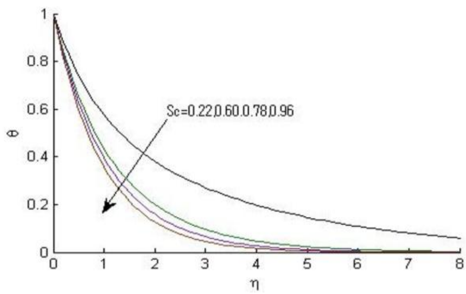

Figure 3: Effects of Sc on temperature profile with $Gr = 5, Gm = 5$
$Kr = 0.1, Q_1 = 1, M = 1, \alpha = 0.1, \phi = 1, E = 0.01, \Pr = 0.71$

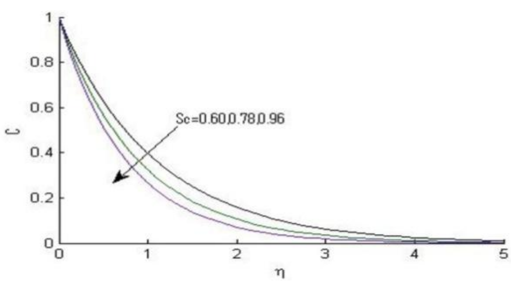

Figure 4: Concentration profile for different value of Sc with
$Gr = 5, Gm = 5, Kr = 0.1, Q_1 = 1, M = 1, \alpha = 0.1, \phi = 1, E = 0.01, \Pr = 0.71$

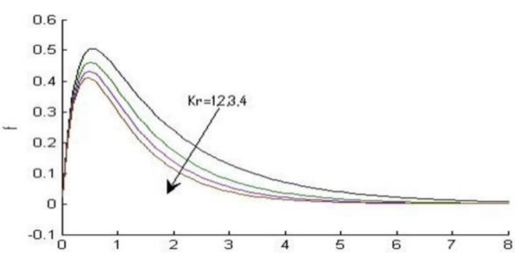

Figure 5: Effect of Kr on velocity profile with $Gr = 5, Gm = 5$
$Q_1 = 1, M = 1, \alpha = 0.1, \phi = 1, E = 0.01, \Pr = 0.71, Sc = 0.22$

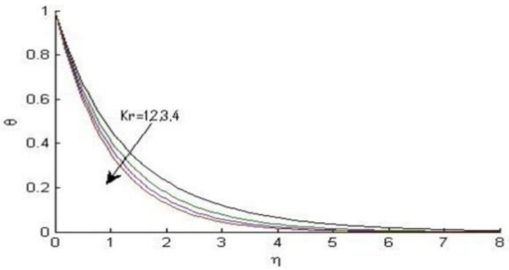

Figure 6: Effect of Kr on temperature profile with $Gr = 5, Gm = 5$
$Q_1 = 1, M = 1, \alpha = 0.1, \phi = 1, \Pr = 0.71, Sc = 0.22 \; E = 0.01$

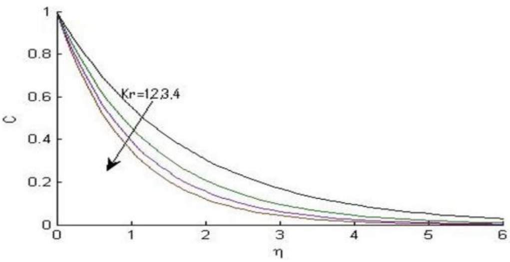

Figure 7: Concentration profile for different values of Kr with
$Gr = 5, Gm = 5, Q_1 = 1, M = 1, \alpha = 0.1, \phi = 1, E = 0.01, \Pr = 0.71, Sc = 0.22$

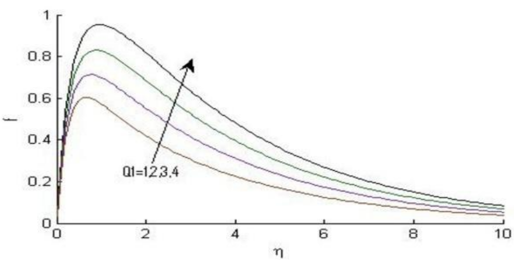

Fig.8 Effect of Q_1 on velocity profile with
$Gr = 5, Gm = 5 \;\; Kr = 0.1, M = 1, \alpha = 0.1, \phi = 1, E = 0.01, \Pr = 0.71, Sc = 0.22$

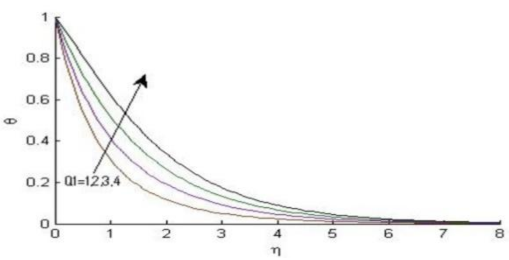

Fig.9 Effect of Q_1 on temperature profile with
$Gr = 10, Gm = 2 \;\; Kr = 0.5, M = 2, \alpha = 0.5, \phi = 2, E = 0.01, \Pr = 0.71, Sc = 0.4$

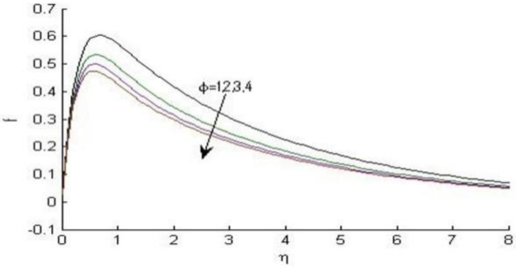

Figure 10: Effects of ϕ on velocity profile with $Gr = 5, Gm = 5$
$Q_1 = 1, Kr = 0.1, M = 1, \alpha = 0.1, E = 0.01, \Pr = 0.71, Sc = 0.22$

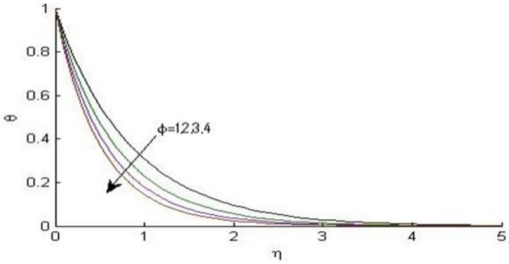

Figure 11: Effects of ϕ on temperature profile with $Gr = 4, Gm = 2$
$Q_1 = 0.5, Kr = 0.5, M = 2, \alpha = 0.5, E = 0.01, \Pr = 0.71, Sc = 0.8$

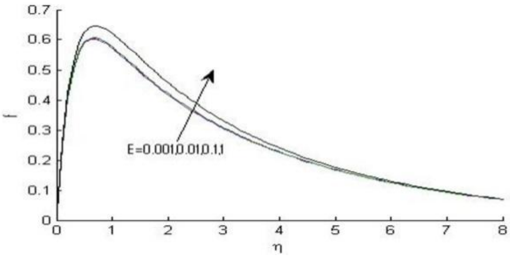

Figure 12: Effects of E on velocity profile with $Gr = 5, Gm = 5$
$Q_1 = 1, Kr = 0.1, M = 1, \alpha = 0.1, \phi = 1, \Pr = 0.71, Sc = 0.22$

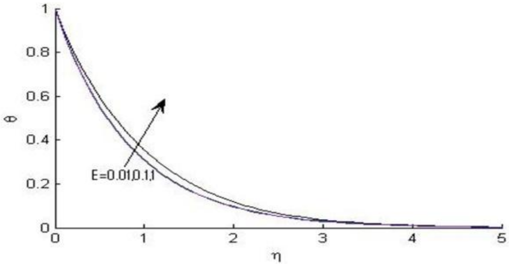

Figure 13: Temperature profile for different values of E with
$Gr = 4, Gm = 2, Q_1 = 0.5, Kr = 0.5, M = 2, \alpha = 0.5, \phi = 1, E = 0.01$ $\Pr = 0.71, Sc = 0.8$

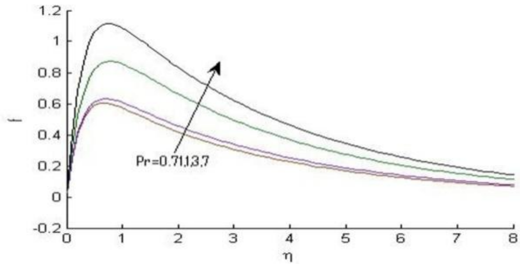

Figure 14: Effects of Pr on velocity profile with $Gr = 5, Gm = 5$
$Q_1 = 1, Kr = 0.1, M = 1, \alpha = 0.1, \phi = 1, E = 0.01, Sc = 0.22$

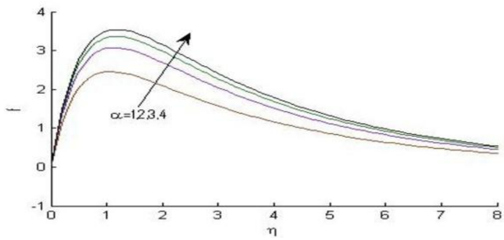

Figure 15: Effects of α on velocity profile with $Gr = 5, Gm = 5$
$Q_1 = 1, Kr = 0.1, M = 1, \phi = 1, E = 0.01, \Pr = 0.71, Sc = 0.22$

3. Conclusions

In this assertion, the effects of viscous dissipative heat, magnetic field parameter, Prandtl number, permeability parameter, Schmidt number, radiation absorption parameter, heat source parameter and chemical reaction parameter on velocity, concentration and temperature are obtained which can be elucidated as follows:

The velocity as well as temperature decreases with an increase in Schmidt number, chemical reaction parameter and heat source parameter but the velocity and temperature increases with radiation absorption parameter and viscous dissipative heat parameter.

The concentration is observed to be decreased when chemical reaction parameter, Schmidt number increase.

The velocity decreases when an increase in magnetic field parameter whereas it increases with Prandtl number and permeability parameter.

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