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Free Convective Oscillatory Couette Flow through Porous Medium in the Presence of Transverse Magnetic Field and Heat Source*

P. R. Sharma and Sushila Choudhary

Department of Mathematics
University of Rajasthan
Jaipur-302004, India
Email: profprsharma@yahoo.com, sumathru11@gmail.com

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Abstract: Free convective oscillatory flow of a viscous incompressible electrically conducting fluid through porous medium between infinite non conducting vertical parallel plates: one is moving and the other is stationary, in the presence of transverse magnetic field and heat source is investigated. The governing equations for velocity and temperature distributions are derived, solved by using perturbation technique and the effects of different physical parameters are shown through graphs. The expressions for the rate of shear stress and the rate of heat transfer are obtained, discussed numerically and their numerical values for various values of physical parameters are presented through Tables.

Keywords: MHD, dissipative, Couette flow, heat source, skin friction and Nusselt number.

2010 Mathematics Classification No.: 76W05, 76S05, 76D09, 76R10.

1. Introduction

The MHD Couette flow of a fluid through a porous medium is encountered in many technical and industrial applications. This field has a broad range of applications in various disciplines, such as cooling of electronic systems, petroleum reservoirs, storage of nuclear wastes, ground hydrology etc. Katagiri[1]studied the flow formation in Couette motion in magnetohydro-dynamics. The flow in a channel due to periodic pressure gradient has been investigated by Drake[2]. Gupta and Arora[3] discussed the hydromagnetic flow between two parallel planes: one oscillating and the other fixed. Boundary and inertia effects on flow and heat transfer in porous media have been analyzed by Vafai and Tien[4]. Sengupta and Ray[5] studied the oscillatory Couette flow of a viscoelastic electrically conducting fluid

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through a porous medium within a parallel plate channel in the presence of a transverse magnetic field in a rotating system. An oscillatory hydromagneticCouette flow in a rotating system has been discussed by Singh[6]. Al-Hadhramiet al.[7] analyzed the flow through horizontal channels of porous materials. Unsteady flow and heat transfer of an electrically conducting viscous incompressible fluid between two non-conducting parallel porous plates under uniform transverse magnetic field has been discussed by Sharma and Chaturvedi[8]. Sharma and Yadav[9] studied the heat transfer through three-dimensional Couette flow between a stationary porous plate bounded by porous medium and a moving plate. Unsteady MHD Stokes' flow of viscous fluid between two parallel porous plates has been investigated by Ganesh and Krishnambal[10]. Sharma et al.[11]discussed three-dimensional Couette flow with heat transfer through a porous medium of variable permeability. The Couette flow between two parallel infinite plates in the presence of transverse magnetic field has been studied by Singh and Okwoyo[12]. Raju and Varma[13] discussed unsteady MHD free convection oscillatory Couette flow through a porous medium with periodic wall temperature. Effect of volumetric heat generation/absorption on convective heat and mass transfer in porous medium between two vertical porous plates has been analyzed by Sharma and Dadheech[14].

The aim of present paper is to investigate free convective oscillatory flow of a viscous incompressible electrically conducting fluid through a porous medium between infinite non conducting vertical parallel plates: one is moving and the other is stationary, in the presence of transverse magnetic field and heat source by employingperturbation method.

2. Mathematical Formulation of the Problem

Consider free convective oscillatory flow and heat transfer of a viscous incompressible electrically conducting fluid through porous medium bounded by infinite non conducting vertical parallel plates in the presence of transverse magnetic field and heat source. One of the plates is suddenly started moving from rest with velocity

$$U^*\left(t^*\right) = U_0\left(1+\varepsilon e^{i\omega^* t^*}\right), \tag{1}$$

where U_0 is the mean constant velocity, $\varepsilon << 1$ is the amplitude of the velocity variation, ω^* is the frequency of oscillations and t^* is the time. The x^*-axis is taken along the moving plate in the vertically upward direction and y^*-axis is taken perpendicular to this plate and temperature of the moving plate fluctuates with time. The stationary plate is placed at $y^* = d$ with temperature T_s . All the flow variables are independent of x^* , since plates are infinite in extent and so their derivatives with respect to x^* vanish. Only non-zero velocity component is in the x^* - direction. The non-zero components of velocity and temperature are functions of y^* and t^* only.

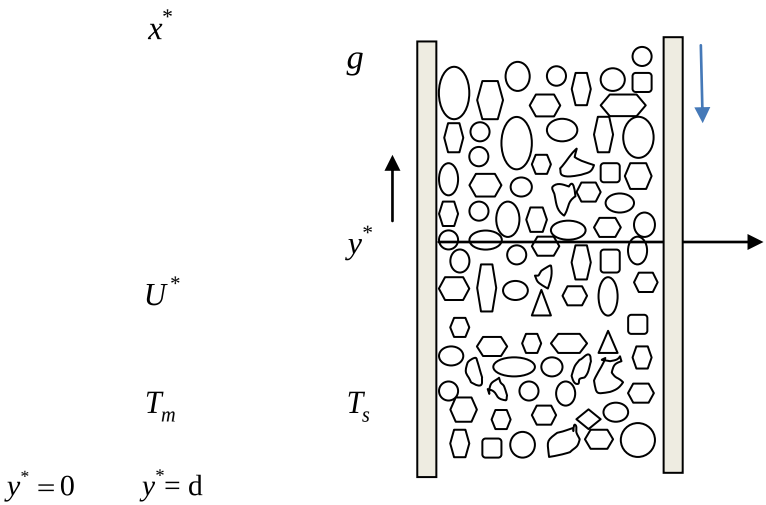

Figure 1: Physical model and the coordinate system

Under these assumptions, the governing equations for free convective oscillatory flow and heat transfer are given by

$$\frac{\partial u^*}{\partial t^*} = \frac{\partial U^*}{\partial t^*} + \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta\left(T^* - T_s\right) - \left(\frac{\nu}{K^*} + \frac{\sigma_e B_0^2}{\rho}\right)\left(u^* - U^*\right), \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu}{\rho C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + Q^*\left(T^* - T_s\right), \tag{3}$$

where u^* is the velocity component in x^*- direction, g is the acceleration due to gravity, β is the thermal expansion coefficient, T^* is the temperature of the fluid in the boundary layer, ρ is the fluid density, ν is the kinematic viscosity, K^* is the permeability of porous medium, σ_e is the electrical conductivity of the fluid, B_0 is the magnetic field intensity, κ is the thermal conductivity, C_p is the specific heat at constant pressure and Q^* is the heat source parameter.The boundary conditions are

(4) $$\begin{cases} y^* = 0:\ u^* = U^*\left(t^*\right) = U_0\left(1+\varepsilon e^{i\omega^* t^*}\right), \\ T^* = T_m + \varepsilon\left(T_m - T_s\right)e^{i\omega^* t^*}; \quad y^* = d:\ u^* = 0, \quad T^* = T_s, \end{cases}$$

where T_m is the temperature of moving plate.

3. Method of Solution

Introducing the following dimensionless quantities

$$y = \frac{y^*}{d}, u = \frac{u^*}{U_0}, t = \frac{\omega \nu t^*}{d^2}, \omega = \frac{\omega^* d^2}{\nu}, U = \frac{U^*}{U_0},$$

(5) $$Gr = \frac{g\beta d^2\left(T_m - T_s\right)}{\nu U_0}, \theta = \frac{T^* - T_s}{T_m - T_s}, M = \frac{\sigma_e B_0^2 d^2}{\rho\nu},$$

$$\Pr = \frac{\mu C_p}{\kappa}, K = \frac{K^*}{d^2}, Ec = \frac{U_0^2}{C_p\left(T_m - T_s\right)}, Q = \frac{Q^* d^2}{\mu C_p},$$

into the equations (2) and (3), we get

(6) $$\omega\frac{\partial u}{\partial t} = \omega\frac{dU}{dt} + \frac{\partial^2 u}{\partial y^2} + Gr\theta - \left(M + \frac{1}{K}\right)\left(u - U\right),$$

(7) $$\omega\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 + Q\theta,$$

where u is the dimensionless velocity along x-axis, U is the velocity of moving plate, θ is the dimensionless temperature, t is the time, Gr is the Grashof number for heat transfer, M is the Hartmann number, K is the permeability parameter, $\Pr$ is the Prandtl number, Ec is the Eckert number and Q is the heat source parameter.

The boundary conditions in dimensionless form are reduced to

(8) $$y = 0: u = U = 1 + \varepsilon e^{it}; y = 1: u = 0, \theta = 0.$$

Since $\varepsilon \ll 1$ is very small, so the velocity and temperature distributions are assumed as given below

(9)
$$\begin{cases} u(x,t) = u_0(y) + \varepsilon e^{it} u_1(y) + O(\varepsilon^2), \\ \theta(y,t) = \theta_0(y) + \varepsilon e^{it} \theta_1(y) + O(\varepsilon^2). \end{cases}$$

Substituting (9) into the equations (6) and (7) and equating the coefficients of like powers of ε and neglecting the terms of $O(\varepsilon^2)$, we obtain

(10)
$$u_0'' - \left(M + \frac{1}{K}\right) u_0 = -Gr\theta_0 - \left(M + \frac{1}{K}\right),$$

(11)
$$u_1'' - \left(M + \frac{1}{K} + i\omega\right) u_1 = -Gr\theta_1 - \left(M + \frac{1}{K} + i\omega\right),$$

(12)
$$\theta_0'' + \Pr Q\theta_0 = -\Pr Ec\left(u_0'\right)^2,$$

(13)
$$\theta_1'' + \Pr(Q - i\omega)\theta_1 = -2\Pr Ec u_0' u_1',$$

where prime denotes the differentiation with respect to y.

Now, the corresponding boundary conditions are reduced to

(14)
$$\begin{cases} y = 0 : u_0 = 1, \quad u_1 = 1, \quad \theta_0 = 1, \quad \theta_1 = 1; \\ y = 1 : u_0 = 0, \quad u_1 = 0, \quad \theta_0 = 0, \quad \theta_1 = 0. \end{cases}$$

The equations (10) to (13) are still coupled ordinary second order differential equations. Since the Eckert number $Ec \ll 1$ for incompressible fluid flows, therefore u_0, u_1, θ_0 and θ_1 can be expanded in the powers of Ec as given below

(15)
$$u_0(y) = u_{00}(y) + Ecu_{01}(y) + O\left(Ec^2\right),$$

(16)
$$u_1(y) = u_{10}(y) + Ecu_{11}(y) + O\left(Ec^2\right),$$

(17)
$$\theta_0(y) = \theta_{00}(y) + Ec\theta_{01}(y) + O(Ec^2),$$

(18)
$$\theta_1(y) = \theta_{10}(y) + Ec\theta_{11}(y) + O\left(Ec^2\right).$$

Substituting equations (15) to (18) into the equations (10) to (13), equating the coefficients of like powers of Ec and neglecting the terms of $O(Ec^2)$, we get

$$u_{00}'' - \left(M + \frac{1}{K}\right)u_{00} = -Gr\theta_0 - \left(M + \frac{1}{K}\right), \tag{19}$$

$$u_{01}'' - \left(M + \frac{1}{K}\right)u_{01} = -Gr\theta_{01}, \tag{20}$$

$$u_{10}'' - \left(M + \frac{1}{K} + i\omega\right)u_{10} = -Gr\theta_{10} - \left(M + \frac{1}{K} + i\omega\right), \tag{21}$$

$$u_{11}'' - \left(M + \frac{1}{K} + i\omega\right)u_{11} = -Gr\theta_{11}, \tag{22}$$

$$\theta_{00}'' + \Pr Q\theta_{00} = 0, \tag{23}$$

$$\theta_{01}'' + \Pr Q\theta_{01} = -\Pr\left(u_{00}'\right)^2, \tag{24}$$

$$\theta_{10}'' + \Pr\left(Q - i\omega\right)\theta_{10} = 0, \tag{25}$$

$$\theta_{11}'' + \Pr\left(Q - i\omega\right)\theta_{11} = -2\Pr u_{00}' u_{10}'. \tag{26}$$

The corresponding boundary conditions are reduced to

$$\begin{cases} y = 0: u_{00} = 1,\ u_{01} = 0,\ u_{10} = 1,\ u_{11} = 0,\ \theta_{00} = 1,\ \theta_{01} = 0,\ \theta_{10} = 1,\ \theta_{11} = 0; \\ y = 1: u_{00} = 0,\ u_{01} = 0,\ u_{10} = 0,\ u_{11} = 0,\ \theta_{00} = 0,\ \theta_{01} = 0,\ \theta_{10} = 0,\ \theta_{11} = 0. \end{cases} \tag{27}$$

Equations (19) to (26) are ordinary coupled second order differential equations and solved under the boundary conditions (27). Through straight forward calculations, the solutions of $u_{00}, u_{01}, u_{10}, u_{11}, \theta_{00}, \theta_{01}, \theta_{10}$ and θ_{11} are known. Finally, the expressions of $u\left(y,t\right)$ and $\theta\left(y,t\right)$ are known and given by

$$\begin{aligned} u = {} & A_{10}e^{\sqrt{A}y} + A_9 e^{-\sqrt{A}y} + A_7 e^{iA_1 y} + A_8 e^{-iA_1 y} + 1 + Ec\Big(A_{70}e^{\sqrt{A}y} + A_{69}e^{-\sqrt{A}y} \\ & + A_{56}e^{iA_1 y} + A_{57}e^{-iA_1 y} + A_{58}e^{2\sqrt{A}y} + A_{59}e^{-2\sqrt{A}y} - A_{60}e^{2iA_1 y} - A_{61}e^{-2iA_1 y} \\ & + A_{62}e^{A_{24}y} + A_{63}e^{-A_{24}y} - A_{64}e^{A_{25}y} - A_{65}e^{-A_{25}y} + A_{66}\Big) + \varepsilon e^{it}\Big\{A_{14}e^{\sqrt{B}y} \\ & + A_{13}e^{-\sqrt{B}y} + A_{11}e^{iA_4 y} + A_{12}e^{-iA_4 y} + 1 + Ec\Big(A_{92}e^{\sqrt{B}y} + A_{91}e^{-\sqrt{B}y} \end{aligned} \tag{28}$$

$$+A_{71}e^{iA_4y}+A_{72}e^{-iA_4y}+A_{73}e^{A_{30}y}+A_{74}e^{-A_{30}y}-A_{75}e^{A_{31}y}-A_{76}e^{-A_{31}y}$$
$$+A_{77}e^{A_{32}y}+A_{78}e^{-A_{32}y}-A_{79}e^{A_{33}y}+A_{80}e^{-A_{33}y}+A_{81}e^{A_{34}y}+A_{82}e^{-A_{34}y}$$
$$-A_{83}e^{A_{35}y}-A_{84}e^{-A_{35}y}-A_{85}e^{A_{36}y}-A_{86}e^{-A_{36}y}+A_{87}e^{A_{33}y}+A_{88}e^{-A_{37}y}\Big)\Big\},$$

where A_1 to A_{92} are constants and their expressions are not included for the sake of brevity.

(29)
$$\theta=A_2e^{iA_1y}+A_3e^{-iA_1y}+Ec\Big(A_{29}e^{iA_1y}+A_{28}e^{-iA_1y}-A_{15}e^{2\sqrt{A}y}-A_{16}e^{-2\sqrt{A}y}$$
$$-A_{17}e^{2iA_1y}-A_{18}e^{-2iA_1y}-A_{19}e^{A_{24}y}-A_{20}e^{-A_{24}y}+A_{21}e^{A_{25}y}+A_{22}e^{-A_{25}y}$$
$$+A_{23}\Big)+\varepsilon e^{it}\Big\{A_5e^{iA_4y}+A_6e^{-iA_4y}+Ec\Big(A_{55}e^{iA_4y}+A_{54}e^{-iA_4y}-A_{38}e^{A_{30}y}$$
$$-A_{39}e^{-A_{30}y}+A_{40}e^{A_{31}y}+A_{41}e^{-A_{31}y}-A_{42}e^{A_{32}y}-A_{43}e^{-A_{32}y}+A_{44}e^{A_{33}y}$$
$$+A_{45}e^{-A_{33}y}-A_{46}e^{A_{34}y}-A_{47}e^{-A_{34}y}+A_{48}e^{A_{35}y}+A_{49}e^{-A_{35}y}+A_{50}e^{A_{36}y}$$
$$+A_{51}e^{-A_{36}y}-A_{52}e^{A_{33}y}-A_{53}e^{-A_{37}y}\Big)\Big\},$$

where A_1 to A_{92} are constants and their expressions are not included for the sake of brevity.

4. Skin friction Coefficient

The coefficient of skin friction at the plates is given by

(30)
$$C_f=\frac{d\tau_\omega}{\mu U_0}=\left(\frac{\partial u}{\partial y}\right)_{y=0\,\&\,1},$$

where τ_ω is the wall shear stress given by $\tau_\omega=\mu\left(\frac{\partial u^*}{\partial y^*}\right)_{y^*=0\,\&\,d}$.

The expression of skin-friction coefficient at the moving plate is given by

(31)
$$C_{f_m}=\sqrt{A}\left(A_{10}-A_9\right)+iA_1\left(A_7-A_8\right)+Ec\Big\{\sqrt{A}\left(A_{70}-A_{69}\right)$$
$$+iA_1\left(A_{56}-A_{57}\right)+2\sqrt{A}\left(A_{58}-A_{59}\right)-2iA_1\left(A_{60}-A_{61}\right)$$
$$+A_{24}\left(A_{62}-A_{63}\right)-A_{25}\left(A_{64}-A_{65}\right)\Big\}+\varepsilon e^{it}\Big[\sqrt{B}\left(A_{14}-A_{13}\right)$$
$$+iA_4\left(A_{11}-A_{12}\right)+Ec\Big\{\sqrt{B}\left(A_{92}-A_{91}\right)+iA_4\left(A_{71}-A_{72}\right)$$
$$+A_{30}\left(A_{73}-A_{74}\right)-A_{31}\left(A_{75}-A_{76}\right)+A_{32}\left(A_{77}-A_{78}\right)$$

$$-A_{33}\left(A_{79}-A_{80}\right)+A_{34}\left(A_{81}-A_{82}\right)-A_{35}\left(A_{83}-A_{84}\right)$$
$$-A_{36}\left(A_{85}-A_{86}\right)+A_{37}\left(A_{87}-A_{88}\right)\}\Big].$$

The expression of skin friction coefficient at stationary plate is given by

(32)
$$\begin{aligned}C_{f_s} =& \sqrt{A}\left(A_{10}e^{\sqrt{A}}-A_9e^{-\sqrt{A}}\right)+iA_1\left(A_7e^{iA_1}-A_8e^{-iA_1}\right)\\
&+Ec\Big\{\sqrt{A}\left(A_{70}e^{\sqrt{A}}-A_{69}e^{-\sqrt{A}}\right)+iA_1\left(A_{56}e^{iA_1}-A_{57}e^{-iA_1}\right)\\
&+2\sqrt{A}\left(A_{58}e^{2\sqrt{A}}-A_{59}e^{-2\sqrt{A}}\right)-2iA_1\left(A_{60}e^{2iA_1}-A_{61}e^{-2iA_1}\right)\\
&+A_{24}\left(A_{62}e^{A_{24}}-A_{63}e^{-A_{24}}\right)-A_{25}\left(A_{64}e^{A_{25}}-A_{65}e^{-A_{25}}\right)\Big\}\\
&+\varepsilon e^{it}\Big[\sqrt{B}\left(A_{14}e^{\sqrt{B}}-A_{13}e^{-\sqrt{B}}\right)+iA_4\left(A_{11}e^{iA_4}-A_{12}e^{-iA_4}\right)\\
&+Ec\Big\{\sqrt{B}\left(A_{92}e^{\sqrt{B}}-A_{91}e^{-\sqrt{B}}\right)+iA_4\left(A_{71}e^{iA_4}-A_{72}e^{-iA_4}\right)\\
&+A_{30}\left(A_{73}e^{A_{30}}-A_{74}e^{-A_{30}}\right)-A_{31}\left(A_{75}e^{A_{31}}-A_{76}e^{-A_{31}}\right)\\
&+A_{32}\left(A_{77}e^{A_{32}}-A_{78}e^{-A_{32}}\right)-A_{33}\left(A_{79}e^{A_{33}}-A_{80}e^{-A_{33}}\right)\\
&+A_{34}\left(A_{81}e^{A_{34}}-A_{82}e^{-A_{34}}\right)-A_{35}\left(A_{83}e^{A_{35}}-A_{84}e^{-A_{35}}\right)\\
&-A_{36}\left(A_{85}e^{A_{36}}-A_{86}e^{-A_{36}}\right)+A_{37}\left(A_{87}e^{A_{37}}-A_{88}e^{-A_{37}}\right)\Big\}\Big].\end{aligned}$$

5. Nusselt number

The rate of heat transfer in terms of Nusselt number at the plates is given by

(33)
$$Nu=-\frac{dq_\omega}{\kappa\left(T_m-T_s\right)}=\left(\frac{\partial\theta}{\partial y}\right)_{y=0\&1},$$

where q_w is the rate of heat transfer given by $q_w=-\kappa\left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=0\&d}$.

The expression of Nusselt number at moving plate is given by

(34)
$$Nu_m=\left(\frac{\partial\theta}{\partial y}\right)_{y=0}$$

$$
\begin{aligned}
&= iA_1\left(A_2 - A_3\right) + Ec\left\{iA_1\left(A_{29} - A_{28}\right) - 2\sqrt{A}\left(A_{15} - A_{16}\right)\right.\\
&\quad \left. - 2iA_1\left(A_{17} - A_{18}\right) - A_{24}\left(A_{19} - A_{20}\right) - A_{25}\left(A_{21} - A_{22}\right)\right\}\\
&\quad + \varepsilon e^{it}\left[iA_4\left(A_5 - A_6\right) + Ec\left\{iA_4\left(A_{55} - A_{54}\right) - A_{30}\left(A_{38} - A_{39}\right)\right.\right.\\
&\quad + A_{31}\left(A_{40} - A_{41}\right) - A_{32}\left(A_{42} - A_{43}\right) + A_{33}\left(A_{44} - A_{45}\right)\\
&\quad - A_{34}\left(A_{46} - A_{47}\right) + A_{35}\left(A_{48} - A_{49}\right) + A_{36}\left(A_{50} - A_{51}\right)\\
&\quad \left.\left. + A_{37}\left(A_{52} - A_{53}\right)\right\}\right].
\end{aligned}
$$

The expression of Nusselt number at stationary plate is given by

(35)
$$
\begin{aligned}
Nu_m &= \left(\frac{\partial\theta}{\partial y}\right)_{y=0}\\
&= iA_1\left(A_2 e^{iA_1} - A_3 e^{-iA_1}\right) + Ec\left\{iA_1\left(A_{29} e^{iA_1} - A_{28} e^{-iA_1}\right)\right.\\
&\quad - 2\sqrt{A}\left(A_{15} e^{2\sqrt{A}} - A_{16} e^{-2\sqrt{A}}\right) - 2iA_1\left(A_{17} e^{2iA_1} - A_{18} e^{-2iA_1}\right)\\
&\quad \left. - A_{24}\left(A_{19} e^{A_{24}} - A_{20} e^{-A_{24}}\right) + A_{25}\left(A_{21} e^{A_{25}} - A_{22} e^{-A_{25}}\right)\right\}\\
&\quad + \varepsilon e^{it}\left[iA_4\left(A_5 e^{iA_4} - A_6 e^{-iA_4}\right) + Ec\left\{iA_4\left(A_{55} e^{iA_4} - A_{56} e^{-iA_4}\right)\right.\right.\\
&\quad - A_{30}\left(A_{38} e^{A_{30}} - A_{39} e^{-A_{30}}\right) + A_{31}\left(A_{40} e^{A_{31}} - A_{41} e^{-A_{31}}\right)\\
&\quad - A_{32}\left(A_{42} e^{A_{32}} - A_{43} e^{-A_{32}}\right) + A_{33}\left(A_{44} e^{A_{33}} - A_{45} e^{-A_{33}}\right)\\
&\quad - A_{34}\left(A_{46} e^{A_{34}} - A_{47} e^{-A_{34}}\right) + A_{35}\left(A_{48} e^{A_{35}} - A_{49} e^{-A_{35}}\right)\\
&\quad \left.\left. + A_{36}\left(A_{50} e^{A_{36}} - A_{51} e^{-A_{36}}\right) - A_{37}\left(A_{52} e^{A_{37}} - A_{53} e^{-A_{37}}\right)\right\}\right].
\end{aligned}
$$

6. Results and discussion

Effect of different physical parameters on velocity and temperature distributions are shown through figures when $\varepsilon = 0.01, \omega = 10$ and $t = 1$. Figures are drawn by using MATLAB software. Figure 2 shows the effect of Grashof number on velocity distribution and it is observed that the velocity increases with increasing values of the thermal Grashof number. The flow is accelerated due to the enhancement in buoyancy force corresponding to an increase in the thermal Grashof number. The positive values of Grashof number correspond to cooling of the plate (aiding flow) and the negative values of Grashof number correspond to heating of the plate (opposing flow). The effect of Prandtlnumber is shown through figure 3 and it is seen that velocity increases with Prandtl number for the buoyancy aiding flow $(Gr > 2)$. On the other hand, if the opposing flow $(Gr < 0)$ is considered then the effect become opposite. From figures 4 and 5, respectively, it is observed that velocity increases with Eckert number and heat source parameter for aiding flow, while opposite behavior is seen for opposing flow.

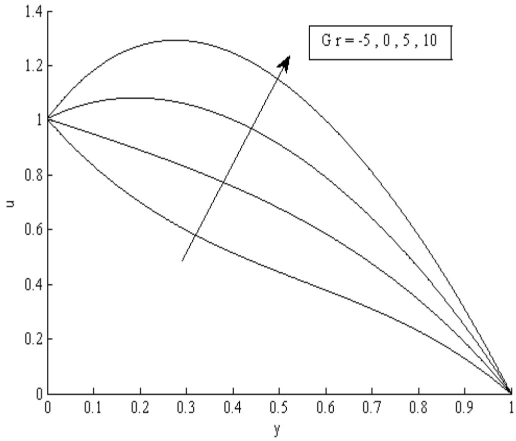

Figure 2: Velocity distribution versus y when
$\Pr = 1,\ Ec = 0.01,\ Q = 0.5,\ M = 2,\ K = 0.5.$

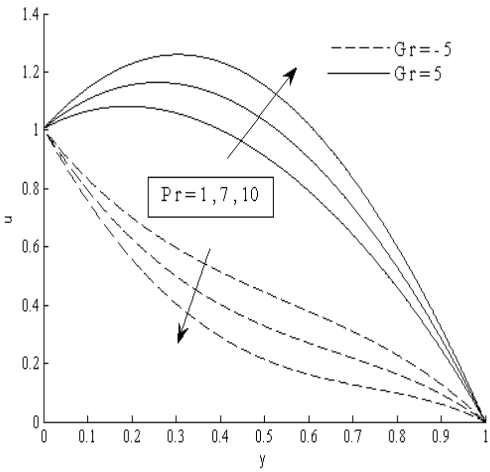

Figure 3: Velocity distribution versus y when
$Ec = 0.01,\ Q = 0.5,\ M = 2,\ K = 0.5.$

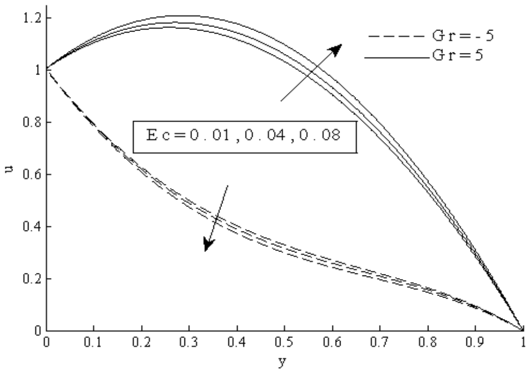

Figure 4: Velocity distribution versus y when
$\Pr = 1,\ Q = 0.5,\ M = 2,\ K = 0.5.$

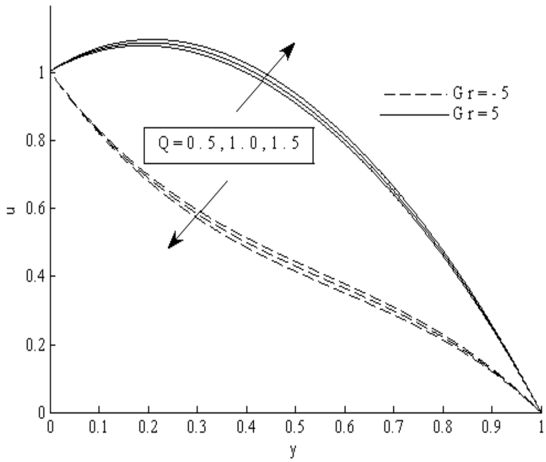

Figure 5: Velocity distribution versus y when
$\Pr = 1,\ Ec = 0.01,\ M = 2,\ K = 0.5.$

The effect of Hartmann number on velocity distribution is shown through figure 6. It is depicted that velocity decreases with Hartmann number for aiding flow. We know that the transverse magnetic field results a resistive type force (Lorentz force) which tends to resist the fluid flow and reduce velocity. On the other hand, if the opposing flow is considered then the effect become opposite. It is noticed from figure 7 that velocity decreases as the permeability parameter increases in the case of aiding flow and opposing flow both.The effect of Grashof number, Prandtl number, Eckert number or heat source parameter is shown from figure 8 to 11 and it is depicted that temperature increases with Grashof number, Prandtl number, Eckert number or heat source parameter in the case of aiding flow and opposing flow both.

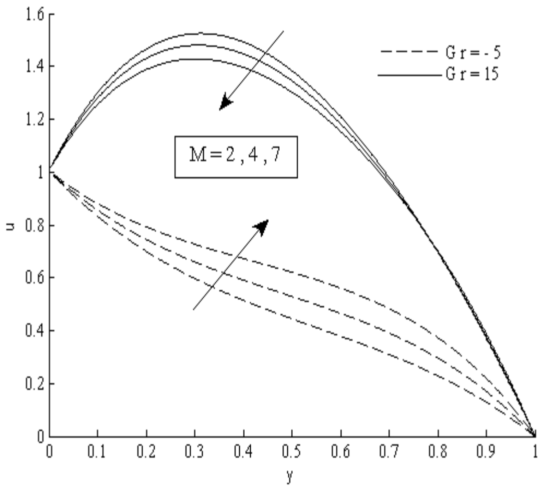

Figure 6: Velocity distribution versus y when $\Pr = 1,\ Ec = 0.01,\ Q = 0.5,\ K = 0.5.$

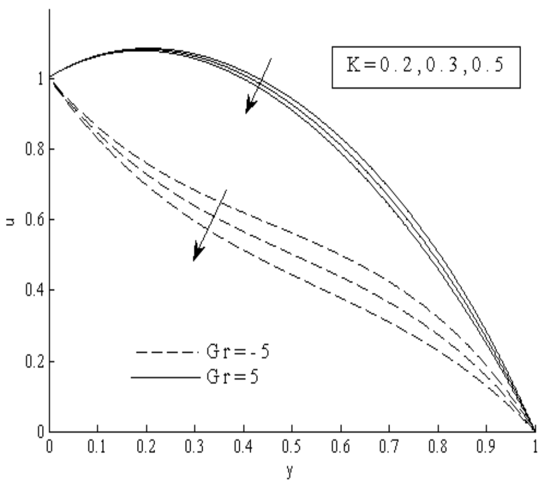

Figure 7: Velocity distribution versus y when $\Pr = 1,\ Ec = 0.01,\ Q = 0.5,\ K = 0.5.$

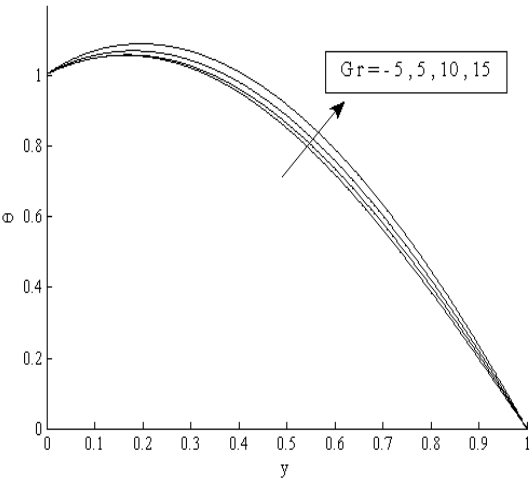

Figure 8: Temperature distribution versus y when
$\Pr = 7,\ Ec = 0.01,\ Q = 0.5,\ M = 2.$

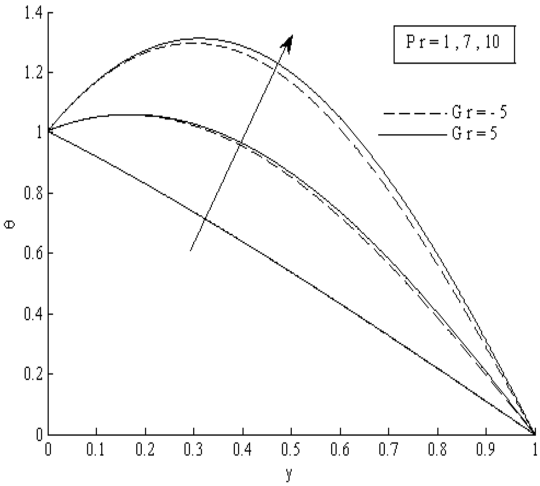

Figure 9: Temperature distribution versus y when
$Ec = 0.01, Q = 0.5,\ M = 2,\ \ K = 0.5.$

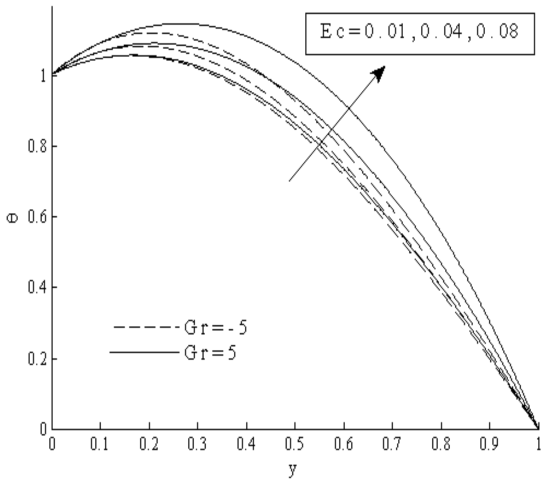

Figure 10: Temperature distribution versus y when $\Pr = 7,\ Q = 0.5,\ M = 2,\ K = 0.5.$

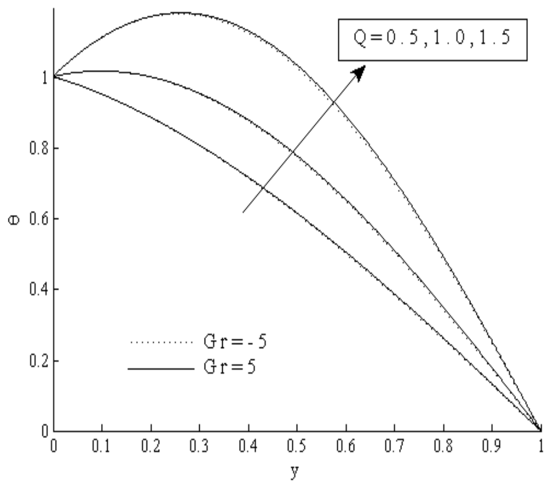

Figure 11: Temperature distribution versus y when $\Pr = 3,\ Ec = 0.01,\ M = 2,\ K = 0.5.$

Numerical values of skin friction coefficient and Nusselt number at the moving plate and stationaryplate are calculated for different values of physical parameters and presented through Table-1 and Table-2, respectively.

Table 1: Numerical values of skin friction coefficient at both the plates for various values of physical parameters when $w = 10$ and $t = 1$

ε	Gr	Pr	Ec	M	K	Q	Cf_m	Cf_s
0.00	5	1	0.01	2	0.5	0.5	0.836300	2.674183
0.01	5	1	0.01	2	0.5	0.5	0.840729	2.675883
0.01	10	1	0.01	2	0.5	0.5	2.242261	3.282657
0.01	5	7	0.01	2	0.5	0.5	1.244207	3.046396
0.01	5	1	0.05	2	0.5	0.5	0.850839	2.687551
0.01	5	1	0.01	4	0.5	0.5	0.856069	3.002527
0.01	5	1	0.01	2	1.0	0.5	0.826140	2.498703
0.01	5	1	0.01	2	0.5	1.5	0.939862	2.761879

From Table-1, it is depicted that skin friction coefficient at the moving plate and stationary plate increases with Grashof number, Prandtl number, Eckert number, Hartmann number or heat source parameter, while it decreases with permeability parameter.

Table 2: Numerical values of Nusselt number at both the plates for various values of physical parameters when $\omega = 10$ and $t = 1$.

ε	Gr	Pr	Ec	M	K	Q	Nu_m	Nu_s
0.00	5	1	0.01	2	0.5	0.5	0.822955	1.103770
0.01	5	1	0.01	2	0.5	0.5	0.815294	1.110195
0.01	10	1	0.01	2	0.5	0.5	0.810072	1.118777
0.01	5	7	0.01	2	0.5	0.5	0.669918	2.124405
0.01	5	1	0.05	2	0.5	0.5	0.797241	1.171647
0.01	5	1	0.01	4	0.5	0.5	0.815349	1.111596
0.01	5	1	0.01	2	1.0	0.5	0.815300	1.109406
0.01	5	1	0.01	2	0.5	1.5	0.426978	1.326175

It is observed from Table-2 that Nusselt number at the moving plate increases with the increase of Hartmann number or permeability parameter, while it decreases with Grashof number, Prandtl number, Eckert number or heat source parameter. Nusselt number at the stationary plate increases with Grashof number, Prandtl number, Eckert number, Hartmann number or heat source parameter, while it decreases with permeability parameter.

7. Conclusions

Free convective oscillatory flow of a viscous incompressible electrically conducting fluid through porous medium bounded by vertical infinite non conducting parallel plates in presence of transverse magnetic field and heat source is investigated using perturbation technique. Effects of different physical parameters on the velocity and temperature distributions are studied. The following observations are made:

1. The velocity profile increases with Grashof number since the flow is accelerated due to the enhancement in buoyancy force.
2. The velocity decreases as Hartmann number increases for $Gr > 0$, while it increases with Hartmann number for $Gr < 0$.
3. An increment in the Prandtl number results an increment in the temperature profile for the aiding flow, while opposite behavior of temperature profile is depicted for the opposing flow.
4. The rate of heat transfer at the stationary plate increases with Prandtl number or heat source parameter, while it decreases with the permeability parameter.
5. Skin friction coefficient increases with Grashof number or Hartmann number, while it decreases with the permeability parameter at both the plates.
6. Nusselt number at the moving plate decreases with Eckert number, while it increases with the Hartmann number.

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