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On Generalized Ricci- Recurrent Indefinite Trans-Sasakian Manifolds

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Abstract: In this paper generalized Ricci-recurrent (ε) -trans-Sasakian maniolds are studied. Among others, it is proved that a generalized Ricci-recurrent Cosymplectic manifold is always recurrent. It is also proved that if M is one of the (ε) -Sasakian, $(\varepsilon)-\alpha$-Sasakian, (ε)-Kenmotsu or $(\varepsilon)-\beta$-Kenmotsu manifolds, which are generalized Ricci-recurrent with cyclic Ricci tensor and non- zero $A(\xi)$ every where then they are Einstein and Ricci symmetric manifolds.

Keywords: (ε) -Sasakian, $(\varepsilon)-\alpha$-Sasakian, (ε)-Kenmotsu, $(\varepsilon)-\beta$ -Kenmotsu, (ε)- cosympletic, (ε)-trans-Sasakian, Ricci-recurrent and Einstein manifolds .

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1. Introduction

In 1985, Oubina[1] had introduced trans-Sasakian manifolds which generalizes both α-Sasakian and β-Kenmotsu manifolds. Sasakian, α-Sasakian, Kenmotsu, β-Kenmotsu manifolds are particular cases of trans-Sasakian manifolds of type (α,β). Trans-Sasakian structures of type $(0,0)$, $(\alpha,0)$ and $(\beta,0)$ are called cosympltic[2], α-Sasakian[3] and β-Kenmotsu[3] strutures respectively. A. Bejancu and K. L. Duggal[4] introduced the notion of (ε)-Ssasakian manifolds with indefinite metric. In1998, Xu Xufeng and Chao Xiaoli[5] proved that (ε)-Sasakian manifold is a hypersurface of an indefinite Kaehlerian manifold. Further, R. Kumar, R. Rani and R. Nagaich[6] studied (ε)-Sasakian manifolds. Since Sasakian manifolds with indefinite metric play significant role in Physics[7], so it is necessary to study them and their generalizations. In 2009, U. C. De and Avijit Sarkar[8] introduced and studied the notion of (ε)-Kenmotsu manifolds with indefinite metric. Prasad, Shukla and Tripathi[9] have studied some special type of trans-Sasakian manifolds. In 2010, S. S. Shukla and D. D. Singh[10] introduced and studied the notion of (ε)-trans- Sasakian manifolds or indefinite trans- Sasakian manifolds. In 2010, R. Prasad, V. Srivastava and S. Kishor[11] studied the generalized $N(k)$- contact metric manifold. Recently, in 2013, R. Prasad, S. kishor and V. Srivastava[12] studied generalized Ricci- recurrent (k,μ)- contact metric manifold.

2. Preliminaries

Let M be an $(2n+1)$-dimensional almost contact metric manifold equipped with almost contact metric struture (ϕ,ξ,η,g), where ϕ is (1,1) tensor field, ξ is a vector field, η is 1- form and g is indefinite metric such that

(2.1) $$\phi^2=-I+\eta\otimes\xi,\ \eta(\xi)=1,\ \phi(\xi)=0\,.$$

(2.2) $$g(\xi,\xi)=\varepsilon,\ \eta(X)=\varepsilon g(X,\xi)\,.$$

(2.3) $$g(\phi X,\phi Y)=g(X,Y)-\varepsilon\eta(X)\eta(Y)\,.$$

for all vector fields X,Y on M, where ε is 1 or -1 according as ξ is space like or light like and rank ϕ is $2n$.

An almost contact metric struture (ϕ,ξ,η,g) on M is called a (ε)-trans-Sasakian manifold or indefinite trans- Sasakian manifold if

(2.4) $$(\nabla_X\phi)Y=\alpha\{g(X,Y)\xi-\varepsilon\eta(Y)X\}+\beta\{g(\phi X,Y)\xi-\varepsilon\eta(Y)\phi X\}\,,$$

for any $X,Y\in\Gamma(TM)$, where ∇ is Levi-Civita connection of semi-Riemannian metric g and α and β smooth functions on M.
From equations (2.1), (2.2), (2.3) and (2.4), we have

(2.5) $$\nabla_X\xi=\varepsilon\{-\alpha\phi X+\beta(X-\eta(X)\xi)\}\,.$$

(2.6) $$(\nabla_X\eta)Y=-\alpha g(\phi X,Y)+\beta\{g(X,Y)-\varepsilon\eta(X)\eta(Y)\}\,.$$

(2.7) $$\nabla_\xi\phi=0\,.$$

Further, on such a (ε)-trans Sasakian manifold M of dimension $(2n+1)$ with structure (ϕ,ξ,η,g) the following relations holds,

(2.8) $$R(X,Y)\xi=(\alpha^2-\beta^2)\{\eta(Y)X-\eta(X)Y\}+2\alpha\beta\{\eta(Y)\phi X-\eta(X)\phi Y\} +\varepsilon\{(Y\alpha)\phi X-(X\alpha)\phi Y+(Y\beta)\phi^2X-(X\beta)\phi^2Y\}.$$

(2.9) $$R(\xi,Y)X=(\alpha^2-\beta^2)\{\varepsilon g(X,Y)\xi-\eta(X)Y\}+2\alpha\beta\{\varepsilon g(\phi X,Y)\xi +\eta(X)\phi Y\}+\varepsilon(X\alpha)\phi Y+\varepsilon g(\phi X,Y)(grad\,\alpha) -\varepsilon g(\phi X,\phi Y)(grad\,\beta)+\varepsilon(X\beta)\{Y-\eta(Y)\xi\}.$$

(2.10) $$R(\xi,Y)\xi=\{\alpha^2-\beta^2-\varepsilon(\xi\beta)\}\{-Y+\eta(Y)\xi\}+\{2\alpha\beta+\varepsilon(\xi\alpha)\}\phi Y.$$

(2.11) $$2\alpha\beta+\varepsilon(\xi\alpha)=0.$$

(2.12) $$S(X,\xi)=\left\{2n(\alpha^2-\beta^2)-\varepsilon(\xi\beta)\right\}\eta(X)-\varepsilon(\phi X)\alpha-\varepsilon(2n-1)X\beta.$$

(2.13) $$Q\xi=\varepsilon\left[\left\{2n(\alpha^2-\beta^2)-\varepsilon\xi\beta\right\}\xi+\phi(grad\,\alpha)-(2n-1)(grad\,\beta)\right].$$

If

$$(2n-1)(grad\,\beta)-\varphi(grad\,\alpha)=(2n-1)(\xi\beta)\xi.$$

From equations (2.12) and (2.13), we have

(2.14) $$S(\xi,\xi)=2n(\alpha^2-\beta^2-\varepsilon\xi\beta).$$

(2.15) $$Q\xi=2\varepsilon n\left\{(\alpha^2-\beta^2)-\varepsilon(\xi\beta)\right\}\xi.$$

(2.16) $$S(\phi X,\xi)=\left[(X\alpha)-\eta(X)(\xi\alpha)-(2n-1)(\phi X)\beta\right].$$

If

$$(2n-1)(grad\,\beta)-\varphi(grad\,\alpha)=(2n-1)(\xi\beta)\xi,$$

(2.17) $$\xi\beta=0.$$

From equations (2.1), (2.12), (2.14) and (2.17), we get

(2.18) $$S(X,\xi)=2n(\alpha^2-\beta^2)\eta(X).$$

(2.19) $$S(\xi,\xi)=2n(\alpha^2-\beta^2).$$

(2.20) $$Q\xi=2\varepsilon n(\alpha^2-\beta^2)\xi.$$

3. Generalized Ricci-Recurrent Manifolds

Definition 3.1: A Riemannian manifold (M, g) is said to be recurrent if $(\nabla_X R)(Y,Z)W = A(X)R(Y,Z)W$, $\forall\ X, Y, Z, W$ tangential to *M*, where *A* is 1-form on *M* such that $A(X) = g(X, A^*)$ and A^* is called associated vector field to the 1- form *A* and ∇ is the Levi- Civita connection on *M*.

If 1- form *A* vanishes identically on *M*, the recurrent manifold reduces to locally symmetric manifold due to Cartan, i.e. $\nabla R = 0$.

Definition 3.2: A Riemannian manifold (M, g) is said to be Ricci-recurrent if it satisfy the relation

$$(\nabla_X S)(Y,Z) = A(X)S(Y,Z),$$

$\forall\ X, Y, Z, W$ tangential to *M*, where ∇ is the Levi- Civita connection on *M* and *A* is 1- form on *M*. If 1- form *A* vanishes identically on *M*, the Ricci-recurrent manifold becomes a Ricci- symmetric manifold, i.e. $\nabla S = 0$.

It is well known that an Einstein manifold is a Ricci-symmetric manifold.

A non-flat Reimannian manifold *M* is called a generalized Ricci-recurrent manifold[9], if its Ricci tensor *S*, satisfies the condition

$$(3.1) \qquad (\nabla_X S)(Y,Z) = A(X)S(Y,Z) + B(X)\,g(Y,Z),$$

where ∇ is the Levi- Civita connection of the Riemannian metric *g* and *A, B* are 1- forms on *M*. In particular, if 1- form *B* vanishes identically, then *M* the reduces to well known Ricci-recurrent manifold[9].

A non-flat semi-Reimannian manifold *M* is called a generalized Ricci-recurrent semi-Reimannian manifold, if its Ricci tensor *S*, satisfies the condition

$$(3.2) \qquad (\nabla_X S)(Y,Z) = A(X)S(Y,Z) + B(X)\,g(Y,Z),$$

where ∇ is the Levi- Civita connection of the semi-Riemannian metric *g* and *A, B* are 1- forms on *M*. In particular, if 1- form *B* vanishes identically, then *M* the reduces to Ricci-recurrent semi- Reimannian manifold.

Theorem 3.1: *Let M be an $(2n+1)$ dimensional generalized Ricci-recurrent on (ε)-trans-Sasakian manifold. Then, 1- forms A and B are related as*

$$B(X)=-\varepsilon\left[2n(\alpha^2-\beta^2-\varepsilon\xi\beta)A(X)-X\left\{2n(\alpha^2-\beta^2-\varepsilon\xi\beta)\right\}\right]$$
$$+2\alpha(X\alpha)-2\alpha(\eta(X)(\xi\alpha))-2\alpha(2n-1)(\phi X)\beta$$
$$+2\beta\varepsilon(\phi X)\alpha+2\beta\varepsilon(2n-1)(X\beta)-2\beta\varepsilon(2n-1)(\xi\beta)\eta(X).$$

In Particular ,we get

$$B(\xi)=-\varepsilon\left[2n(\alpha^2-\beta^2-\varepsilon\xi\beta)A(\xi)-\xi\left\{2n(\alpha^2-\beta^2-\varepsilon\xi\beta)\right\}\right].$$

Proof: we have

(3.3) $$(\nabla_X S)(Y,Z)=X\,S(Y,Z)-S(\nabla_X Y,Z)-S(Y,\nabla_X Z),$$

from (3.1) and (3.3), we get

(3.4) $$A(X)S(Y,Z)+B(X)\,g(Y,Z)=X\,S(Y,Z)-S(\nabla_X Y,Z)-S(Y,\nabla_X Z).$$

Putting $Y=Z=\xi$ in above equation, we obtain

(3.5) $$A(X)S(\xi,\xi)+B(X)\,g(\xi,\xi)=X\,S(\xi,\xi)-2S(\nabla_X\xi,\xi),$$

from equations (2.2), (2.5), (2.14) and (3.5), we get

(3.6) $$2n(\alpha^2-\beta^2-\varepsilon\xi\beta)A(X)+\varepsilon B(X)=X\left\{2n(\alpha^2-\beta^2-\varepsilon\xi\beta)\right\}$$
$$-2\left[S\left\{\varepsilon(-\alpha\phi X+\beta(X-\eta(X)\xi)),\xi\right\}\right],$$

(3.7) $$2n(\alpha^2-\beta^2-\varepsilon\xi\beta)A(X)+\varepsilon B(X)=X\left\{2n(\alpha^2-\beta^2-\varepsilon\xi\beta)\right\}$$
$$+2\varepsilon\alpha S(\phi X,\xi)-2\varepsilon\beta S(X,\xi)+2\varepsilon\beta\eta(X)S(\xi,\xi),$$

from (2.14), (2.16) and (3.7), we get

(3.8) $$2n(\alpha^2-\beta^2-\varepsilon\xi\beta)A(X)+\varepsilon B(X)=X\left\{2n(\alpha^2-\beta^2-\varepsilon\xi\beta)\right\}$$
$$+2\varepsilon\alpha\left[(X\alpha)-\eta(X)(\xi\alpha)-(2n-1)(\phi X)\beta\right]$$

$$-2\varepsilon\beta\Big[\big\{2n(\alpha^2-\beta^2)-\varepsilon(\xi\beta)\big\}\eta(X)-\varepsilon(\phi X)\alpha-\varepsilon(2n-1)X\beta\Big]$$
$$+2\varepsilon\beta\eta(X)\{2n(\alpha^2-\beta^2-\varepsilon\xi\beta)\}.$$

(3.9) $$B(X)=-\varepsilon\Big[2n(\alpha^2-\beta^2-\varepsilon\xi\beta)A(X)-X\big\{2n(\alpha^2-\beta^2-\varepsilon\xi\beta)\big\}\Big]$$
$$+2\alpha(X\alpha)-2\alpha(\eta(X)(\xi\alpha))-2\alpha(2n-1)(\phi X)\beta$$
$$+2\beta\varepsilon(\phi X)\alpha+2\beta\varepsilon(2n-1)(X\beta)-2\varepsilon\beta(2n-1)(\xi\beta)\eta(X).$$

Putting $X=\xi$ in equation (3.9), we obtain

(3.10) $$B(\xi)=-\varepsilon\Big[2n(\alpha^2-\beta^2-\varepsilon\xi\beta)A(\xi)-\xi\big\{2n(\alpha^2-\beta^2-\varepsilon\xi\beta)\big\}\Big].$$

Let A^* and B^* be the associated vector fields of A and B respectively, so

$$g(X,A^*)=A(X) \qquad \text{and} \qquad g(X,B^*)=B(X)$$

If

$$\varphi(grad\,\alpha)=(2n-1)(grad\,\beta),$$

from equations (2.2), (2.5), (2.18), (2.19) and (3.5), we get

(3.11) $$2n(\alpha^2-\beta^2)A(X)+B(X)\varepsilon=X\{2n(\alpha^2-\beta^2)\}.$$

(3.12) $$B(X)=\varepsilon\Big[X\{2n(\alpha^2-\beta^2)\}-2n(\alpha^2-\beta^2)A(X)\Big].$$

Putting $X=\xi$ in equation (3.12), we get

(3.13) $$B(\xi)=\varepsilon\Big[\xi\{2n(\alpha^2-\beta^2)\}-2n(\alpha^2-\beta^2)A(\xi)\Big].$$

For (ε)-Sasakian manifold, the equation (3.9) becomes

(3.14) $$B(X)=-2n\varepsilon A(X),$$

which implies

(3.15) $$B^*=-2n\varepsilon A^*.$$

For $(\varepsilon)-\alpha$-Sasakian manifold, the equation (3.9) becomes

(3.16) $$B(X) = -2n\varepsilon\alpha^2 A(X),$$

which implies

$$B^* = -2n\varepsilon\alpha^2 A^*.$$

Hence we have the following Lemma:

Lemma 3.1: *In a Ricci- recurrent* (ε) *-Sasakian or* $(\varepsilon)-\alpha$ *-Sasakian manifold,* A^* *and* B^* *have same or opposite directions if* ε *is -1 and 1 respectively.*

For (ε) - Kenmotsu manifold, the equation (3.9) becomes

(3.17) $$B(X) = 2n\varepsilon A(X),$$

which implies

(3.18) $$B^* = 2n\varepsilon A^*.$$

For $(\varepsilon)-\beta$ - Kenmotsu manifold, the equation (3.9) becomes

(3.19) $$B(X) = 2n\varepsilon\beta^2 A(X),$$

which implies

(3.20) $$B^* = 2n\varepsilon\beta^2 A^*.$$

Hence we have the following Lemma:

Lemma 3.2: *In a Ricci- recurrent* (ε) *- Kenmotsu or* $(\varepsilon)-\beta$ *- Kenmotsu manifold,* A^* *and* B^* *have same or opposite directions if* ε *is -1 and 1 respectively.*

For (ε) -Cosymplectic manifold, the equation (3.9) reduces to

$$B(X) = 0, \qquad \forall X.$$

Hence for Cosymplectic manifold generalized Ricci recurrent manifold reduces to recurrent manifold.

4. Generalised Ricci- Recurrent Indefinite Trans-Sasakian Manifolds with Cyclic Ricci Tensor

A Riemannian manifold is said to admit cyclic Ricci tensor, if a Riemannian manifold is Ricci recurrent

(4.1) $$(\nabla_X S)(Y,Z)+(\nabla_Y S)(Z,X)+(\nabla_Z S)(X,Y)=0.$$

Theorem 4.1: *In a* $(2n+1)$ *-dimensional generalized Ricci-recurrent on* (ε) *- trans- Sasakian manifolds with cyclic Ricci tensor, the Ricci tensor satisfies,*

$$\begin{aligned}A(\xi)S(X,Y)=2n\varepsilon\Big[(\alpha^2-\beta^2-\varepsilon\xi\beta)A(\xi)-\xi((\alpha^2-\beta^2-\varepsilon\xi\beta))\Big]g(X,Y)\\ -\varepsilon(2n-1)(\xi\beta)\{A(X)\eta(Y)+A(Y)\eta(X)\}+\varepsilon\{A(X)(\phi Y)\alpha\\ +A(Y)(\phi X)\alpha\}+\varepsilon(2n-1)\{A(X)Y\beta+A(Y)X\beta\}\\ -2n\{\eta(X)Y(\alpha^2-\beta^2-\varepsilon\xi\beta)+\eta(Y)X(\alpha^2-\beta^2-\varepsilon\xi\beta)\}\\ -2\alpha\varepsilon\{(X\alpha)\eta(Y)+(Y\alpha)\eta(X)\}+4\varepsilon\alpha(\xi\alpha)\eta(Y)\eta(X)\\ +2\alpha\varepsilon(2n-1)\{((\phi X)\beta)\eta(Y)+((\phi Y)\beta)\eta(X)\}\\ -2\beta\{((\phi X)\alpha)\eta(Y)+(\phi Y)\alpha)\eta(X)\}-2\beta(2n-1)\{(X\beta)\eta(Y)\\ +(Y\beta)\eta(X)\}-4\beta(2n-1)(\xi\beta\eta(X)\eta(Y).\end{aligned}$$

Proof: Suppose that M is a generalized Ricci symmetric manifold admitting cyclic Ricci tensor. Then in view of (3.1) and (4.1), we get

(4.2) $$\begin{aligned}A(X)S(Y,Z)+A(Y)S(Z,X)+A(Z)S(X,Y)+B(X)g(Y,Z)\\ +B(Y)\,g(Z,X)+B(Z)\,g(X,Y)=0.\end{aligned}$$

If M is (ε)-trans-Sasakian manifold, putting $Z=\xi$ in equation (4.2), we get

(4.3) $$\begin{aligned}A(\xi)S(X,Y)=-\big[A(X)S(Y,\xi)+A(Y)S(\xi,X)+B(X)g(Y,\xi)\\ +B(Y)\,g(\xi,X)+B(\xi)\,g(X,Y)\big],\end{aligned}$$

from equations (2.1), (2.2), (2.12), (3.8), (3.9) and (4.3), we get

(4.4) $$A(\xi)S(X,Y)=-A(X)\Big[\{2n(\alpha^2-\beta^2)-\varepsilon(\xi\beta)\}\eta(Y)-\varepsilon(\phi Y)\alpha$$

$$-\varepsilon(2n-1)(Y\beta)\big]-A(Y)\big[\{2n(\alpha^2-\beta^2)-\varepsilon(\xi\beta)\}\eta(X)-\varepsilon(\phi X)\alpha$$
$$-\varepsilon(2n-1)(X\beta)\big]+\big[\{2n(\alpha^2-\beta^2)-\varepsilon(\xi\beta)\}A(X)-X\{2n(\alpha^2-\beta^2)$$
$$-\varepsilon(\xi\beta)\}-2\alpha\varepsilon(X\alpha)+2\varepsilon\alpha(\eta(X)(\xi\alpha))+2\alpha\varepsilon(2n-1)(\phi X)\beta$$
$$-2\beta(\phi X)\alpha+2\varepsilon\beta(2n-1)(\xi\beta)\eta(X)\ \ +2\beta(2n-1)(X\beta)\big]\eta(Y)$$
$$\big[\{2n(\alpha^2-\beta^2)-\varepsilon(\xi\beta)\}A(Y)-Y\{2n(\alpha^2-\beta^2)-\varepsilon(\xi\beta)\}$$
$$-2\alpha\varepsilon(Y\alpha)+2\varepsilon\alpha(\eta(Y)(\xi\alpha))+2\alpha\varepsilon(2n-1)(\phi Y)\beta-2\beta(\phi Y)\alpha$$
$$+2\varepsilon\beta(2n-1)(\xi\beta)\eta(Y)+2\beta(2n-1)(Y\beta)\big]\eta(X)+\varepsilon\big[\{2n(\alpha^2-\beta^2)$$
$$-\varepsilon(\xi\beta)\}A(\xi)-\xi\{\{2n(\alpha^2-\beta^2)-\varepsilon(\xi\beta)\}\big]g(X,Y).$$

$$A(\xi)S(X,Y)=2n\varepsilon\{(\alpha^2-\beta^2-\varepsilon\xi\beta)A(\xi)-\varepsilon(\alpha^2-\beta^2-\varepsilon\xi\beta)\}g(X,Y) \tag{4.5}$$
$$-\varepsilon(2n-1)(\xi\beta)\{A(X)\eta(Y)+A(Y)\eta(X)\}+\varepsilon\{A(X)(\phi Y)\alpha$$
$$+A(Y)(\phi X)\alpha\}+\varepsilon(2n-1)\{A(X)Y\beta+A(Y)X\beta\}-2n\{\eta(X)Y(\alpha^2$$
$$-\beta^2-\varepsilon\xi\beta)+\eta(Y)X(\alpha^2-\beta^2-\varepsilon\xi\beta)\}-2\alpha\varepsilon\{(X\alpha)\eta(Y)$$
$$+(Y\alpha)\eta(X)\}+4\varepsilon\alpha(\xi\alpha)\eta(X)\eta(Y)+2\alpha\varepsilon(2n-1)\{((\phi X)\beta)\eta(Y)$$
$$+((\phi Y)\beta)\eta(X)\}-2\beta\{((\phi X)\alpha)\eta(Y)+((\phi Y)\alpha)\eta(X)\}$$
$$-2\beta(2n-1)\{(X\beta)\eta(Y)+(Y\beta)\eta(X)\}-4\beta(2n-1)(\xi\beta)\eta(X)\eta(Y).$$

Hence proved.

If

$$\varphi(grad\,\alpha)=(2n-1)(grad\,\beta),$$

from equations (2.1), (2.2), (2.18), (3.12), (3.13) and (4.3), we get

$$A(\xi)S(X,Y)=-A(X)\{2n(\alpha^2-\beta^2)\eta(Y)\}-A(Y)\{2n(\alpha^2-\beta^2)\eta(X)\} \tag{4.6}$$
$$-X\{2n(\alpha^2-\beta^2)\eta(Y)\}-\{2n(\alpha^2-\beta^2)\}A(X)\eta(Y)$$
$$-Y\{2n(\alpha^2-\beta^2)\eta(X)\}-\{2n(\alpha^2-\beta^2)\}A(Y)\eta(X)$$
$$-\varepsilon\big[\xi\{2n(\alpha^2-\beta^2)\}-\{2n(\alpha^2-\beta^2)\}A(\xi)\big]g(X,Y).$$

$$A(\xi)S(X,Y)=\varepsilon\big[\{2n(\alpha^2-\beta^2)\}A(\xi)-\xi\{2n(\alpha^2-\beta^2)\}\big]g(X,Y). \tag{4.7}$$
$$-X\{2n(\alpha^2-\beta^2)\}\eta(Y)+Y\{2n(\alpha^2-\beta^2)\}\eta(X)$$

Corollary 4.2: *For a* $(2n+1)$ *-dimensional generalized Ricci-recurrent manifold M with cyclic Ricci tensor, we have the following statements:*

1. *If* $\alpha \neq 0$, $\beta = 0$, M *is a* $(\varepsilon)-\alpha$ *- Sasakian manifold, then from equation (4.5)*

$$A(\xi)S(X,Y) = 2n\varepsilon\alpha^2 A(\xi)g(X,Y),$$

$$S(X,Y) = 2n\varepsilon\alpha^2 g(X,Y), \quad if \quad A(\xi) \neq 0.$$

2. *If* $\alpha = 1$, $\beta = 0$, M *is a* (ε) *-Sasakian manifold, then from equation (4.5)*

$$A(\xi)S(X,Y) = 2n\varepsilon A(\xi)g(X,Y),$$

$$S(X,Y) = 2n\varepsilon g(X,Y), \quad if \quad A(\xi) \neq 0.$$

3. *If* $\alpha = 0$, $\beta \neq 0$, M *is a* $(\varepsilon)-\beta$ *- Kenmotsu manifold, then from equation (4.5)*

$$A(\xi)S(X,Y) = -2n\varepsilon\beta^2 A(\xi)g(X,Y),$$

$$S(X,Y) = -2n\varepsilon\beta^2 g(X,Y), \quad if \quad A(\xi) \neq 0.$$

4. *If* $\alpha = 0$, $\beta = 1$, M *is a* (ε) *-Kenmotsu manifold, then from equation (4.5)*

$$A(\xi)S(X,Y) = -2n\varepsilon A(\xi)g(X,Y),$$

$$S(X,Y) = -2n\varepsilon g(X,Y), \quad if \quad A(\xi) \neq 0.$$

Theorem 4.3: *Let M be a generalized Ricci- recurrent manifold with cyclic Ricci tensor. If M is one of* (ε) *-Sasakian,* $(\varepsilon)-\alpha$ *- Sasakian,* (ε) *- Kenmotsu and* $(\varepsilon)-\beta$ *- Kenmotsu manifolds with non-zero* $A(\xi)$ *everywhere, then M is Einstein and Ricci symmetric.*

If $\alpha = 0$, $\beta = 0$, M is a (ε) -cosymplectic manifold, then from equation (4.5)

$$A(\xi)S(X,Y) = 0,$$

$$S(X,Y) = 0, \quad if \quad A(\xi) \neq 0.$$

Hence we have a lemma:

Lemma 4.4: . *If M be a generalized Ricci- recurrent* (ε)*-cosymplectic manifold with cyclic Ricci tensor and non-zero* $A(\xi)$ *everywhere, then M is Ricci flat.*

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