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Effect of Viscosity of Fluid on its Velocity through a Horizontal channel having porous medium placed in an Inclined Magnetic Field

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Abstract: This present paper deals with effect of viscosity of fluid on its velocity through a horizontal channel having porous medium placed in an inclined magnetic field. After forming the governing equations for the flow, they are solved with the help of non-dimensional parameters under assumed boundary conditions, the expression for velocity of fluid has been obtained and using it, graph has been plotted between magnetic field and velocity for different values of viscosity. It has been found that the viscosity has a great effect on velocity profile as it shows a resonance character in the graph, and this can be used for industrial purpose in maintaining the flow of fluid through channels.

Keywords: MHD Flow, Magnetic Field, porous medium, velocity pro - file.

1. Introduction

The flow of fluid, metal ion and ionize gasses etc. is very useful in industrial field which is studied in magneto hydrodynamics and this field is related with the motion of an electrically conducting fluid in the presence of magnetic field. It is the fluid dynamics of electrically conducting fluid. Various types of studies have been made in this field because of its growing applications. Barletta A, et.al.[1] studied the viscous dissipation and double-diffusive convection instability in a fluid with saturated porous layer in horizontal forced flow. Chamkha A.J.[2] Examined the magneto hydrodynamic free convection by a vertical plate embedded in a thermally stratified porous medium and found similarity solutions for the laminar boundary layer equations describing steady hydro magnetic two dimensional flows.

Rahman[3] et al. examined the MHD flow of mixed convection through a horizontal channel with a open cavity. Mounuddin K and Pattabhiramacharyulu N. C.[4] studied the steady flow of a viscous fluid through a saturated porous medium of finite thickness, impermeable and thermally insulated bottom keeping the other side stress free at a constant temperature. Ram et al.[5] studied the Hall effects on the hydro magnetic free convection and mass transfer flow through porous media. Al-Hadhrami[6] considered the flow of fluid through horizontal channels with porous medium and obtained expressions bearing Reynolds number. M. Syamala et al.[7] studied MHD flow of a couple stress fluid through a porous medium in a parallel plate channel in presence of an inclined magnetic field. Rapits[8] examined the flow of a polar fluid assuming the medium to be porous and derived the angular velocity for the flow. El-Shehawey and El-Sebaei[9] discussed the problem of peristaltic transport in the cylindrical tube through the porous medium and found that axial velocity of fluid increased with increasing the value of permeability. Afifi and Gad[10] discussed the flow of a Newtonian, incompressible fluid flowing between infinite parallel walls filled with porous medium under the transversely applied magnetic field. Dash et.al.[11] examined the characteristics of the flow of fluid field in a tube having porous homogenous medium. Ahmadi and Manvi[12] derived the general equation of motion through porous medium and extended it for few fundamental flows of fluid problems. Raju, et al.[13] studied steady magneto hydrodynamic forced convective flow of a viscous fluid of finite depth in a saturated porous medium over a fixed horizontal channel with thermally insulated and impermeable bottom wall in the presence of viscous dissipation and joule heating. The governing equation was solved using non dimensional parameters and the exact solutions were obtained for velocity and temperature distributions when the temperatures on the fixed bottom and on the free surface were prescribed. The expressions for flow rate, mean velocity, temperature, mean temperature, mean mixed temperature in the flow region and the Nusselt number on the free surface have been obtained. The cases of large and small values of porosity coefficients have been discussed as limiting cases. Further, the cases of small depth (shallow fluid) and large depth (deep fluid) were also discussed.

2. Mathematical Formulation

Consider a steady viscous electrically conducting flow of fluid through a plane channel having saturated porous medium with finite depth H. In rectangular Cartesian co-ordinates system with the origin on the bottom, assume $x-$axis to be in the direction of the flow. The boundary at the bottom is represented as $y=0$ and the free surface as $y=H$. An inclined magnetic field of uniform strength B_0 is applied at an angle β to the flow. The flow occurs due to a constant pressure gradient at the mouth of the channel. Let the flow be characterized by a velocity u in the x direction. The fluid is assumed to be ionized. However within any small but finite volume, then number of particles with positive and negative charges is nearly equal. Hence the total excess charge density and imposed electric field intensity have been assumed to be zero.

By considering the above assumptions the governing equations of the flow are given below.

(2.1) $$-\frac{\partial \bar{p}}{\partial \bar{x}}+\mu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}-\mu\frac{\bar{u}}{k^*}-\sigma Bo^2 \operatorname{Sin}^2 \beta\bar{u}=0,$$

where $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates k^* is dimensional porous parameter.

σ : Electrical conductivity,
β : Angle of Inclination,
p : Pressure of fluid,
ρ : Density of fluid,
B_0 : Applied Magnetic field,
$\wp$: Pressure gradient,
H : Depth of channel,
μ : Kinematic viscosity,

It is solved subject to boundary condition, $y=0$, $u=0$, and $y=H$, then $\frac{\partial u}{\partial y}=0$. Let ‘ a ’ be the characteristics length non-dimensional quantities are follows

(2.2) $$\bar{x}=xa,\quad \bar{y}=ya,\quad \bar{u}=\frac{\mu u}{\rho a^2},$$

(2.3) $$\bar{p}=\frac{\mu^2 p}{\rho a^2},\quad M=\frac{\sigma Bo^2 a^2}{\mu},\quad K^*=\frac{a^2}{\alpha^2},$$

(2.4) $$-\frac{\partial\overline{p}}{\partial\overline{x}}=\frac{\mu^2}{\rho a^2}\wp,$$

where $\left(\wp=-\frac{\partial p}{\partial x}\right)$.

M is the magnetic parameter and α is the permeability parameter in dimensionless form.
Using equation (2.2), (2.3) and (2.4) in (2.1) and solving equation

(2.5) $$\frac{\mu^2}{\rho a^3}\wp+\frac{\mu^2}{\rho a^4}\frac{\partial^2 u}{\partial y^2}-\frac{\mu^2\alpha^2}{\rho a^4}u-\sigma Bo^2\,\text{Sin}^2\beta\frac{\mu}{\rho a^2}u=0,$$

(2.6) $$\frac{\partial^2 u}{\partial y^2}-\alpha^2 u-\frac{\sigma Bo^2 a^2}{\mu}\text{Sin}^2\beta\ u=-a\wp,$$

(2.7) $$\frac{\partial^2 u}{\partial y^2}-\alpha_1^2\ \text{Sin}^2\beta\ u=-a\wp\ ,$$

where

(2.8) $$\alpha_1^2=\frac{\alpha^2}{\text{Sin}^2\beta}+M \text{ and } M=\frac{\sigma Bo^2 a^2}{\mu}.$$

Solving above differential equation and obtaining

(2.9) $$U=C_0\,e^{\alpha_1\,\text{Sin}\beta\,y}+C_1 e^{-\alpha_1\,\text{Sin}\beta\,y}+\frac{a\wp}{\alpha_1^2\ \text{Sin}^2\beta},$$

where C_0 and C_1 are arbitrary constant. Whose solution under the boundary conditions is $y=0$, $U=0$.

(2.10) $$C_0+C_1+\frac{a\wp}{\alpha_1^2\ \text{Sin}^2\beta}=0$$

Equation (2.9), differentiating with respect to y then

(2.11) $$\frac{\partial U}{\partial y} = C_0\, \alpha_1 \operatorname{Sin}\beta e^{\alpha_1 \operatorname{Sin}\beta y} - C_1 \alpha_1 \operatorname{Sin}\beta\, e^{-\alpha_1 \operatorname{Sin}\beta y}$$

and $y = H$, then $\frac{\partial U}{\partial y} = 0$

(2.12) $$C_0\, \alpha_1 \operatorname{Sin}\beta e^{\alpha_1 \operatorname{Sin}\beta H} - C_1 \alpha_1 \operatorname{Sin}\beta\, e^{-\alpha_1 \operatorname{Sin}\beta H} = 0,$$

(2.13) $$C_0 = C_1\, e^{-2\alpha_1 \operatorname{Sin}\beta H},$$

C_0 value put in equation (2.12), then

(2.14) $$C_1 = -\frac{a\wp}{\alpha_1^{\,2}\ \operatorname{Sin}^2\beta\left(1 + e^{-2\alpha_1 \operatorname{Sin}\beta\, H}\right)}\ .$$

From equation (2.12) and (2.14)

(2.15) $$C_0 = -\frac{a\wp\, e^{-2\alpha_1 \operatorname{Sin}\beta H}}{\alpha_1^{\,2}\ \operatorname{Sin}^2\beta\left(1 + e^{-2\alpha_1 \operatorname{Sin}\beta\, H}\right)}\ .$$

The values of C_0 and C_1 putting in equation (2.11), obtain complete solution as

(2.16) $$U(y) = \frac{a\wp}{\alpha_1^{\,2}\ \operatorname{Sin}^2\beta}\left[1 - \left\{\frac{e^{-2\alpha_1 \operatorname{Sin}\beta H}}{\left(1 + e^{-2\alpha_1 \operatorname{Sin}\beta H}\right)} e^{\alpha_1 \operatorname{Sin}\beta\, y} + \frac{e^{-\alpha_1 \operatorname{Sin}\beta y}}{\left(1 + e^{-2\alpha_1 \operatorname{Sin}\beta H}\right)}\right\}\right],$$

(2.17) $$U(y) = \frac{a\wp}{\alpha_1^{\,2}\ \operatorname{Sin}^2\beta}\left[1 - \left\{\frac{e^{-\alpha_1 \operatorname{Sin}\beta H}}{\left(e^{\alpha_1 \operatorname{Sin}\beta H} + e^{-\alpha_1 \operatorname{Sin}\beta H}\right)} e^{\alpha_1 \operatorname{Sin}\beta\, y} + \frac{e^{-\alpha_1 \operatorname{Sin}\beta y} e^{\alpha_1 \operatorname{Sin}\beta H}}{\left(e^{\alpha_1 \operatorname{Sin}\beta H} + e^{-\alpha_1 \operatorname{Sin}\beta H}\right)}\right\}\right],$$

(2.18) $$U(y) = \frac{a\wp}{\alpha_1^{\,2}\ \operatorname{Sin}^2\beta}\left[1 - \left\{\frac{e^{\alpha_1 \operatorname{Sin}\beta(H-y)} + e^{-\alpha_1 \operatorname{Sin}\beta(H-y)}}{\left(e^{\alpha_1 \operatorname{Sin}\beta H} + e^{-\alpha_1 \operatorname{Sin}\beta H}\right)}\right\}\right].$$

Since

(2.19) $$\alpha_1^2 = \frac{\alpha^2}{\mathrm{Sin}^2\beta} + M \quad \text{or} \quad \alpha_1 = \frac{1}{\mathrm{Sin}\,\beta}\sqrt{\alpha^2 + M\,\mathrm{Sin}^2\beta}\,.$$

From equation (2.18) and (2.19) then

(2.20) $$U(y) = \frac{a\wp}{\left(\alpha^2 + M\ \mathrm{Sin}^2\beta\right)}\left[1 - \left\{\frac{e^{\sqrt{\alpha^2+\mathrm{Sin}^2\beta}\ (H-y)} + e^{-\sqrt{\alpha^2+\mathrm{Sin}^2\beta}\ (H-y)}}{\left(e^{\sqrt{\alpha^2+\mathrm{Sin}^2\beta}\ H} + e^{-\sqrt{\alpha^2+\mathrm{Sin}^2\beta}\ H}\right)}\right\}\right],$$

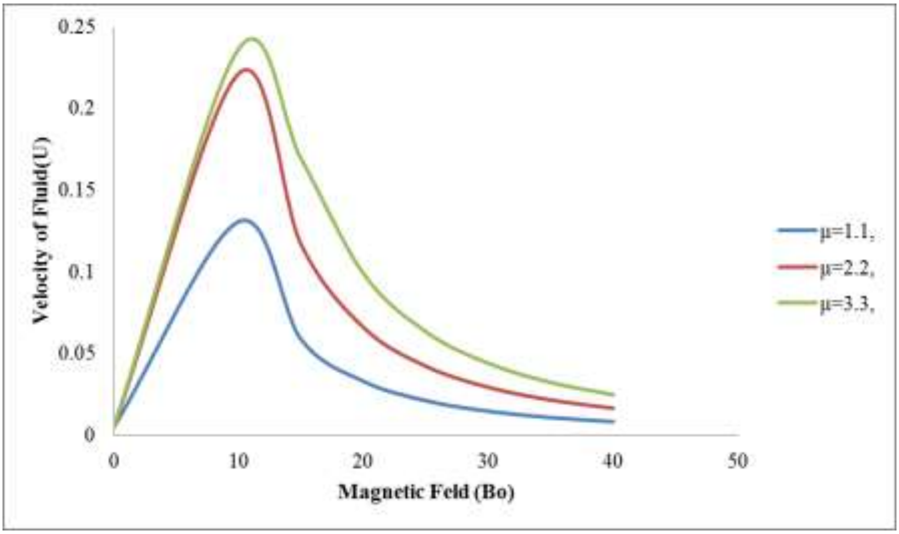
(2.21) $$U(y) = \frac{a\wp}{\left(\alpha^2 + M\,\mathrm{Sin}^2\beta\right)}\left[1 - \left\{\frac{\mathrm{Cos}\,h\left\{\left(\sqrt{\alpha^2 + M\,\mathrm{Sin}^2\beta}\right)(H-y)\right\}}{\mathrm{Cos}\,h\left\{\left(\sqrt{\alpha^2 + M\,\mathrm{Sin}^2\beta}\right)H\right\}}\right\}\right],$$

Using this derived relation, suitable values are given to various parameters to plot the curves and then, a graphical study has been made to derive the result.

3. Results and Discussion

Velocity Profile between Magnetic Field (Bo) and Fluid Vviscosity (μ)

Sr. No	**Magnetic Field(Bo)**	**Velocity of Fluid(U)**		
		μ=1.1	**μ=2.2**	**μ=3.3**
1	0	0.004971	0.004971	0.004971
2	10	0.130765	0.221422	0.236628
3	15	0.058623	0.116754	0.169248
4	20	0.032984	0.065945	0.098791
5	25	0.021112	0.042215	0.063309
6	30	0.014662	0.02932	0.043973
7	35	0.010773	0.021543	0.032311
8	40	0.008248	0.016495	0.02474



The graph shows that the velocity profile of fluid with respect to magnetic field for different values of fluid viscosity as magnetic field increases, velocities also increase from the values nearly equal to zero and take a maximum value around $M = 10$, and then decreases sharply following a resonance character.

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