ISSN 0974 - 9373

Vol. 19 No. 2 (2015) **Journal of International Academy of Physical Sciences** pp. 103-116

# Effects of Magnetic Field on Heat Transfer in Peristaltic Flow of Micropolar Fluid in an Asymmetric Channel

**T. S. Pal and D. K. Singh**
Department of Mathematics
K. P. G. College, Chakkey, Jaunpur (U. P.) India
Email: ddsing2010@gmail.com

(Received February 26, 2015 )

**Abstract:** In this paper we investigate the effects of magnetic field on heat transfer in peristaltic flow of micropolar fluid in an asymmetric channel. The expressions for velocity micro-rotation velocity volume flow rate and temperature distribution are obtained. The effect of different values of magnetic parameter ($M_1$) amplitude of wave ($\phi_1$) and microrotational parameter ($n_3$) are discussed through graphs. These results may be useful for diagonostic purpose.

**Keywords:** Magnetic field, Micropolar fluid, Magnetic parameter, Amplitude of wave.

**2010 MS Classification No.:** 76S05

## 1. Introduction

Eringen[1] has introduced the theory of micropolar fluids. These fluids possess microstructure, in particular spin inertia and have a capacity of sustaining stress and body moments, which rapidly effect the microscopic motions of the fluid. Here we deal with the motion of micropolar fluids which is a subclass of microfluids. Eringen[2] suggested that the theory of micro-polar fluids might serve as a satisfactory model for the description of the flow properties of polymeric fluids, fluids containing certain additives and in particular the animal blood. The classical Navier-Stokes theory is inadequate, especially in predicting the behaviour of such type of suspended particles or movement of blood cells and the boundary layer flows of such fluids. He also works out an example for the flow of micropolar fluids in a circular pipe. Initially Laxmana Rao[3] studied the general solution of field equations of micro-polar fluids. Later on Wilson[4] dealt with the basic flows of micropolar fluids. He particularly emphasized the case in which $\mu_R$, the coupling constant relating to the microstructure to the microscopic flow, is greater than $\mu_s$, the usual coefficient of viscosity. He analysed shearing flow between two infinite plates in steady relative motion, rotational flow in circular tubes, propagation and decay of plane wave disturbances, oscillatory flow of an infinite plate immersed in the fluid and surface flow problems. Renuka[5] has studied some problems of a micro-polar fluid between two parallel plates and in a circular cylinder. Also Renuka et al [6] studied the wave propagation of micropolar fluid in viscoelastic membrane.

The peristaltic flow of biofluids in different geometries has many applications in mathematics, biology and engineering. The initial mathematical models of peristalsis obtained by a train of sinusoidal waves in an infinitely, long symmetric channel or tube have been investigated by Shapiro et al[7] and Fung and Yih[8]. Several studies have been made to analyse both theoretical and experimental aspects of the peristaltic motion of non-Newtonian fluids in different situations. Bohme and Friendrich[9] studied the mechanism of peristaltic transport of an incompressible viscoelastic fluid by means of an infinite train of sinusoidal waves travelling along the wall of the duct in the case of plane flow. El-Shehawey et al[10] studied the peristaltic motion of an incompressible non-Newtonian fluid through a porous medium. They showed that the pressure rise increases as the permeability decreases and noted that both pressure rise and friction force does not depend on permeability parameter at a certain value of flow rate. The effects of an oldroyd-B fluid on the peristaltic mechanism are examined by Hayat et al[11], under the long wave length assumption. They noticed that in the narrow part of the channel, the behaviour of an oldroyd-B fluid is much different from that of Newtonian fluid than in the wide part of the channel. After these studies, many investigations were done to understand the peristaltic action for Newtonian and non-Newtonian fluids in different situations. Several authors Eytan and Elad[12], Rao and Mishra[13], Hasoun[14, 15], Hayat et al[16], Subha Reddy et al[17], Ali and Hayat[18], Sobh[19], have studied peristaltic motion of Newtonian and non-Newtonian fluid flow in asymmetric channel.

Mekheimer[20] studied the effect of a uniform magnetic field on a peristaltic transport of a blood in a non-uniform two dimensional channels, when blood is represented by a couple-stress fluid.

Muthu et al[21] carried out a study of the peristaltic motion of an incompressible micro-polar fluid in two dimensional channel. They investigated the effects of viscoelastic wall properties and micropolar fluid parameters on the flow using the equation of the fluid as well as the deformable boundaries. In view of the above study, wide range of practical importance of magnetic field and non-Newtonian behaviour of physiological fluids. We have made an attempt to analyse the flow development and effect of magnetic field on heat transfer in peristaltic flow of micro-polar fluid in an asymmetric channel.

## 2. The Governing Equations

Using the assumptions made in formulating the problems are as follows:

1. The fluid simulating blood is steady and incompressible
2. The flow is asymmetric.
3. The red blood cell are neutrally buoyant, under flow condition the settling tendency of erythrocyte is negligible.
4. Nobody forces and no body complexes are present.
5. The tube simulating the arterial wall is thin walled, stenotic, homogeneous and isotropic.

The governing equations which describe the laminar flow of a micropolar fluid are given by the following equations [Eringen[2]].

(2.1) $$\frac{\partial \rho}{\partial t} + \bar{\nabla}(\rho \bar{v}) = 0,$$

(2.2) $$(\lambda v + 2\mu_0)\bar{\nabla}(\bar{\nabla}\bar{v}) - (\mu_0 + k)\bar{\nabla}\times\bar{\nabla}\times\bar{v} + 2k\bar{\nabla}\times\bar{v} - \bar{\nabla}\pi_t + \rho f_i = \frac{\rho D\bar{v}}{Dt},$$

(2.3) $$(\alpha_v + \beta_v + \gamma)\bar{\nabla}(\bar{\nabla}.\bar{v}) - \gamma\bar{\nabla}\times(\bar{\nabla}\times\bar{v}) + 2k\bar{\nabla}\times\bar{v} - 4k\bar{v} + \rho I = \rho j\frac{D\bar{v}}{Dt}.$$

The first equation is the principle of conservation of mass and the others are the conservation of linear and angular momentum, ρ is the mass density, $\bar{v}$ is the velocity vector, $\bar{v}$ is micro-rotational velocity vector, $\pi_t$ is the thermodynamic pressure, $f_i$ is the body force per unit mass, *I* body couple, *j* is the micro-inertia constant, $\lambda_v$, $\mu_0$ are the viscosity coefficients of classical fluid mechanics and $k, \alpha_v, \beta_v, \gamma$ are the new viscosity coefficients for micropolar fluid. We get the field equations approximate to

(2.4) $$(\mu_s + \mu_R)\frac{\partial^2 u_x}{\partial y'^2} + 2\mu_R\frac{\partial v_x}{\partial y'} = \frac{\partial p_x}{\partial x'} + M_1 u,$$

(2.5) $$\gamma\frac{\partial^2 v_x}{\partial y'^2} - 2\mu_R\frac{\partial^2 u_x}{\partial y'} - 4\mu_R v_x = 0.$$

Energy equation is given by

(2.6) $$K_1\frac{\partial^2 T}{\partial y'^2} + (2\mu_s + \mu_R)\left(\frac{\partial u_x}{\partial y'}\right)^2 + 2\mu_R\left(\frac{1}{2}\frac{\partial v_x}{\partial y'} + v_x\right)^2 + y\left(\frac{\partial v_x}{\partial y'}\right)^2 = 0,$$

where $\mu_s$, $\mu_R$ and $\gamma$ are the shear viscosity, rational viscosity and rotational gradient coefficient respectively. $u_x$, $v_x$, and $p_x$ are axial velocity, rotational velocity and pressure respectively. $\sigma$ is the electrical conductivity, $\beta_0$ is an external magnetic field, $\rho$ is the fluid density, x' and y' are the axial and vertical co-ordinates along the length and thickness of the channel respectively. $K_1$ is the thermal conductivity and $T$ is temperature distribution in the channel.

## 3. Formulation of the Problem

Let the vertical displacement of the upper and lower walls be $\eta$ and $-\eta$ respectively, where $\eta(X,t) = a\cos\frac{2\pi}{\lambda}(X-ct) = a\cos\frac{2\pi}{\lambda}x' = a\cos 2\pi x$.

Take $X - ct = x'$ where $a$ is the amplitude of the wave, $c$ is the phase speed of the wave, $\lambda$ is the wave length and $t$ is time.

**Boundary Conditions:**

$$(3.1)\qquad \begin{cases} u = 0, y' = \pm h_1 = \pm\left(d + \eta(X,t)\right) = \pm d\left(1 + \phi_1 \cos 2\pi x\right) \\ \dfrac{\partial v_x}{\partial y'} = 0, \quad y' = \pm h_1 \\ T = T_0, \quad y' = \pm h_1 \end{cases}$$

By using the non-dimensional equations

$$(3.2)\qquad \begin{cases} x = \dfrac{x'}{\lambda}, y' = \dfrac{y}{d}, u_x' = \dfrac{u}{c}, v_x' = \dfrac{v}{c\delta}, \delta = \dfrac{d}{\lambda} \\ n_2 = \dfrac{\mu_s d^2}{\gamma}, n_3 = \dfrac{\mu_R d^2}{\gamma}, \rho = \dfrac{p_x d^2}{\mu_s c\lambda}, \theta = \dfrac{T - T_0}{T_1 - T_0}, h = \dfrac{h_1}{d}, \phi_1 = \dfrac{a}{d} \\ Gr = \dfrac{\rho g \alpha d^2 \left(T_1 - T_0\right)}{c\mu_s} \end{cases}$$

where $d$ is the width of the channel, $\delta$ is the ratio between width of the channel and wave length, $G_r$ is free convention parameter and $T_1$, $T_0$ are the constant temperatures.

The governing equations take the form after, using (3.2)

$$(3.3)\qquad \left(n_2 + n_3\right)\frac{\partial^2 u}{\partial y^2} + 2d\delta n_3 \frac{\partial v}{\partial y} = n_2 \frac{\partial p}{\partial x} + \frac{M_1}{\gamma} d^4 u\,,$$

(3.4) $$\frac{\partial^2 v}{\partial y^2}-\frac{2n_3}{d\delta}\frac{\partial u}{\partial y}-4n_3 v=0\,,$$

(3.5) $$\frac{G_r K_1 n_2}{cd^4\rho g\alpha}\frac{\partial^2\theta}{\partial y^2}+\frac{(2n_2+n_3)}{d^2}\left(\frac{\partial u}{\partial y}\right)^2+2\left(\frac{1}{2d}\frac{\partial u}{\partial y}+\delta v\right)^2+\delta^2\left(\frac{\partial v}{\partial y}\right)^2=0\,,$$

where

$$M_1=\frac{\sigma\beta_0^2}{\rho}\,.$$

The Boundary conditions (3.1) become

(3.6) $$\begin{cases} u=0, & at \quad y=\pm h, \\ \dfrac{\partial v}{\partial y}=0, & at \quad y=\pm h, \\ \theta=0, & at \quad y=\pm h. \end{cases}$$

## 4. Solutions

The velocity profiles are obtained from the equations (3.3), (3.4) by using boundary conditions (3.6) and are as follows

(4.1) $$u=\frac{\gamma n_2}{M_1}\frac{\partial p}{\partial x}\left[\frac{\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_2^2\right)}{2C_2\gamma(n_2+n_3)}\left(-\frac{CoshA_1 y}{CoshA_1 h}+\frac{2\sin hA_1 y}{\sin hA_1 h}\right)+\right.$$
$$\left.\frac{\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_1^2\right)}{2\gamma(n_2+n_3)C_2}\left(-\frac{CoshA_2 y}{CoshA_2 h}-\frac{2\sin hA_2 y}{\sin hA_2 h}\right)-\frac{1}{d^4}\right],$$

(4.2) $$v=\frac{1}{4}\frac{d^2}{M_1(n_2+n_3)\gamma}\left(\frac{\partial p}{\partial x}\right)\frac{1}{\delta C_2}\left[A_3\left(\frac{-\sin hA_1 y}{CoshA_1 h}+\frac{2\cos hA_1 y}{\sin hA_1 h}\right)\right.$$
$$\left.+A_4\left(-\frac{\sin hA_2 y}{coshA_2 h}-\frac{2\cos hA_2 y}{\sin hA_2 h}\right)\right],$$

where

$$A_1=\sqrt{(C_1+C_2)},\ A_2=\sqrt{(C_1-C_2)},\ C_1=\frac{1}{2(n_2+n_3)}\left(\frac{-4n_3^2}{d}+M_1\frac{d^4}{\gamma}+4n_3(n_2+n_3)\right),$$

$$C_2=\frac{d^3}{2\gamma^2\delta(n_2+n_3)}\sqrt{\begin{array}{l}d^2\delta^2\left(\frac{16n_3^4\gamma^4}{d^{10}}+M_1^2\gamma^2-M_1\frac{8n_3^2\gamma^3}{d^5}\right)+4\delta n_3\gamma^2(n_2+n_3)\times\\ \left(\frac{\gamma}{d^2}(n_2+n_3)+\frac{2M_1\gamma\delta}{d^2}-\frac{16\delta\gamma^2n_2^2}{d^7}-4\delta\gamma M_1\right)\end{array}},$$

$$A_3=A_1\left(M_1-\frac{\gamma(n_2+n_3)A_2^2}{d^4}\right)d\left[\frac{d^3}{4\gamma n_3^2}\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_1^2\right)-1\right],$$

$$A_4=A_2\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_1^2\right)\left[\frac{d^3}{4\gamma n_3^2}\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_2^2\right)-1\right].$$

The volumetric flow rate is given by $Q=2\int_0^h u\,dy$.

(4.3)
$$Q=\frac{2\gamma n_2}{M_1}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{M_1-(n_2+n_3)\frac{\gamma}{d^4}A_2^2}{2C_2(n_2+n_3)\gamma A_1}\right)\left(\begin{array}{l}-\tanh A_1h+2CothA_1h\\ -2\cos echA_1h\end{array}\right)\right.$$
$$\left.+\left(\frac{M_1-(n_2+n_3)\frac{\gamma}{d^4}A_1^2}{2C_2(n_2+n_3)\gamma A_2}\right)\left(\tanh A_2h-2CothA_2h+2\cos echA_2h\right)-\frac{h}{d^4}\right].$$

## 5. Heat Transfer

Putting the value of $\frac{\partial u}{\partial y}, \frac{\partial v}{\partial y}$ and $v$ in equation (3.5) and integrating twice and using boundary conditions (3.6), we get

(5.1)
$$\theta=\frac{\rho g\alpha d^6}{G_r c\gamma n_2 k_1}\Big[B_1^*\left(\cosh 2A_1h-\cosh 2A_1y\right)+B_2^*\left(\cosh 2A_2h-\cosh 2A_2y\right)$$
$$+B_3^*\left(h^2-y^2\right)+B_4^*\left(\frac{y}{h}\sinh 2A_1h-\sinh 2A_1y\right)+B_5^*\left(\begin{array}{l}\frac{y}{h}\sinh 2A_2h\\ -\sinh 2A_2y\end{array}\right)$$

$$+B_6^*\left(\cosh\left(A_1+A_2\right)h-\cosh\left(A_1+A_2\right)y\right)+B_7^*\left(\cosh\left(A_1-A_2\right)h\right.$$
$$\left.-\cosh\left(A_1-A_2\right)y\right)+B_8^*\left(\sinh\left(A_1+A_2\right)h-\sinh\left(A_1+A_2\right)y\right)$$
$$\left.+B_9^*\left(\sinh\left(A_1-A_2\right)h-\sinh\left(A_1-A_2\right)y\right)\right],$$

where

$$B_1^*=\frac{1}{32C_2^2}\left(\frac{1}{4\cosh^2 A_1h}+\frac{4}{\sinh^2 A_1h}\right)\frac{c^2}{M_1^2}\left(\frac{\partial p}{\partial x}\right)^2\frac{d^2}{\gamma\left(n_2+n_3\right)^2}\left[\left(M_1-\frac{\left(n_2+n_3\right)A_2^2}{d^4}\right)\times\right.$$
$$\left.\left\{\left(M_1-\frac{\left(n_2+n_3\right)A_2^2}{d^4}\right)\frac{\gamma^2}{d^6}n_2^2\frac{\left(4n_2+3n_3\right)}{2}+\frac{\gamma}{d^3}n_2n_3\frac{A_3}{A_1}\right\}+A_3^2\frac{\left(2n_3+A_1^2\right)}{4A_1^2}\right],$$

$$B_2^*=\frac{d^2c^2}{32C_2^2\gamma\left(n_2+n_3\right)^2}\frac{1}{M_1^2}\left(\frac{\partial p}{\partial x}\right)^2\left[\frac{1}{\cosh^2 A_2h}\left(\left(M_1-\left(n_2+n_3\right)\frac{\gamma}{d^4}A_1^2\right)\times\right.\right.$$
$$\left.\left\{\frac{\gamma^2}{2d^6}n_2^2\left(4n_2+3n_3\right)\left(M_1-\frac{\gamma}{d^2}\left(n_2+n_3\right)A_1^2\right)+\frac{\gamma}{d^3}n_2.n_3\frac{A_4}{A_2}\right\}+\frac{A_4^2}{4A_2^2}\left(A_2^2+2n_3\right)\right)$$
$$+\frac{1}{\sinh^2 A_2h}\left(\left(M_1-\left(n_2+n_3\right)\frac{\gamma^2}{d^4}A_1^2\right)\left\{\frac{2\gamma^2n_2}{d^6}\left(4n_2+3n_3\right)\left(M_1-\frac{\gamma}{d^4}\left(n_2+n_3\right)A_1^2\right)\right.\right.$$
$$\left.\left.\left.+\frac{4n_2n_3\gamma A_3}{d^3A_2}\right\}+A_4^2+2n_3\frac{A_3^2}{A_2^2}\right)\right],$$

$$B_3^*=\frac{c^2}{16C_2^2M_1^2}\left(\frac{\partial p}{\partial x}\right)^2\frac{1}{\left(n_2+n_3\right)}\frac{d^2}{\gamma}\left[\frac{1}{\cosh^2 A_1h}\left[\left(M_1-\left(n_2+n_3\right)\frac{\gamma}{d^4}A_2^2\right)\right.\right.$$
$$\left\{-\left(2n_2+n_3\right)n_2^2\frac{\gamma^2}{d^6}\left(M_1-\frac{\gamma}{d^4}\left(n_2+n_3\right)A_2^2\right)-n_3n_2^2\frac{\gamma^2}{2d^6}A_1^2\left(M_1-\frac{\gamma}{d^4}\left(n_2+n_3\right)A_2^2\right)\right.$$
$$\left.\left.\left.-2n_2A_1A_3\right\}-\frac{A_3^2}{4}\frac{d^2}{\gamma}-\frac{A_3^2}{4}\right]+\frac{1}{\sinh^2 A_1h}\left[\left(M_1-\left(n_2+n_3\right)\frac{\gamma}{d^4}A_2^2\right)\right.$$

$$\left\{4\left(2n_2+n_3\right)\frac{\gamma^2}{d^6}n_2^2A_1^2\left(M_1-\left(n_2+n_3\right)\frac{\gamma A_2^2}{d^4}\right)+\frac{\gamma}{d^4}n_2^2A_1^2\left(M_1-\frac{\left(n_2+n_3\right)\gamma A_2^2}{d^4}\right)\right.$$

$$\left.+\frac{4n_2A_1A_3}{d}\right\}+A_3^2\frac{d^2}{\gamma}-A_1^2A_3^2\Bigg]+\frac{1}{\cosh^2 A_2h}\Bigg[\left(M_1-\left(n_2+n_3\right)\frac{\gamma}{d^4}A_1^2\right)\Big\{-\left(2n_2+n_3\right)$$

$$\times\frac{\gamma^2}{d^6}n_2^2\left(M_1-\left(n_2+n_3\right)\frac{\gamma}{d^4}A_1^2\right)A_2^2-\frac{1}{2}\frac{\gamma^2}{d^6}n_2^2n_3A_2^2\left(M_1-\frac{\gamma}{d^4}\left(n_2+n_3\right)A_1^2\right)$$

$$\left.-\frac{\gamma}{d^3}n_2n_3A_2A_4\right\}+\frac{A_2^2A_4^2}{4}\frac{d^2}{\gamma}-\frac{n_3A_4^2}{2}\frac{d^2}{\gamma}\Bigg]+\frac{1}{\sinh^2 A_2h}\Bigg[\Bigg(\left(M_1-\left(n_2+n_3\right)\frac{\gamma}{d^4}A_1^2\right)$$

$$\left\{\frac{4\left(2n_2+n_3\right)A_2^2n_3^2\gamma^2}{d^6}\left(M_1-\left(n_2+n_3\right)\frac{\gamma}{d^4}A_1^2\right)+\frac{n_3^2\gamma A_2^2}{d^4}\left(M_1-\frac{\gamma}{d^4}\left(n_2+n_3\right)A_1^2\right)\right.$$

$$\left.+\frac{2n_3}{d^2}A_2A_3\right\}-A_2^2A_1+\frac{d^2}{Y}A_3\Bigg]\Bigg],$$

$$B_4^*=\frac{-1}{32\cosh A_1h.\sinh A_1h}\frac{1}{M_1^2}\left(\frac{\partial p}{\partial x}\right)^2\frac{c^2}{C_2^2}\frac{d^2}{\gamma\left(n_2+n_3\right)^2}\Bigg[\left(M_1-\frac{\gamma}{d^4}\left(n_2+n_3\right)A_2^2\right)^2$$

$$\left(8n_2+6n_3\right)\frac{\gamma n_2}{d^4}+\left(M-\frac{\gamma}{d^4}\left(n_2+n_3\right)A_2^2\right).2n_2n_3\frac{\gamma}{d^2}\frac{A_3}{A_1}\left(\frac{1}{d}+1\right)+\frac{A_3^2}{A_1^2}\left(A_1^2+n_3\right)\Bigg],$$

$$B_5^*=\frac{-1}{32\sinh A_2h.\cosh A_2h}\frac{1}{M_1^2}\left(\frac{\partial p}{\partial x}\right)^2\frac{c^2}{C_2^2}\frac{d^2}{\gamma\left(n_2+n_3\right)^2}\Bigg[\frac{\gamma^2}{d^6}n_2^2\left(M_1-\frac{\gamma}{d^4}\left(n_2+n_3\right)A_1^2\right)^2$$

$$\left(8n_2+6n_3\right)+\left(M_1-\frac{\gamma}{d^4}\left(n_2+n_3\right)A_1^2\right)\frac{n_2}{2A_2}\left(A_3+A_4\right)+\left(A_1^2+\frac{d^2}{2\gamma}\frac{A_3A_4}{A_2^2}\right)\Bigg],$$

$$B_6^*=\frac{-1}{8\left(A_1+A_2\right)^2}\frac{d^2}{\gamma\left(n_2+n_3\right)^2}\left(\frac{1}{M_1^2}\right)\left(\frac{\partial p}{\partial x}\right)^2\frac{c^2}{C_2^2}\Bigg[\frac{1}{\cosh A_1h.\cosh A_2h}\Bigg[\Bigg(M_1$$

$$\left.-\frac{\gamma}{d^4}\left(n_2+n_3\right)A_2^2\right)\left(M_1-\frac{\gamma}{d^4}\left(n_2+n_3\right)A_1^2\right)\times 2A_1A_2\left(\frac{\gamma^2}{d^6}n_2^2\left(2n_2+n_3\right)+4n_3\right)$$

$$+\frac{A_1A_2A_3A_4}{2}+\frac{\gamma}{d^3}n_2n_3A_2A_3\times\left(M_1-\left(n_2+n_3\right)\frac{\gamma}{d^4}A_1^2\right)+\frac{\gamma}{d^3}n_2n_3A_1A_4$$

$$\times\left(M_1-\frac{(n_2+n_3)\gamma}{d^4}A_2^2\right)+K_1A_3A_4\ \Bigg]+\ \frac{1}{\sinh A_1h.\sinh A_2h}\left[\left(M_1-\frac{(n_2+n_3)\gamma A_2^2}{d^4}\right)\right.$$

$$\times\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_1^2\right)\frac{\gamma}{d^2}n_2^2\left(32A_1A_2n_3\frac{\gamma^2}{d^6}(2n_2+n_3)+\frac{A_1A_2}{d}\right)+2A_1A_2A_3A_4$$

$$+\frac{4n_2n_3\gamma A_2A_3}{d^3}\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_1^2\right)+\frac{8\gamma}{d^3}n_2n_3A_1A_3\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_2^2\right)$$

$$\left.+4n_3A_3^2\ \right],$$

$$B_7^*=\frac{C^2}{8C_2^2}\frac{1}{M_1^2}\left(\frac{\partial p}{\partial x}\right)^2\frac{d^2}{\gamma(n_2+n_3)^2}\frac{1}{(A_1-A_2)^2}\left[\frac{1}{\cosh A_1h\cosh A_2h}\Bigg[\ \left(M_1\right.\right.$$

$$\left.-\frac{\gamma}{d^4}(n_2+n_3)A_2^2\right)\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_1^2\right)\left(\frac{2A_1A_2\gamma^2n_2^2(2n_2+n_3)}{d^6}+4n_3A_1A_2\right)$$

$$-\frac{A_1A_2A_3A_4}{2}+\frac{\gamma}{d^3}n_2n_3A_2A_3\left(M_1-(n_2+n_3)\frac{\gamma}{d^4}A_1^2\right)+\frac{\gamma n_2n_3A_1A_4}{d^3}\left(M_1\right.$$

$$\left.-\frac{(n_2+n_3)\gamma}{d^4}A_2^2\right)-\frac{4n_3}{d}A_3A_4\Bigg]+\frac{1}{\sinh A_1h.\sinh A_2h}\left[\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_2^2\right)\right.$$

$$\left(M_1-\frac{(n_2+n_3)\gamma A_1^2}{d^4}\right)\left(32A_1A_2n_3(2n_2+n_3)\frac{\gamma^3}{d^8}n_2^2-\frac{n_2^2}{d^4}A_1A_2\right)-2A_1A_2A_3A_4$$

$$-4n_2n_3\frac{\gamma}{d^3}A_1A_3\left(M_1-\frac{(n_2+n_3)\gamma A_1^2}{d^4}\right)-\frac{8}{d^3}\gamma n_2n_3A_1A_3\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_2^2\right)$$

$$\left.\left.-4n_2A_3^2\right]\right],$$

$$B_8^*=\frac{1d^2}{8\gamma}\left(\frac{1}{M_1^2}\right)\left(\frac{\partial p}{\partial x}\right)^2\frac{1}{(n_2+n_3)^2C_2^2(A_1-A_2)^2}\left[\frac{4}{\cosh A_1h\sinh A_2h}\right.$$

$$\left[\left(M_1-\frac{\gamma}{d^4}(n_2+n_3)A_2^2\right)\left\{\frac{(2n_2+n_3)}{d^6}A_1A_2\gamma n_2^2\left(M_1-\gamma\frac{(n_2+n_3)}{d^4}A_1^2\right)+\frac{n_3n_2^2\gamma^2A_1A_2}{2d^4}\right.\right.$$

$$\left.\times\left(M_1-\gamma\frac{(n_2+n_3)A_1^2}{d^4}\right)+\frac{\gamma n_2n_3}{2d^3}A_1A_3\right\}+\frac{1}{4}A_1A_2A_3A_4+\frac{1}{2}\frac{\gamma}{d^3}n_2n_3A_2A_3\left(M_1\right.$$

$$-\frac{\gamma}{d^4}(n_2+n_3)A_1^2\Big)+\frac{A_3^2 n_3}{d^2}\Big]+\frac{4}{\sinh A_1 h.\cosh A_2 h}\Big[\Big(M_1-\gamma\frac{(n_2+n_3)}{d^4}A_2^2\Big)$$
$$\Big\{(2n_2+n_3)\frac{\gamma^2}{d^6}A_1A_2\Big(M_1-(n_2+n_3)\frac{\gamma}{d^4}A_1^2\Big)n_2^2+\frac{\gamma}{2d^6}n_3n_2^2A_1A_2\Big(M_1$$
$$-\frac{(n_2+n_3)\gamma}{d^4}A_1^2\Big)+\frac{\gamma}{2d^3}n_2n_3A_1A_4\Big\}+\frac{A_1A_2A_3A_4}{4}+\frac{\gamma}{2d^3}A_1A_2n_2n_3\Big(M_1$$
$$-(n_2+n_3)\frac{\gamma}{d^4}A_1^2\Big)+n_2A_3A_4)\Big] \quad ,$$

$$B_9^*=\frac{1}{8M_1^2}\Big(\frac{\partial p}{\partial x}\Big)^2\frac{d^2c^2}{\gamma(n_2+n_3)^2C_2^2(A_1-A_2)^2}\Big[\frac{4}{\cosh A_1 h\sinh A_1 h}\Big[\Big(M_1$$
$$-(n_2+n_3)\frac{\gamma}{d^4}A_2^2\Big)\Big\{(2n_2+n_3)\frac{\gamma^2}{d^6}A_1A_2n_2^2\Big(M_1-\frac{(n_2+n_3)\gamma}{d^4}A_1^2\Big)\frac{n_3n_2^2A_1A_2\gamma^2}{2d^5}$$
$$\Big(M_1-\frac{(n_2+n_3)\gamma}{d^4}A_1^2\Big)+\frac{A_1A_3n_2n_3\gamma}{2d^3}\Big\}-\frac{1}{4}A_1A_2A_3A_4+\frac{1}{2}\frac{\gamma n_2n_3A_2A_3}{d^3}\Big(M_1$$
$$-\frac{(n_2+n_3)\gamma}{d^4}A_1^2\Big)+\frac{A_3^2n_3}{2}\Big]+\frac{4}{\sinh A_2 h.\cosh A_2 h}\Big[\Big(M_1-(n_2+n_3)\frac{\gamma}{d^4}A_2^2\Big)$$
$$\Big\{(2n_2+n_3)\frac{\gamma}{d^4}A_1A_2\Big(M_1-(n_2+n_3)\frac{\gamma}{d^4}A_1^2\Big)\times\frac{\gamma}{d^2}n_2^2-\frac{\gamma^2}{2d^6}n_2^2n_3A_1A_2\Big(M_1$$
$$-(n_2+n_3)\frac{\gamma}{d^4}A_1^2\Big)-\frac{1}{2d^3}\gamma n_2n_3A_1A_4\Big\}-\frac{A_1A_2A_3A_4}{4}-\frac{n_2n_3\gamma A_2A_3}{2d^3}\Big(M_1$$
$$M_1-(n_2+n_3)\frac{\gamma}{d^4}A_1^2\Big)-\ A_3A_4n_3\Big]\Big] .$$

## 6. Results and Discussion

To give a good insight of the physical problem addressed in this paper, we discuss the effect of the various parameters [i.e. Magnetic parameter ($M_1$) amplitude of the wave ($\phi_1$), free convection parameter ($G_r$) and micro-rotation parameter ($n_3$)] of the problem on the axial velocity, micro-rotational velocity and temperature distributions.

Fig. 1 depicts the variation of axial velocity with $y$ for different values of $\phi_1$ and $M_1$ and fixed value of $G_r$ = 4.0. It is clear from the figure that the velocity increases with $\phi_1$ and decreases with $M_1$. Therefore we can conclude that there is considerable reduction in axial velocity with increasing $M_1$. So the magnetic field can be effectively utilized to deaccelerate the blood flow in accelerated flow problems like stenotic condition of the artery.

Fig. 2 Shows the variation of micro-rotational velocity with $y$ for different value of $\phi_1$ and $M_1$ and fixed value of $G_r = 4.0$. We can conclude from the figure that the microrotational velocity decreases with increasing the magnetic parameter. But it increases with the increase of the amplitude of the wave ($\phi_1$).

Fig. 3 Shows the variation of temperature with y for different values of $\phi_1$, $M_1$ and $n_3$ and fixed value of $G_r = 4.0$. It is clear from that the temperature increases with $\phi_1$ and $M_1$. It is also observed from the figure that the temperature is higher in the case of micropolar fluid in comparison with Newtonian fluid. Application of a transverse magnetic field normal to the flow direction gives rise to a resistive drag like a force acting in a direction opposite to that of flow. This has a tendency to reduce both the fluid velocity and angular velocity and increase in the fluid temperature.

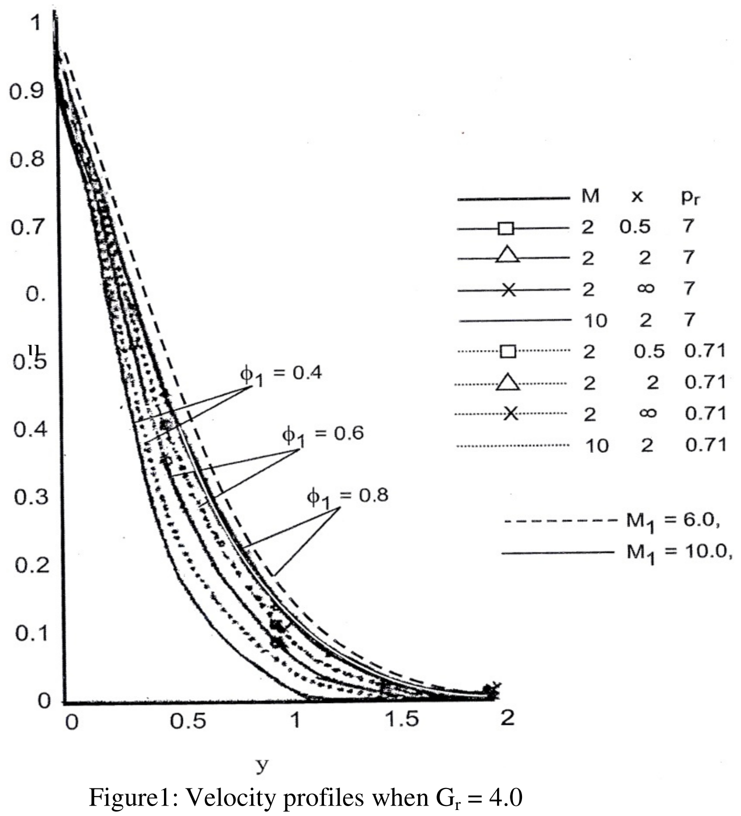


Figure1: Velocity profiles when $G_r = 4.0$

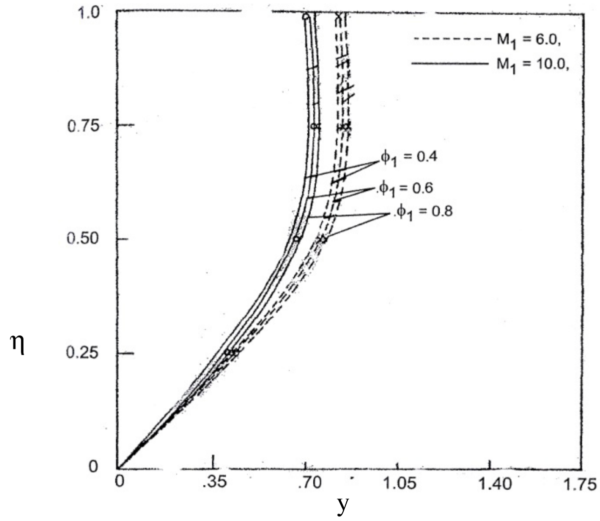


Figure 2: Micro rotation velocity profiles when $G_r = 4.0$

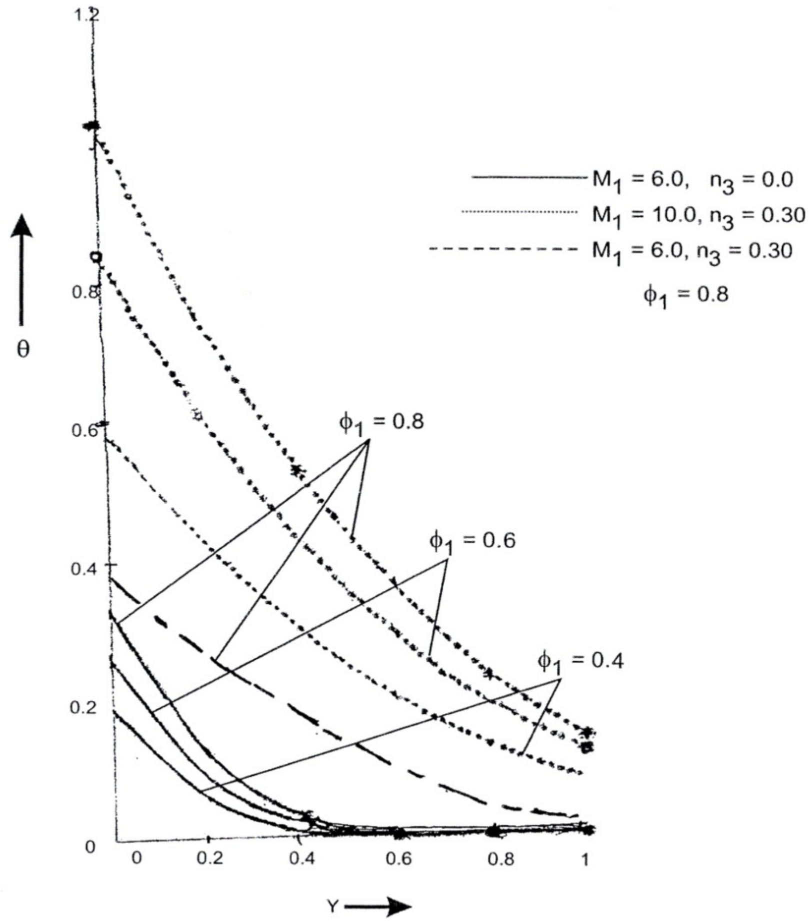


Figure 3: Temperature profiles when $G_r = 4.0$

## References

1. A. C. Eringen, Simple Micropolar Fluid, *Int. J. Engg. Sci.*, **2** (1964) 205-217.

2. A. C. Eringen, Theory of Micro-Polar Fluid, *J. Maths. and Mech.*, **16** (1966) 1.

3. S. K. Laxmana Rao, A General Solution of the Field Equation of Micro-polar Fluids, *Proc. 11th Cong. on Theo. and Applied Mech.*, (1967) 10-21.

4. A. J. Willson, Basic flow of Micro-polar Fluid, *Applied Sci. Res.*, **20** (1969) 338-355.

5. Ranuka and Raj Gopalan, Some flow problems in micropolar fluids, *J. Ind. Inst. Sci.,* **50** (1968) 57-77.

6. Ranuka, Ravindrandan and R. Girja Devi, Wave propagation on a micropolar fluid contained in a viscoelastic membrane, *Proc. Ind. Acad. Sci. India*, **71** (1970) 108-120.

7. A. N. Shapiaro, M. Y. Jaffrin and S. L. Weinberg, Peristaltic pumping with long wave-lengths at low Reynolds number, *J. Fluid Mech.*, **37** (1969) 799-825.

8. Y. C. Fung and C. S. Yih, Peristaltic transport, *J. Applied Mech.*, **35** (1968) 669-675.

9. G. Bohme and R. Friedrich, Peristaltic Flow of viscoelastic fluids, *J. fluid Mech.*, **128** (1983) 109.

10. E. F. Elshehawey, A. M. Sobh and E. M. Elbarbary, Peristaltic motion of a generalised Newtonian Fluid through a Porous Medium, *J. Phys. Soc. Jpn.*, **69** (2000) 401-407.

11. T. Hayat, Y. Waug, K. Hutter, S. Asghar and A. M. Siddiqui,. Peristaltic Transport of an oldroyd-B Fluid in a planner channel, *Math. Prob. in Engg.*, **4** (2004) 347.

12. O. Eytan and D. Elad, Analysis of Intra-Utering fluid motion induced by utering contraction, *Bull. Math. Bio.*, **61** (1999), 221-238.

13. A. R. Rao and M. Mishra, Nonlinear and curvature effects on peristaltic flow of a viscous fluid in an Asymmetric channel, *Acta, Mech.*, **168** (2004) 35-59.

14. M. H. Haroun, Effect of Wall compliance on peristaltic transport of a Newtonian Fluid in an asymmetric channel, *Math. Prob. in Engg.,* Article ID **61475** (2006) 1-19.

15. M. H. Haroun, Effect of Deborah Number and Phase Difference on Peristaltic Transport of Third Order Fluid in an Asymmetric Channel, *Commun. Nonlinear Sci. Num. Simul.*, **12** (2007) 1464.

16. T. Hayat, N. Alvi and N. Ali, Peristaltic mechanism of a Maxwell fluid in an asymmetric channel, *Non-linear Anal. Real World, Appl.*, **9** (2008) 1474-1490.

17. M. V. Subha Reddy, A. Ramachandra Rao and S. Sreenadh, Peristaltic Motion of a Power Law Fluid in an Asymmetric Channel, *Int. J. of Non-linear Mech.*, **42** (2007) 1153-1161.

18. N. Ali and T. Hayat, Peristaltic motion of a Carreau fluid in an asymmetric channel, *Appl. Math. Computer*, **193** (2007) 535-552.

19. A. M. Sobh, Slip Flow in Peristaltic Transport of a Carreau Fluid in an Asymmetric Channel, *Canadian Journal of Physics*, **87** (2009) 1-9.

20. K. H. S. Mekheimer, Peristaltic Flow of Blood under Effect of a Magnetic Field in a non Uniform Channels, *Appl. Math. Compt*., **153** (2004) 763.

21. P. Muthu, B. V. Kumar and P. Chandra, On the Influence of Wall Properties in the Peristaltic Motion of Micropolar Fluid, *ANZIAM J.,* **45** (2003) 245.